

NIST POLYTECHNIC



SUBJECT- HYDRAULIC MACHINES & INDUSTRIAL FLUID POWER

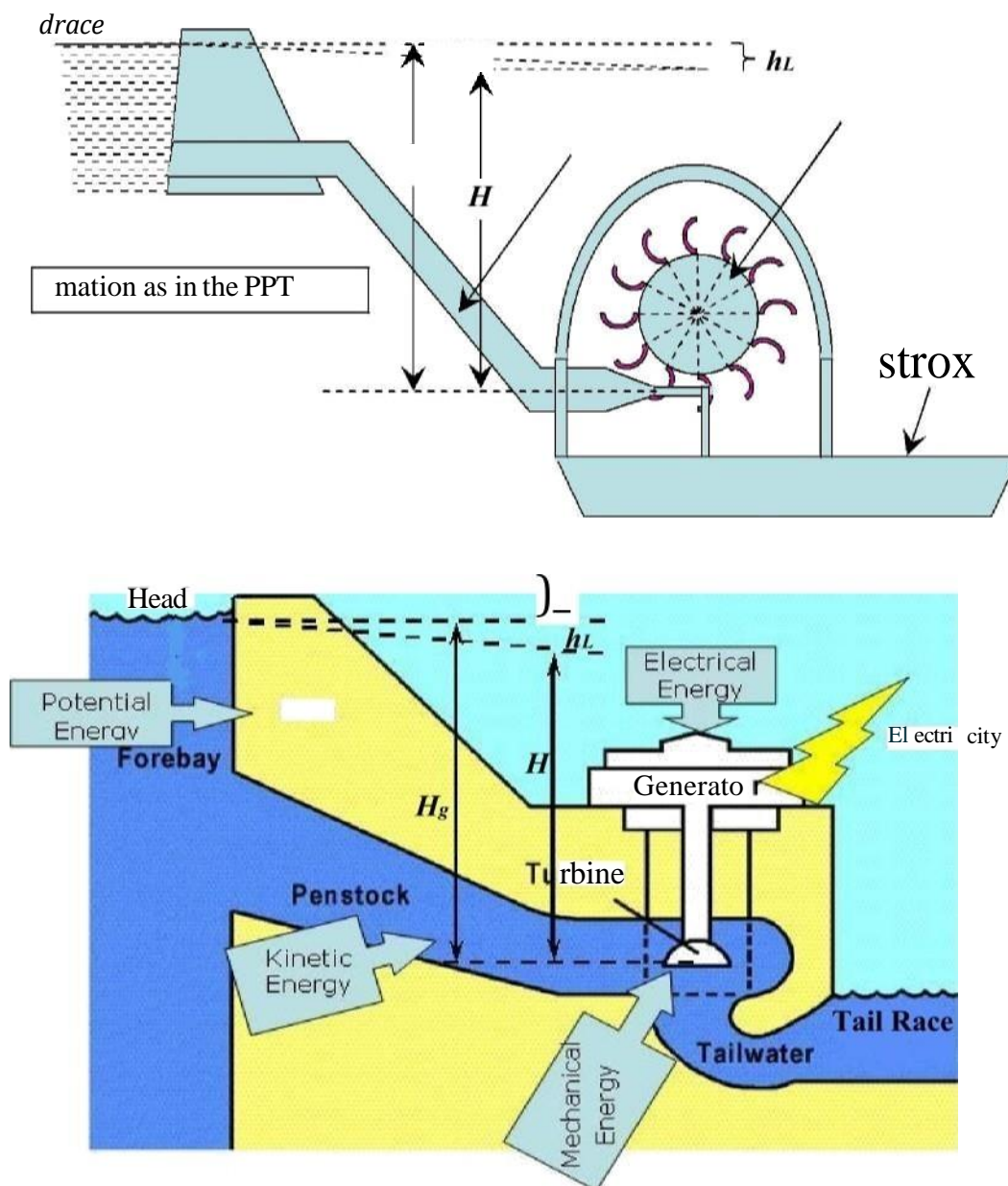
HYDRAULIC TURBINES

Introduction:

The device which converts Hydraulic energy into mechanical energy or vice versa is known as *Hydraulic Machines* . The hydraulic machines which convert hydraulic energy into mechanical energy are known as

Turbines and that convert mechanical energy into hydraulic energy is known as *Pumps* .

Fig . shows a general layout of a hydroelectric plant .



It consists of the following:

- 1 . A *Dam* constructed across a river or a channel to store water. The reservoir is also

known as **Headrace**

- 2 Pipes of large diameter called **Penstocks** which carry water under pressure from storage reservoir to the turbines . These pipes are usually made of steel or reinforced concrete.
- 3 . **Turbines** having different types of vanes or buckets or blades mounted on a wheel called runner.
- 4 . **Tailrace** which is a channel carrying water away from the turbine after the water has worked on the turbines . The water surface in the tailrace is also referred to as tailrace

important Terms:

Gross Head (H_g) • It is the vertical difference between headrace and tailrace.

Net Head or effective head (H): Net head or effective head is the actual head available at the inlet of the turbine to work on the turbine

$$H = H_g - h$$

Where h is the total head loss during the transit of water from the headrace to tailrace which is mainly head loss due to friction, and is given by

$$h = \frac{4 f L U^2}{2 g d}$$

Where f is the coefficient of friction of penstock depending on the type of material of penstock

L is the total length of penstock

U is the mean flow velocity of water through the penstock

d is the diameter of penstock and

g is the acceleration due to gravity

TYPES OF EFFICIENCIES

Depending on the considerations of input and output, the efficiencies can be classified as

- (i) Hydraulic Efficiency
- (ii) Mechanical Efficiency
- (iii) Overall efficiency

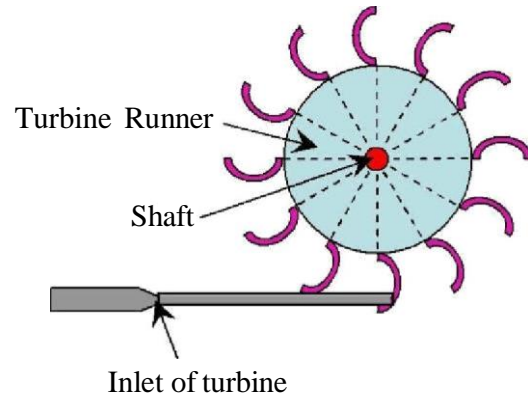
(i) Hydraulic Efficiency: (η_h)

It is the ratio of the power developed by the runner of a

turbine to the power supplied at the inlet of a turbine. Since the power supplied is hydraulic, and the probable loss is between the striking jet and vane it is rightly called hydraulic efficiency.

If R.P. is the Runner Power and W.P. is the Water Power

$$\eta_h = \frac{R.P.}{W.P.} \quad (01)$$



1. Mechanical Efficiency: (η_m)

It is the ratio of the power available at the shaft to the power developed by the runner of a turbine. This depends on the slips and other mechanical problems that will create a loss of energy between the runner in the annular area between the nozzle and spear, the amount of water reduces as the spear is pushed forward and vice-versa.

and shaft which is purely mechanical and hence mechanical efficiency.

If S.P. is the Shaft Power

$$\eta_m = \frac{S.P.}{R.P.} \quad (02)$$

(iii) Overall Efficiency: (η_o)

It is the ratio of the power available at the shaft to the power supplied at the inlet of a turbine. As this covers overall problems of losses in energy, it is known as overall efficiency. This depends on both the hydraulic losses and the slips and other mechanical problems

that will create a loss of energy between the jet power supplied and the power generated at the shaft available for coupling of the generator.

–S.P.

(03)

From Eqs 1,2 and 3, we have

$$2 \text{ th } X \text{ km}$$

Classification of Turbines

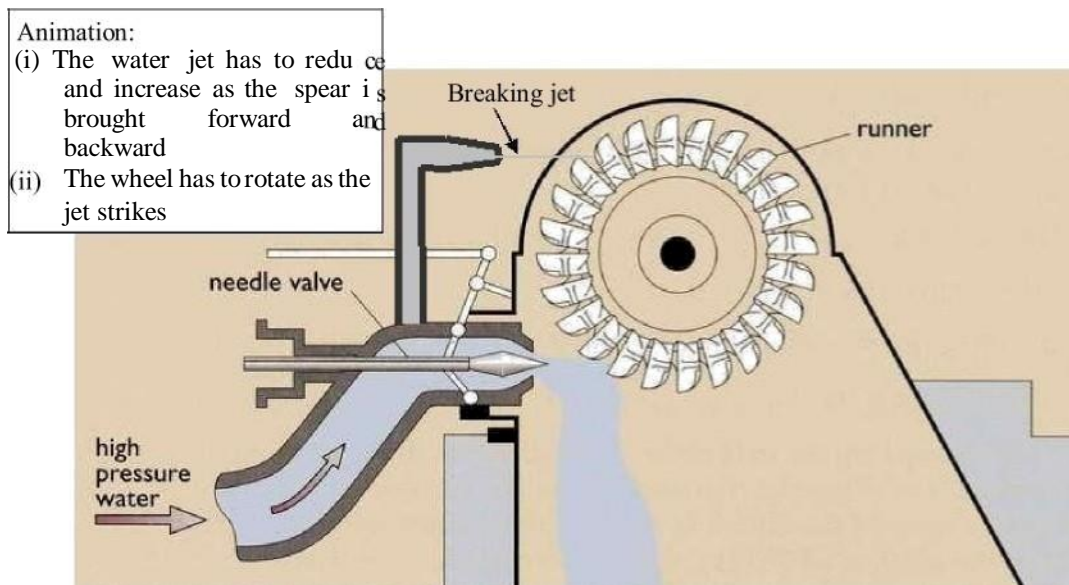
The hydraulic turbines can be classified based on type of energy at the inlet, direction of flow through the vanes, head available at the inlet, discharge through the vanes and specific speed. They can be arranged as per the following table:

Turbine		Type of energy	Head	Discharge	Direction of flow	Specific Speed
Name	Type					
Pelton Wheel	Impulse	Kinetic	High Head > 250m to 1000m	Low	Tangential to runner	Low <35 Single jet 3 - 60 Multiple jet
Francis Turbine	Reaction Turbine	Kinetic * Pressure	Medium 60 m to 150 m	Medium	Radial flow	Medium 60 to 300
Kaplan Turbine			Low 30 m	High	Axial Flow	

As can be seen from the above table, any specific type can be explained by suitable construction of sentences by selecting the other items in the table along the row.

PELTON WHEEL OR TURBINE

Pelton wheel, named after an eminent engineer, is an impulse turbine wherein the flow is tangential to the runner and the available energy at the entrance is completely kinetic energy. Further, it is preferred at a very high head and low discharges with low specific speeds. The pressure available at the inlet and the outlet is atmospheric.



The main components of a Pelton turbine are:

- (i) *Nozzle and flow regulating arrangement.*

Water is brought to the hydroelectric plant site through large penstocks at the end of which there will be a nozzle, which converts the pressure energy completely into kinetic energy. This will convert the liquid flow into a high-speed jet, which strikes the buckets or vanes mounted on the runner, which in-turn rotates the runner of the turbine. The amount of water striking the vanes is controlled by the forward and backward motion of the spear. As the water is flowing in the annular area between the annular area between the

Penstock

Nozzle

jet,

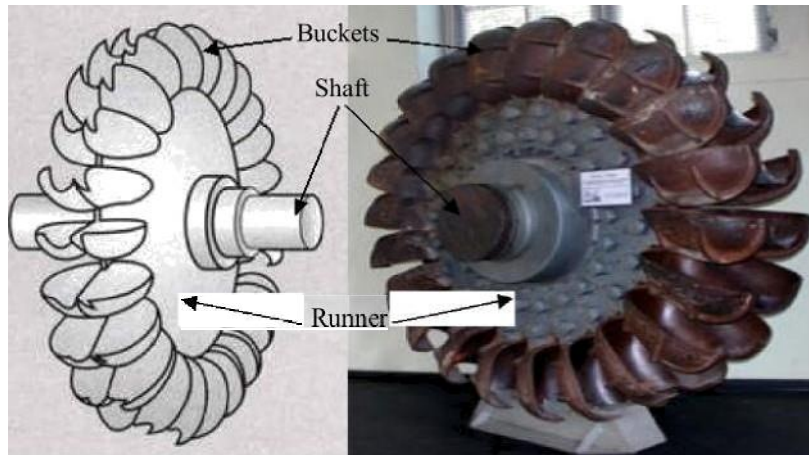
Wheel

Spear

nozzle opening and the spear, the flow gets reduced as the spear moves forward and vice - versa.

(ii) *Runner with buckets.*

Runner is a circular disk mounted on a shaft on the periphery of



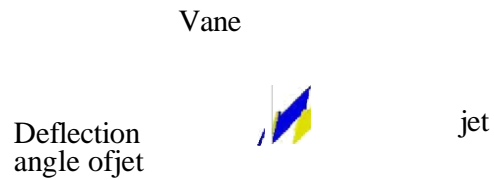
which a number of buckets are fixed equally spaced as shown in Fig. The buckets are made of cast-iron, cast-steel, bronze or stainless steel depending upon the head at the inlet of the turbine. The water jet strikes the bucket on the splitter of the bucket and gets deflected through an angle of $160 - 170^\circ$.

(iii) *Casing.*

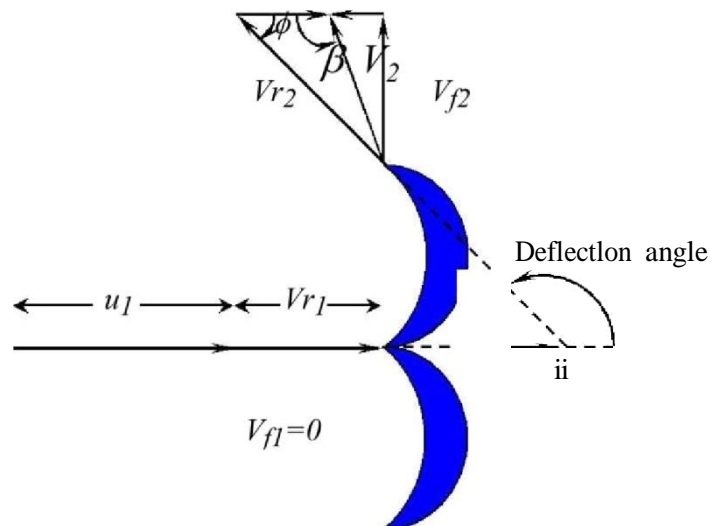
It is made of cast-iron or fabricated steel plates. The main function of the casing is to prevent splashing of water and to discharge the water into tailrace.

(iv) *Breaking jet.*

Even after the amount of water striking the buckets is completely stopped, the runner goes on rotating for a very long time due to inertia. To stop the runner in a short time, a small nozzle is provided which directs the jet of water on the back of bucket with which the rotation of the runner is reversed. This jet is called as breaking jet.



3 D Picture of a jet striking the splitter and getting split in to two parts and deviating.



Velocity triangles for the jet striking the bucket

From the impulse-momentum theorem, the force with which the jet strikes the bucket along the direction of vane is given by

— rate of change of momentum of the jet along the direction of vane motion

= (Mass of water / second) x change in velocity along the x direction

$$= \rho a V_1 [V_{w1} - (-V_{w2})]$$

$$= \rho a V_1 [V_{w1} + V_{w2}]$$

Work done per second by the jet on the vane is given by the product of Force exerted on the vane and the distance moved by the vane in one second

$$W.D./S = F_x \times u$$

$$= \rho a V_1 [V_{w1} + V_{w2}] u$$

Input to the jet per second = Kinetic energy of the jet per second

$$= \frac{1}{2} \rho a V_1^3$$

$$\text{Efficiency of the jet} = \frac{\text{Output / sec ond}}{\text{Input / sec ond}} = \frac{\text{Workdone / sec ond}}{\text{Input / sec ond}}$$

$$= \frac{\rho a V_1 [V_{w1} + V_{w2}] u}{\frac{1}{2} \rho a V_1^3}$$

$$\eta = \frac{2 u [V_{w1} + V_{w2}]}{V_1^2}$$

From inlet velocity triangle, $V_{w1} = V_1$

Assuming no shock and ignoring frictional losses through the vane, we have $V_{r1} = V_{r2}$
 $= (V_1 - u)$

In case of Pelton wheel, the inlet and outlet are located at the same radial distance from the centre of runner and hence $u_1 = u_2 = u$

From outlet velocity triangle, we have $V_{w2} = V_{r2} \cos \phi - u_2$
 $= (V_1 - u) \cos \phi - u$

$$F_x = \rho a V_1 [V_1 + (V_1 - u) \cos \phi - u]$$

$$F_x = \rho a V_1 (V_1 - u) [1 + \cos \phi]$$

Substituting these values in the above equation for efficiency, we have

$$\eta = \frac{2u [V_1 + (V_1 - u) \cos \phi - u]}{V_1^2}$$

$$\eta = \frac{2u}{V_1^2} [(V_1 - u) + (V_1 - u) \cos \phi]$$

$$P' = \frac{2u}{2} (1 - \cos \phi) [1 + \cos \phi]$$

The above equation gives the efficiency of the jet striking the vane in case of Pelton wheel.

To obtain the maximum efficiency for a given jet velocity and vane angle, from maxima-minima, we have

$$\frac{d\eta}{du} = 0$$

$$\Rightarrow \frac{d\eta}{du} = 0$$

$$\Rightarrow \frac{d\eta}{du} = \frac{2}{2u} [1 + \cos \phi] \frac{d}{du} (uV_1 - u^2) = 0$$

or

i.e. When the bucket speed is maintained at half the velocity of the jet, the efficiency of a Pelton wheel will be maximum. Substituting we get,

$$\eta_{\max} = \left(\frac{2u}{2u}\right)^2 (2u - u) [1 + \cos \phi]$$

$$\eta_{\max} = \frac{1}{2} [1 + \cos \phi]$$

From the above it can be seen that more the value of $\cos \phi$, more will be the efficiency.

For maximum efficiency, the value of $\cos \phi$ should be 1 and the value of ϕ should be 0°.

This condition makes the jet to completely deviate by 180° and this, forces the jet striking the bucket to strike the successive bucket on the back of it acting like a breaking jet. Hence to avoid this situation, at least a small angle of $\phi = 5^\circ$ should be provided.

- 6 a. i) Sketch the layout of a PELTON wheel turbine showing the details of nozzle, buckets and wheel when the turbine axis is horizontal (04) ii) Obtain an expression for maximum efficiency of an impulse turbine. (06)

July 06

- 6 (a) With a neat sketch explain the layout of a hydro-electric plant (06)
(b) With a neat sketch explain the parts of an Impulse turbine. (06)

Jan 06

- 6 (a) What is specific speed of turbine and state its significance. (04)
(b) Draw a neat sketch of a hydroelectric plant and mention the function of each component (08)

Jan 05

- 6 (a) Classify the turbines based on head, specific speed and hydraulic actions. Give examples for each. (06)
(b) What is meant by governing of turbines? Explain with a neat sketch the governing of an impulse turbine (06)

July 04

- 5 (a) Explain the classification of turbines (08)

The head at the base of the nozzle of a Pelton wheel is 640 m . The outlet vane angle of the bucket is 15° . The relative velocity at the outlet is reduced by 15% due to friction along the vanes . If the discharge at outlet is without whirl find the ratio of bucket speed to the jet speed . If the jet diameter is 100 mm while the wheel diameter is 1.2 m, find the speed of the turbine in rpm, the force exerted by the jet on the wheel, the Power developed and the hydraulic efficiency. Take $C_v = 0.97$.

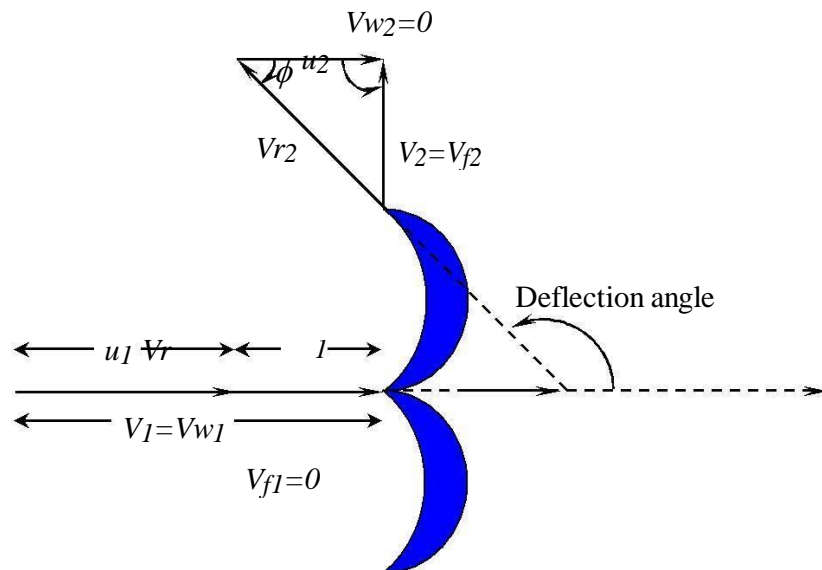
Solution:

$$H = 640 \text{ m}; \phi = 15^\circ; V_{r1} = 0.85 V_{r2}; V_{w2} = 0; d = 100 \text{ mm}; D = 1.2 \text{ m};$$

$$C_v = 0.97; K_u = ?; N = ?; F_x = ?; P = ?; \eta_h = ?$$

We know that the absolute velocity of jet is given by

$$V = C_v \sqrt{2gH} = 0.97 \times \sqrt{2 \times 9.81 \times 640} = 109.74 \text{ m/s}$$



Let the bucket speed be u

$$\text{Relative velocity at inlet} = V_{r1} = V_1 - u = 109.74 - u$$

$$\text{Relative velocity at outlet} = V_{r2} = (1 - 0.15)V_{r1} = 0.85(109.74 - u)$$

$$\text{But } V_{r2} \cos \phi = u \Rightarrow 0.85(109.74 - u) \cos 15^\circ$$

$$\text{Hence } u = 49.48 \text{ m/s}$$

$$\text{But } u = \frac{\pi D N}{60} \text{ and hence}$$

$$N = \frac{60 u}{\pi D} = \frac{60 \times 49.48}{\pi \times 1.2} = 787.5 \text{ rpm (Ans)}$$

$$\text{Jet ratio} = m = \frac{u}{V} = \frac{49.48}{109.74} = 0.45$$

$$\text{Weight of water supplied} = \gamma Q = 10 \times 1000 \times \frac{\pi}{4} \times 0.1^2 \times 109.74^2 = 8.62 \text{ kN/s}$$

$$\text{Force exerted} = F_x = \rho V_1 (V_{w1} - V_{w2})$$

But $V_{w1} = V_1$ and $V_{w2} = 0$ and hence

$$F = 1000 \times \frac{\pi}{4} \times 0.1^2 \times (109.74)^2 = 94.58 \text{ kN}$$

$$\text{Work done/second} = F_x \times u = 94.58 \times 49.48 = 4679.82 \text{ kN/s}$$

$$\text{Kinetic Energy/second} = \frac{1}{2} \rho V^3 = \frac{1}{2} \times 1000 \times \frac{\pi}{4} \times 0.1^2 \times 109.74^3 = 5189.85 \text{ kN/s}$$

$$\text{Hydraulic Efficiency} = \frac{\text{Work done/s}}{\text{Kinetic Energy/s}} = \frac{4679.82}{5189.85} \times 100 = 90.17\%$$

Dec 06 -Jan 07

A PELTON wheel turbine is having a mean runner diameter of 1.0 m and is running at 1000 rpm. The net head is 100.0 m. If the side clearance is 20° and discharge is $0.1 \text{ m}^3/\text{s}$, find the power available at the nozzle and

hydraulic efficiency of the turbine. (10)

Solution:

$D = 1.0 \text{ m}$; $N = 1000 \text{ rpm}$; $H = 100.0 \text{ m}$; $\phi = 20^\circ$; $Q = 0.1 \text{ m}^3/\text{s}$; $WD/s = ?$ and $\eta_h = ?$

Assume $C_v = 0.98$

We know that the velocity of the jet is given by

$$V = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 9.81 \times 100} = 43.83 \text{ m/s}$$

The absolute velocity of the vane is given by

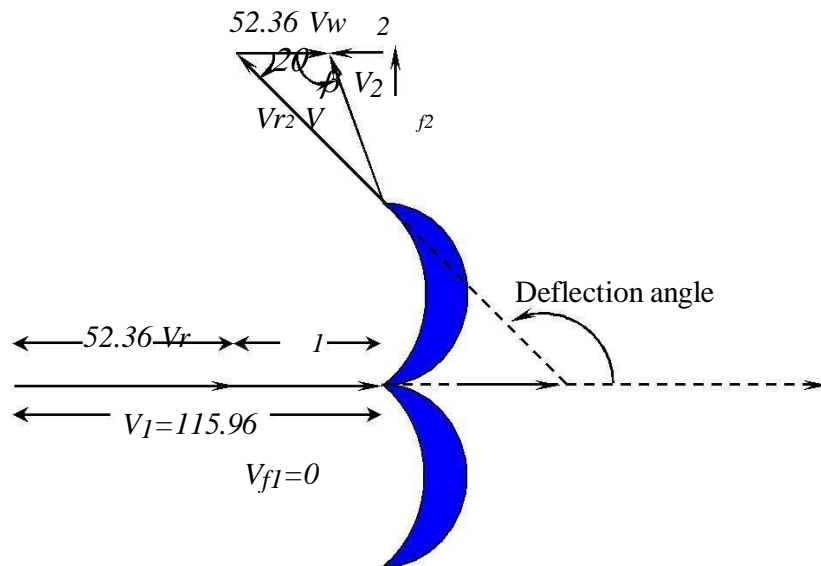
$$u = \frac{\pi D N}{60} = \frac{\pi \times 1.0 \times 1000}{60} = 52.36 \text{ m/s}$$

This situation is impracticable and hence the data has to be modified. Clearly state the assumption as follows:

Assume $H = 700$ m (Because it is assumed that the typing and seeing error as 100 for 700)

Absolute velocity of the jet is given by

$$V = C_v \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 700} = 115.96 \text{ m/s}$$



Power available at the nozzle is the given by work done per second

$$\text{WD/second} = \gamma Q H = \rho g Q H = 1000 \times 10 \times 0.1 \times 700 = 700 \text{ kW}$$

Hydraulic Efficiency is given by

$$\eta_h = \frac{2u(V-u)[1+\cos\phi]}{V^2} = \frac{2 \times 52.36 (115.96 - 52.36)(1 + \cos 20^\circ)}{115.96^2} = 96.07\%$$

July 06

A Pelton wheel has a mean bucket speed of 10 m/s with a jet of water flowing at the rate of 700 lps under a head of 30 m. The buckets deflect the jet through an angle of 160° . Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume the coefficient of nozzle as 0.98. (08)

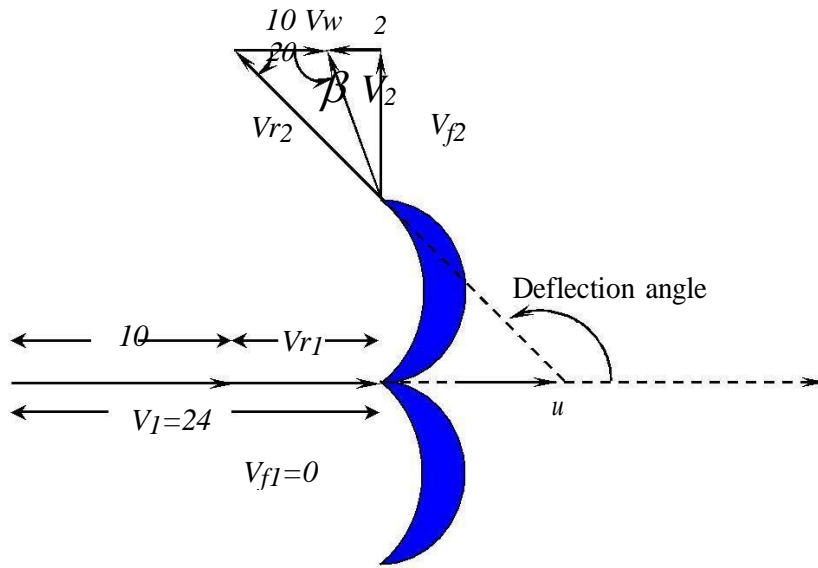
Solution:

$$u = 10 \text{ m/s}; Q = 0.7 \text{ m}^3/\text{s}; \phi = 180 - 160 = 20^\circ; H = 30 \text{ m}; C_v = 0.98;$$

$$\text{WD/s} = ? \text{ and } \eta_h = ?$$

$$\text{Assume } g = 10 \text{ m/s}^2$$

$$V = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 10 \times 30} = 24 \text{ m/s}$$



$$V_{r1} = V_1 - u = 24 - 10 = 14 \text{ m/s}$$

Assuming no shock and frictional losses we have $V_{r1} = V_{r2} = 14 \text{ m/s}$

$$V_{w2} = V_{r2} \cos \phi - u = 14 \times \cos 20 - 10 = 3.16 \text{ m/s}$$

We know that the Work done by the jet on the vane is given by $WD/s = \rho$

$$aV_1 [V_{w1} + V_{w2}]u = \rho Q u [V_{w1} + V_{w2}] \text{ as } Q = aV_l$$

$$=1000 \times 0.7 \times 10 [24 + 3.16] = 190.12 \text{ kN -m/s (Ans)}$$

$$\text{IP/s} = \text{KE/s} = \frac{1}{2} a V^3 = \frac{1}{2} Q V^2 = \frac{1}{2} \times 1000 \times 0.7 \times 24^2 = 201.6 \text{ kN-m/s}$$

Hydraulic Efficiency = Output/ Input = $190 \cdot 12 / 201 \cdot 6 = 94 \cdot 305\%$ It can

also be directly calculated by the derived equation as

$$= \frac{2u}{V \geq 1} (V - u) [1 + \cos \phi] = \frac{2 \times 10}{24} (24 - 10) [1 + \cos 20] = 94.29\% \text{ (Ans)}$$

Jan 06

A Pelton wheel has to develop 13230 kW under a net head of 800 m while running at a speed of 600 rpm . If the coefficient of Jet $C_v = 0.97$, speed

ratio $\phi=0.46$ and the ratio of the Jet diameter is

1 /16 of wheel diameter . Calculate

- i) Pitch circle diameter ii) the diameter of jet
iii) the quantity of water supplied to the wheel

iv) the number of Jets required .

Assume over all efficiency as 85%. (08)

Solution:

$P = 13239 \text{ kW}$; $H = 800 \text{ m}$; $N = 600 \text{ rpm}$; $C_v = 0.97$; $\phi = 0.46$ (Speed ratio) $d/D = 1/16$; $\eta_o = 0.85$; $D = ?$; $d = ?$; $n = ?$;

Assume $g = 10 \text{ m/s}^2$ and $\rho = 1000 \text{ kg/m}^3$

We know that the overall efficiency is given by

$$\eta_o = \frac{\text{Output}}{\text{Input}} = \frac{P}{QH} = \frac{13239 \times 10^3}{10 \times 1000 \times Q \times 800} = 0.85$$

Hence $Q = 1.947 \text{ m}^3/\text{s}$ (Ans)

Absolute velocity of jet is given by

$$V = C_v \sqrt{2gH} = 0.97 \sqrt{2 \times 10 \times 800} = 122.696 \text{ m/s}$$

Absolute velocity of vane is given by

$$u = \phi \sqrt{2gH} = 0.46 \sqrt{2 \times 10 \times 800} = 58.186 \text{ m/s}$$

The absolute velocity of vane is also given by

$$u = \frac{DN}{60} \text{ and hence}$$

$$D = \frac{60u}{\pi N} = \frac{60 \times 58.186}{\pi \times 600} = 1.85 \text{ m (Ans)}$$

$$d = \frac{1.85}{16} = 115.625 \text{ mm (Ans)}$$

$$\text{Discharge per jet} = q = \frac{\pi}{4} d^2 \times V = \frac{\pi}{4} \times 0.115625^2 \times 122.696 = 1.288 \text{ m}^3/\text{s}$$

$$\text{No. of jets} = n = \frac{Q}{q} = \frac{1.947}{1.288} \approx 2 \text{ (Ans)}$$

July 05

Design a Pelton wheel for a head of 80m . and speed of 300 RPM . The Pelton wheel develops 110 kW . Take coefficient of velocity = 0.98, speed ratio = 0.48 and overall efficiency = 80%. (10)

Solution:

$H = 80 \text{ m}$; $N = 300 \text{ rpm}$; $P = 110 \text{ kW}$; $C_v = 0.98$, $K_u = 0.48$; $\eta_o = 0.80$

Assume $g = 10 \text{ m/s}^2$ and $\rho = 1000 \text{ kg/m}^3$

We know that the overall efficiency is given by

$$\eta_o = \frac{\text{Output}}{\text{Input}} = \frac{P}{\gamma Q H} = \frac{110 \times 10^3}{10 \times 1000 \times Q \times 80} = 0.8$$

Hence $Q = 0.171875 \text{ m}^3/\text{s}$

Absolute velocity of jet is given by

$$V = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 10 \times 80} = 39.2 \text{ m/s}$$

Absolute velocity of vane is given by $u = \phi$

$$\sqrt{2gH} = 0.48 \sqrt{2 \times 10 \times 80} = 19.2 \text{ m/s}$$

The absolute velocity of vane is also given by

$$u = \frac{\pi D N}{60} \text{ and hence}$$

$$D = \frac{60 u}{\pi N} = \frac{60 \times 19.2}{\pi \times 300} = 1.22 \text{ m (Ans)}$$

Single jet Pelton turbine is assumed

The diameter of jet is given by the discharge continuity equation

$$Q = \frac{\pi}{4} d^2 \times V = \frac{\pi}{4} \times d^2 \times 39.2 \Rightarrow$$

$$0.171875 = \frac{\pi}{4} d^2 \times 39.2$$

Hence $d = 74.7 \text{ mm}$

The design parameters are

Single jet

Pitch Diameter = 1.22 m

Jet diameter = 74.7 mm

$$\text{Jet Ratio} = m = \frac{D}{d} = \frac{1.22}{0.0747} = 16.32$$

$$\text{No. of Buckets} = 0.5m + 15 = 24$$

Jan 05

It is desired to generate 1000 kW of power and survey reveals that 450 m of static head and a minimum flow of $0.3 \text{ m}^3/\text{s}$ are available. Comment whether the task can be accomplished by installing a Pelton wheel run at 1000 rpm and having an overall efficiency of 80%.

Further, design the Pelton wheel assuming suitable data for coefficient of velocity and coefficient of drag.

Solution.

(08)

$P = 1000 \text{ kW}$; $H = 450 \text{ m}$; $Q = 0.3 \text{ m}^3/\text{s}$; $N = 1000 \text{ rpm}$; $\eta_o = 0.8$

Assume $C_v = 0.98$; $A_p = 0.45$; $\rho = 1000 \text{ kg/m}^3$; $g = 10 \text{ m/s}^2$

$$\eta_o = \frac{\text{Output}}{\text{Input}} = \frac{P}{\rho Q H} = \frac{1000 \times 10^3}{1000 \times 0.3 \times 450} = 0.74$$

For the given conditions of P , Q and H , it is not possible to achieve the desired efficiency of 80%.

To decide whether the task can be accomplished by a Pelton turbine compute the specific speed N_s

$$N_s = \frac{N \sqrt{P}}{H^{5/4}};$$

where N is the speed of runner, P is the power developed in kW and H is the head available at the inlet

$$N_s = \frac{1000 \sqrt{1000}}{450^{5/4}} = 15.25$$

Hence the installation of single jet Pelton wheel is justified. Absolute velocity of jet is given by

$$U = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 10 \times 450} = 92.97 \text{ m/s}$$

Absolute velocity of vane is given by

$$u = C_u \sqrt{2gH} = 0.48 \sqrt{2 \times 10 \times 450} = 19.2 \text{ m/s}$$

The absolute velocity of vane is also given by

$$u = \frac{D N}{60} \text{ and hence}$$

$$D = \frac{60 \times 19.2}{N \times 300} = 1.22 \text{ m (Ans)}$$

Single jet Pelton turbine is assumed

The diameter of jet is given by the discharge continuity equation

$$Q = \frac{\pi}{4} d^2 \times V = \frac{\pi}{4} \times d^2 \times 39.2 \Rightarrow$$

Hence $d = 74.7 \text{ mm}$

The design parameters are

Single jet

Pitch Diameter = 1.22 m

Jet diameter = 74.7 mm

$$\text{Jet Ratio} = m = \frac{D}{d} = \frac{1.22}{0.0747} = 16.32$$

$$\text{No. of Buckets} = 0.5m + 15 = 24$$

July 04

A double jet Pelton wheel develops 895 MKW with an overall efficiency of 82% under a head of 60m. The speed ratio = 0.46, jet ratio = 12 and the nozzle coefficient = 0.97.

Find the jet diameter, wheel diameter and wheel

speed in RPM.

(12)

Solution:

No. of jets = $n = 2$; $P = 895 \text{ kW}$; $\eta_o = 0.82$; $H = 60 \text{ m}$; $K_u = 0.46$; $m = 12$;

$C_v = 0.97$; $D = ?$; $d = ?$; $N = ?$

We know that the absolute velocity of jet is given by

$$V = C_v \sqrt{2gH} = 0.97 \sqrt{2 \times 9.81 \times 60} = 33.76 \text{ m/s}$$

The absolute velocity of vane is given by

$$u = K_u \sqrt{2gH} = 0.46 \sqrt{2 \times 9.81 \times 60} = 15.93 \text{ m/s}$$

Overall efficiency is given by

$$\eta_o = \frac{P}{\rho Q H} \text{ and hence } Q = \frac{P}{\rho H \eta_o} = \frac{895 \times 10^3}{1000 \times 60 \times 0.82} = 1.819 \text{ m}^3/\text{s}$$

$$\text{Discharge per jet} = q = \frac{Q}{n} = \frac{1.819}{2} = 0.9095 \text{ m}^3/\text{s}$$

From discharge continuity equation, discharge per jet is also given by

$$q = \frac{\pi d^2}{4} V = 0.9095 \Rightarrow d = 0.186 \text{ m}$$

$$d = 0.186 \text{ m}$$

Further, the jet ratio $m = 12 = \frac{D}{d}$

Hence $D = 2.232 \text{ m}$

Also $u = \frac{\pi D N}{60}$ and hence $N = \frac{60 u}{D} = \frac{60 \times 15.93}{\pi \times 2.232} = 136 \text{ rpm}$

Note: Design a Pelton wheel: Width of bucket = $5 d$ and depth of bucket is $1.2 d$

The following data is related to a Pelton wheel:

Head at the base of the nozzle = 80m; Diameter of the jet = 100 mm;

Discharge of the nozzle = $0.3 \text{ m}^3/\text{s}$; Power at the shaft = 206 kW; Power absorbed in mechanical resistance = 4.5 kW. Determine (i) Power lost in the nozzle and (ii) Power lost due to hydraulic resistance in the runner.

Solution:

$H = 80 \text{ m}$; $d = 0.1 \text{ m}$; $a = \frac{1}{4} \pi d^2$
 $d = 0.007854 \text{ m}$; $Q = 0.3 \text{ m}^3/\text{s}$; SP = 206 kW; Power

absorbed in mechanical resistance = 4.5 kW.

From discharge continuity equation, we have, $Q = a \times$

$V = 0.007854 \times V \Rightarrow 0.3$

$V = 38.197 \text{ m/s}$

Power at the base of the nozzle = $\rho g Q H$

$= 1000 \times 10 \times 0.3 \times 80 = 240 \text{ kW}$ Power

corresponding to the kinetic energy of the jet = $\frac{1}{2} \rho a V^3$

$= 218.85 \text{ kW}$

(i) Power at the base of the nozzle = Power of the jet + Power lost in the nozzle

Power lost in the nozzle = $240 - 218.85 = 21.15 \text{ kW}$ (Ans)

(ii) Power at the base of the nozzle = Power at the shaft + Power lost in the
 (nozzle + runner + due to mechanical
 resistance)

Power lost in the runner = $240 - (206 + 21.15 + 4.5) = 5.35 \text{ kW}$ (Ans)

The water available for a Pelton wheel is 4 m³/s and the total head from reservoir to the nozzle is 250 m. The turbine has two runners with two jets per runner. All the four jets have the same diameters. The pipeline is 3000 m long. The efficiency of power transmission through the pipeline and the nozzle is 91% and efficiency of each runner is 90%. The velocity coefficient of each nozzle is 0.975 and coefficient of friction $4f$ for the pipe is 0.0045. Determine:

- (i) The power developed by the turbine; (ii) The diameter of the jet and (iii) The diameter of the pipeline.

Solution.

$Q = 4 \text{ m}^3/\text{s}$; $H_g = 250 \text{ m}$; No. of jets = $n = 2 \times 2 = 4$; Length of pipe = $L = 3000 \text{ m}$;

Efficiency of the pipeline and the nozzle = 0.91 and Efficiency of the runner =

$\eta_h = 0.9$; $C_p = 0.975$; $4f = 0.0045$

Efficiency of power transmission through pipelines and nozzle =

$$\eta = \frac{H}{H_g} = \frac{250 - h_f}{250} = 0.91$$

Hence $h_f = 22.5 \text{ m}$

Net head on the turbine = $H - H_g - h_f = 227.5 \text{ m}$

Velocity of jet = $U_t = C_p \sqrt{2gH} = 0.975 \sqrt{2 \times 10 \times 227.5} = 65.77 \text{ m/s}$

- (i) Power at inlet of the turbine = $WP = \text{Kinetic energy/second} = \frac{1}{2} \rho a U_t^3$

$$WP = \frac{1}{2} \times 1000 \times 4 \times 65.77^3 = 8651.39 \text{ kW}$$

$$\eta = \frac{\text{Power developed by turbine}}{WP} = \frac{\text{Power developed by turbine}}{8651.39} = 0.9$$

Hence power developed by turbine = $0.9 \times 8651.39 = 7786.25 \text{ kW}$ (Ans)

- (ii) Discharge per jet = $\frac{Q}{\text{No. of jets}} = \frac{4.0}{4} = 1.0 \text{ m}^3/\text{s}$

$$\text{But } Q = \frac{\pi d^2}{4} \times H_i \times U_t = 1.0 = \frac{\pi d^2}{4} \times 65.77$$

Diameter of jet = $d = 0.14 \text{ m}$ (Ans)

- (iii) If D is the diameter of the pipeline, then the head loss through the pipe is given by = h_f

$$h_f = \frac{4fLV^2}{2gD} = \frac{fLQ^2}{3D^5} \quad (\text{From } Q = aV)$$

$$h_f = \frac{0.0045 \times 3000 \times 4^2}{3D^5} \Rightarrow 22.5$$

Hence $D = 0.956 \text{ m}$ (Ans)

The three jet Pelton wheel is required to generate 10,000 kW under a net head of 400 m.

The blade at outlet is 15° and the reduction in the relative velocity while passing over the blade is 5%. If the overall efficiency of the wheel is 80%, $C_v = 0.98$ and the speed ratio = 0.46, then find: (i) the diameter of the jet, (ii) total flow (iii) the force exerted by a jet on the buckets (iv) The speed of the runner.

Solution:

No of jets = 3; Total Power $P = 10,000 \text{ kW}$; Net head $H = 400 \text{ m}$; Blade

angle = $\phi = 15^\circ$; $V_{r2} = 0.95 V_{r1}$; Overall efficiency = $\eta_o = 0.8$; $C_v = 0.98$;
Speed ratio = $K_u = 0.46$; Frequency = $f = 50 \text{ Hz/s}$.

We know that $\eta_o = \frac{P}{gQH} \Rightarrow 0.8 = \frac{10,000 \times 10^3}{1000 \times 10 \times Q \times 400}$

$Q = 3.125 \text{ m}^3/\text{s}$ (Ans)

Discharge through one nozzle = $q = \frac{Q}{n} = \frac{3.125}{3} = 1.042 \text{ m}^3/\text{s}$

Velocity of the jet = $V_1 = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 10 \times 400} = 87.65 \text{ m/s}$

But $q = \frac{\pi}{4} d^2 \times V_1 \Rightarrow 1.042 = \frac{\pi}{4} d^2 \times 87.65$

$d = 123 \text{ mm}$ (Ans)

Velocity of the Vane = $u = K_u \sqrt{2gH} = 0.46 \sqrt{2 \times 10 \times 400} = 41.14 \text{ m/s}$

$V_{r1} = (V_1 - u) = 87.65 - 41.14 = 46.51 \text{ m/s}$

$V_{r2} = 0.95 V_{r1} = 0.95 \times 46.51 = 44.18 \text{ m/s}$

$V_{w1} = V_1 = 87.65 \text{ m/s}$

$V_{w2} = V_{r2} \cos \phi - u_2 = 44.18 \cos 15^\circ - 41.14 = 1.53 \text{ m/s}$

Force exerted by the jet on the buckets = $F_x = \rho q (V_{w1} + V_{w2})$

$$F = 1000 \times 1.042 (87.65 + 1.53) = 92.926 \text{ kN (Ans)}$$

$$\text{Jet ratio} = m = \frac{D}{M10 \text{ (Assumed)} d}$$

$$D = 1.23 \text{ m}$$

$$= \frac{DN}{60}$$

$$60$$

$$\text{Hence } N = \frac{60 \times 60}{\pi D n \times 1.23} \times 41.14 = 638.8 \text{ rpm (Ans)}$$

Reaction Turbines

Reaction turbines are those turbines which operate under hydraulic pressure energy and part of kinetic energy. In this case, the water reacts with the vanes as it moves through the vanes and transfers its pressure energy to the vanes so that the vanes move in turn rotating the runner on which they are mounted.

The main types of reaction turbines are

2. ***Radially outward flow reaction turbine:*** This reaction turbine consists a cylindrical disc mounted on a shaft and provided with vanes around the perimeter. At inlet the water flows into the wheel at the centre and then glides through radially provided fixed guide vanes and then flows over the moving vanes. The function of the guide vanes is to direct or guide the water into the moving vanes in the correct direction and also regulate the amount of water striking the vanes. The water as it flows along the moving vanes will exert a thrust and hence a torque on the wheel thereby rotating the wheel. The water leaves the moving vanes at the outer edge. The wheel is enclosed by a water-tight casing. The water is then taken to draft tube.

3. ***Radially inward flow reaction turbine:*** The constitutional details of this turbine are similar to the outward flow turbine but for the fact that the guide vanes surround the moving vanes. This is preferred to the outward flow turbine as this turbine does not develop racing. The centrifugal force on the inward moving body of water decreases the relative velocity and thus the speed of the turbine can be

controlled easily.

The main component parts of a reaction turbine are:

(1) Casing, (2) Guide vanes (3) Runner with vanes (4) Draft tube

Casing: This is a tube of decreasing cross-sectional area with the axis of the tube being of geometric shape of volute or a spiral. The water first fills the casing and then enters the guide vanes from all

sides radially inwards. The decreasing cross-sectional area helps the velocity of the entering water from all sides being kept equal. The geometric shape helps the entering water avoiding or preventing the creation of eddies..

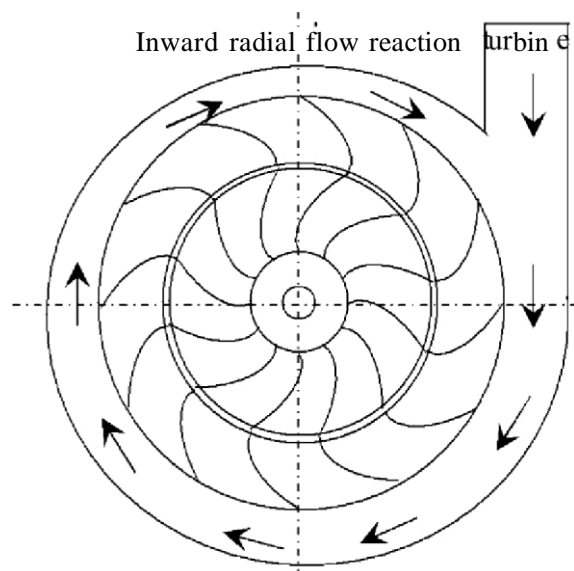
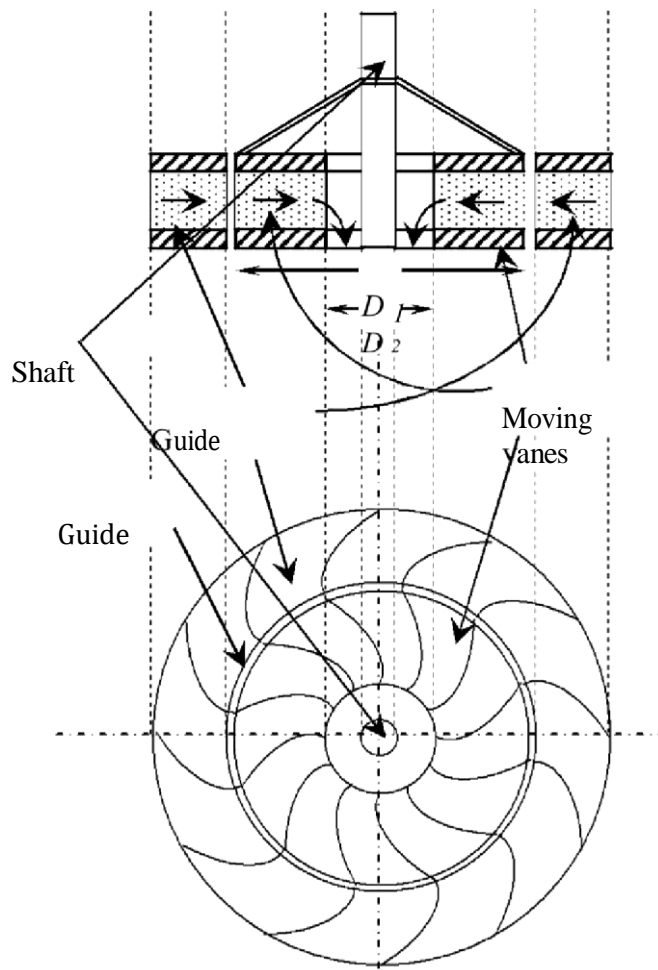
Guide vanes: Already mentioned in the above sections.

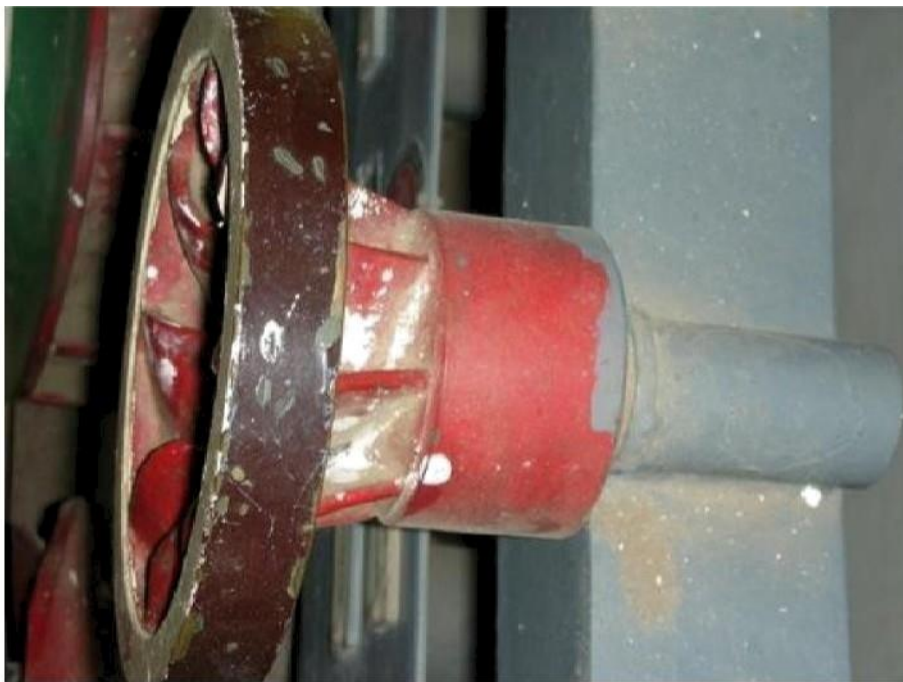
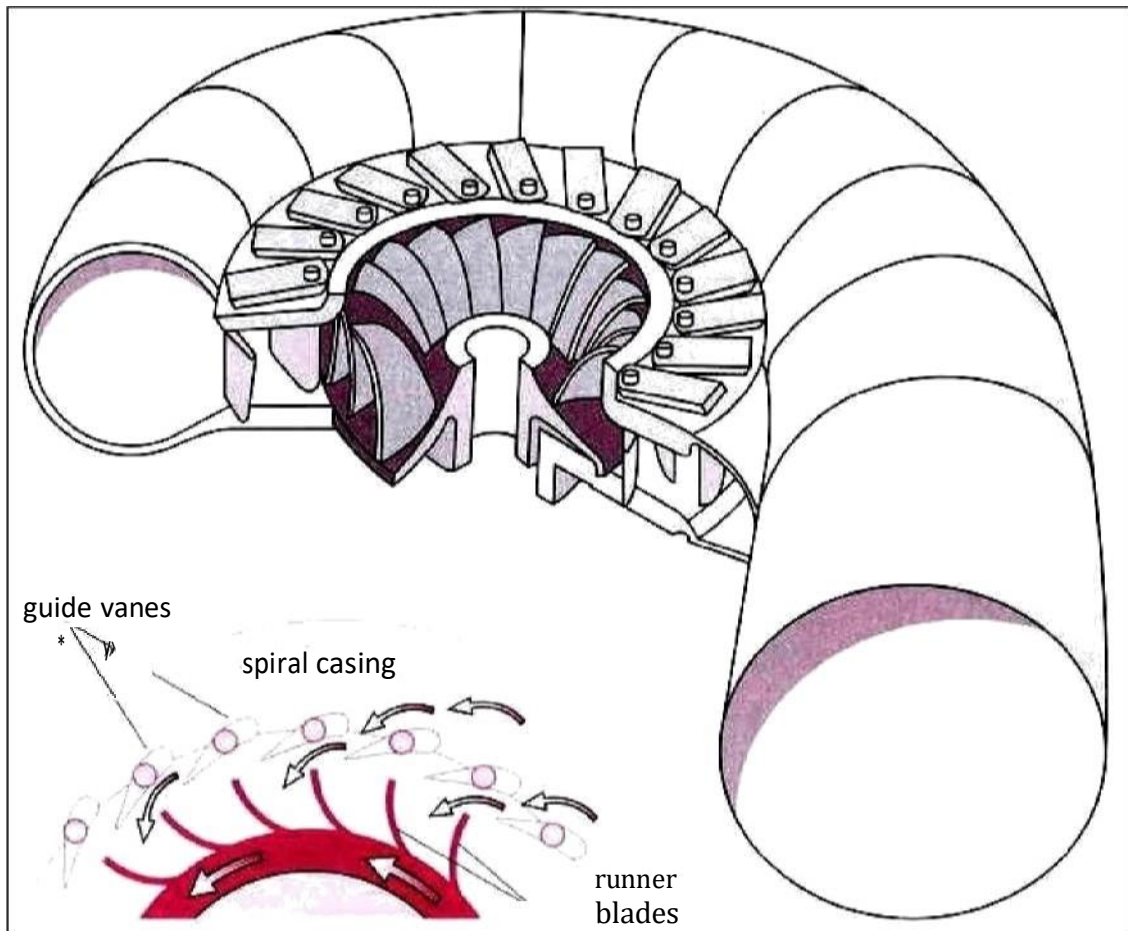
Runner with vanes: The runner is mounted on a shaft and the blades are fixed on the runner at equal distances. The vanes are so shaped that the water reacting with them will pass through them thereby passing their pressure energy to make it rotate the runner.

Draft tube: This is a divergent tube fixed at the end of the outlet of the turbine and the other end is submerged under the water level in the tail race. The water after working on the turbine, transfers the pressure energy there by losing all its pressure and falling below atmospheric pressure. The draft tube accepts this water at the upper end and increases its pressure as the water flows through the tube and increases more than atmospheric pressure before it reaches the tailrace.

(iv) **3fixed oir reaction turbine:** This is a turbine wherein it is similar to inward flow reaction turbine except that when it leaves the moving vane, the direction of water is turned from radial at entry to axial at outlet. The rest of the parts and functioning is same as that of the inward flow reaction turbines.

(v) **dint noir reaction turbine:** This is a reaction turbine in which the water flows parallel to the axis of rotation. The shaft of the turbine may be either vertical or horizontal. The lower end of the shaft is made larger to form the *boss* or the *hub*. A number of vanes are fixed to the boss. When the vanes are composite with the boss the turbine is called **propeller turbine**. When the vanes are adjustable the turbine is called **a Kaplan turbine**.







Runn

Shaft

Guide vanes

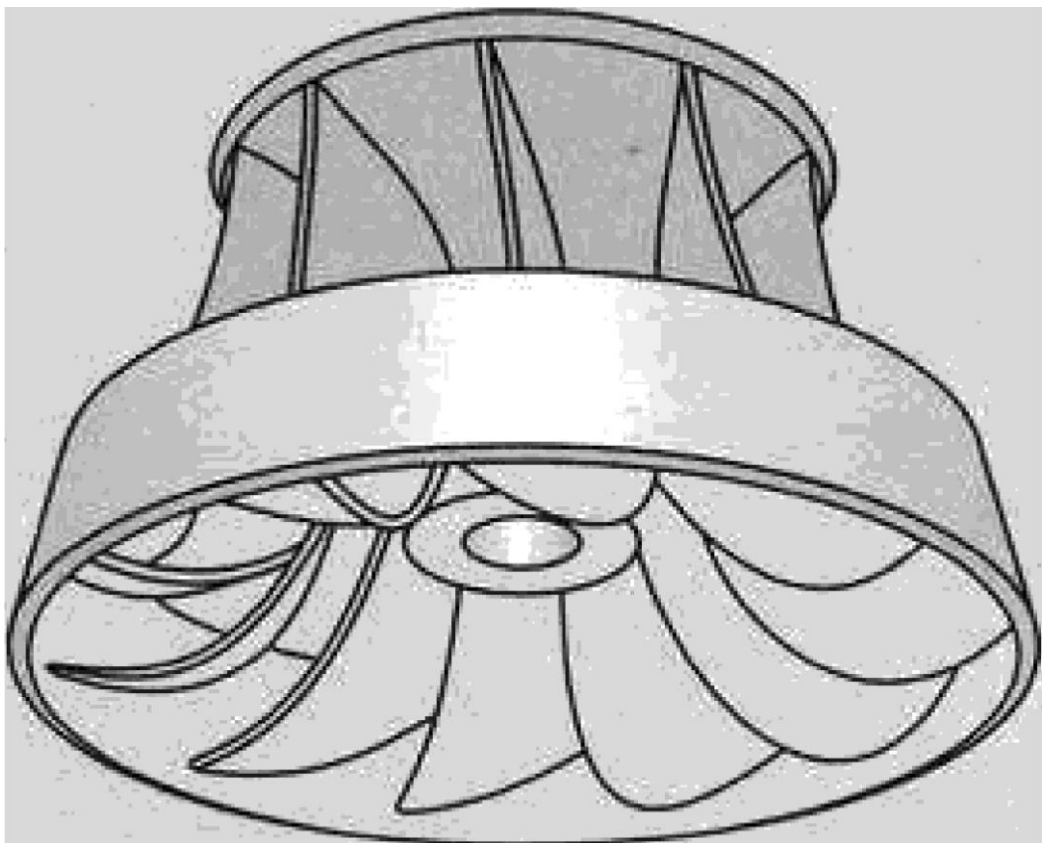
Volute

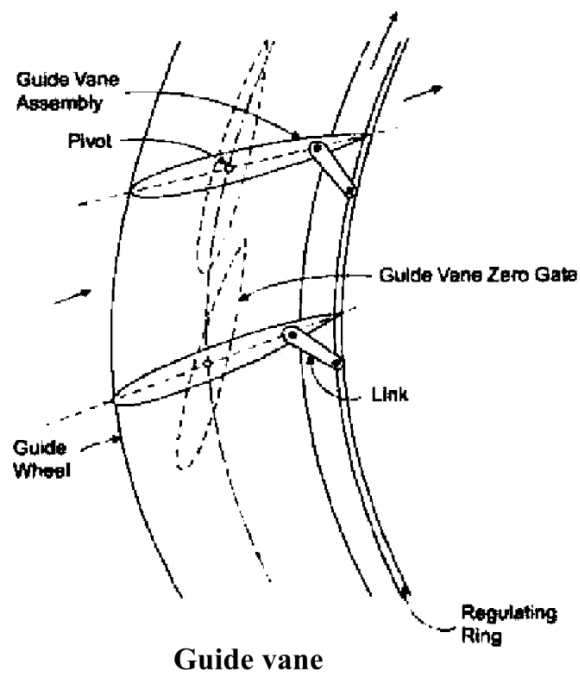
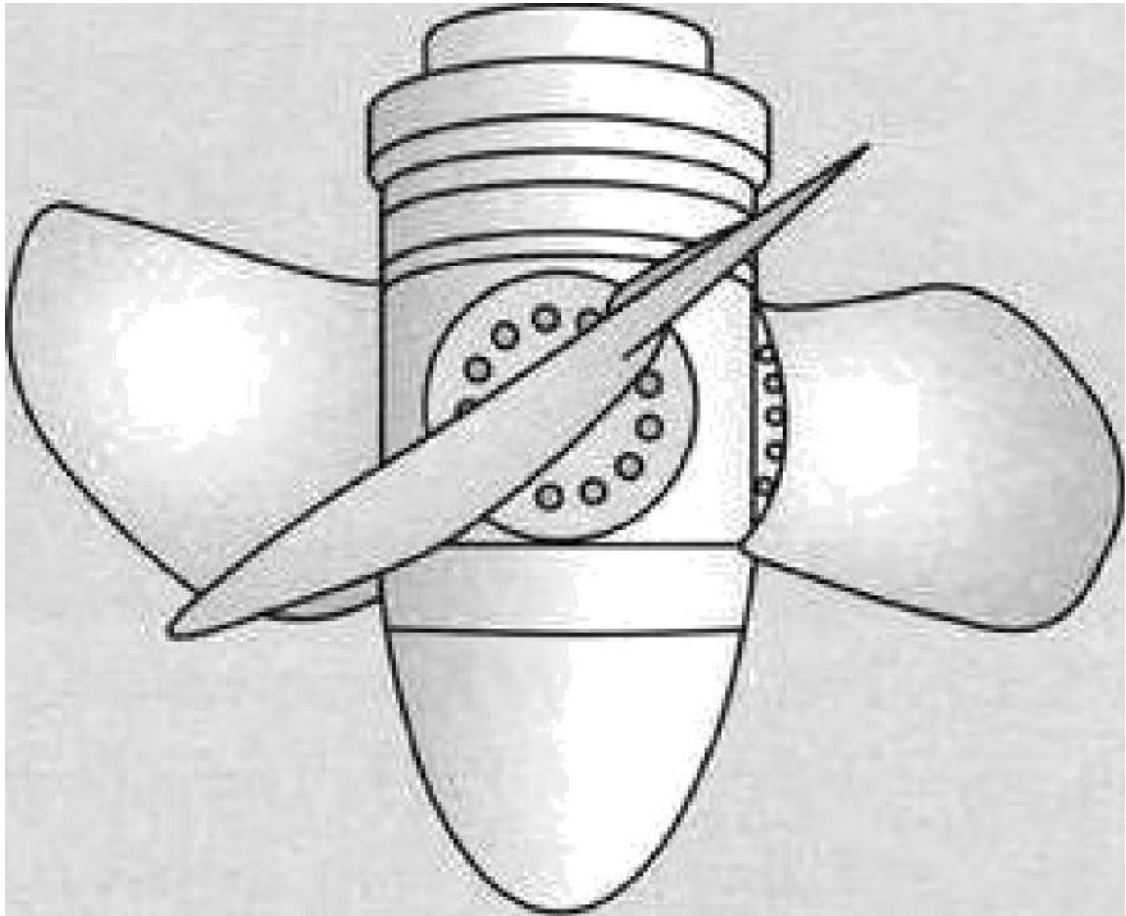
Volute

Moving

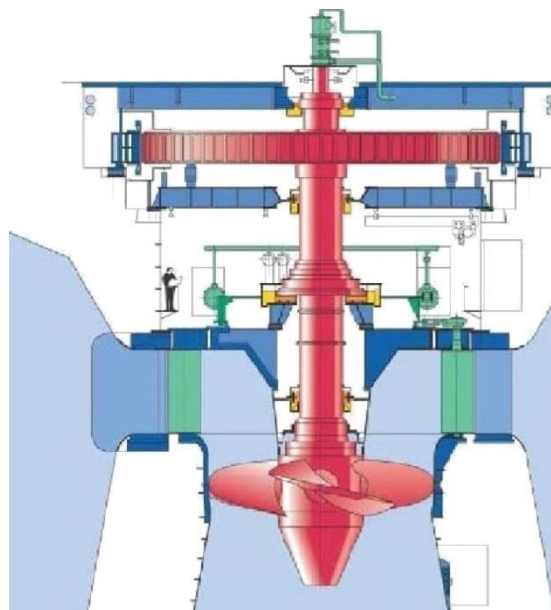
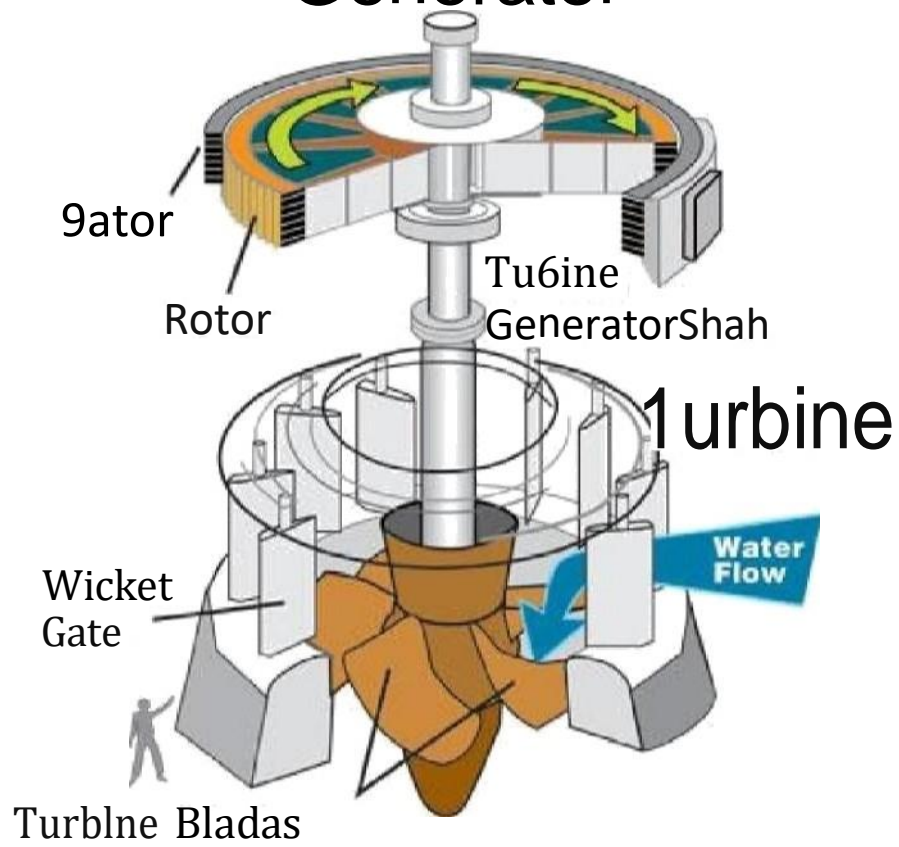
Draft Tube

Francis Turbine Cross -
section

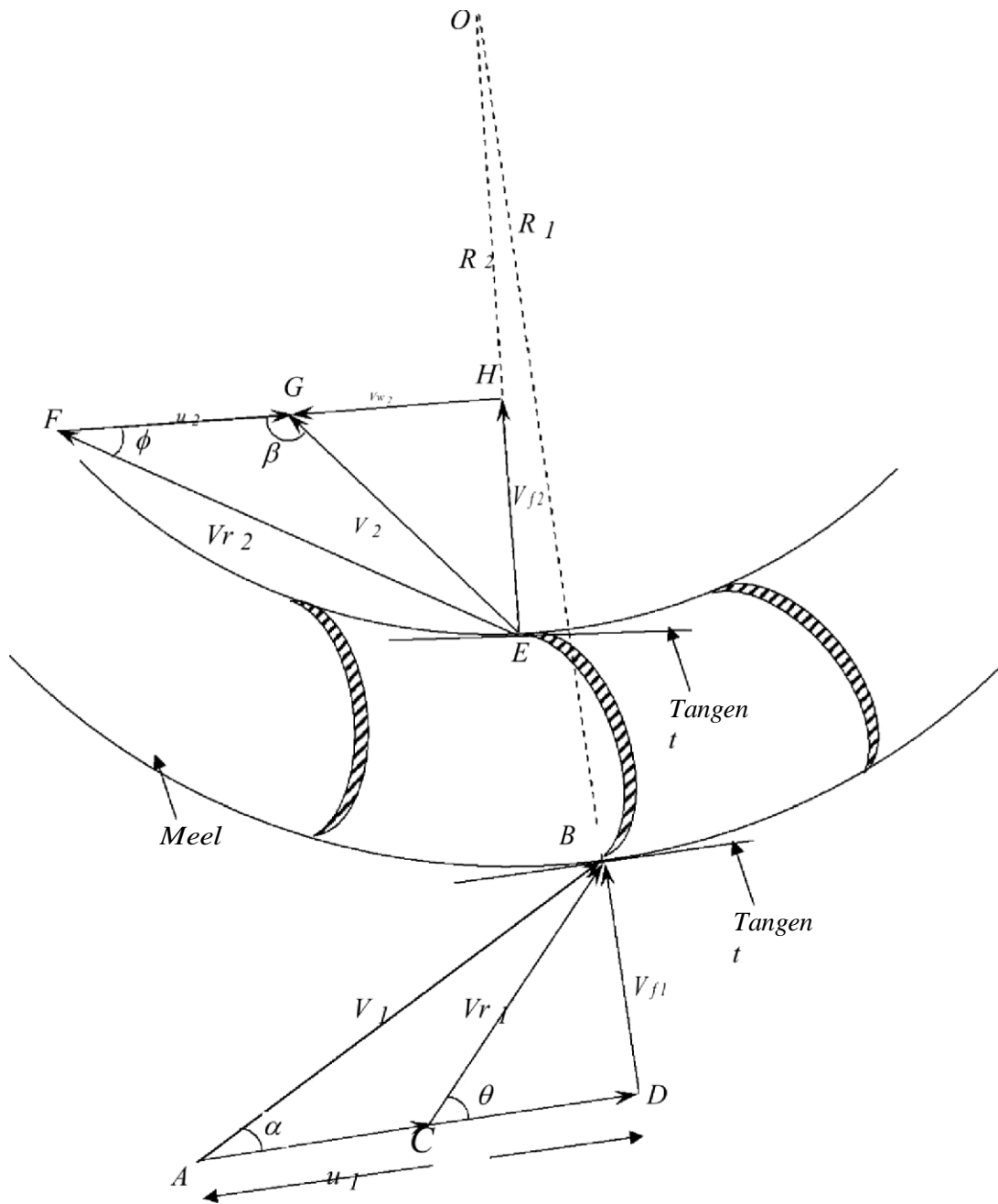




Generator



Derivation of the efficiency of a reaction turbine



Let

R_1 — Radius of wheel at inlet of the vane

R_2 = Radius of wheel at outlet of the vane

ω = Angular speed of the wheel

Tangential speed of the vane at inlet = u_1 $= \omega R_1$

Tangential speed of the vane at outlet $= u_2 = \omega R_2$

The velocity triangles at inlet and outlet are drawn as shown in Fig .

α and β are the angles between the absolute velocities of jet and vane at inlet and outlet respectively

θ and ϕ are vane angles at inlet and outlet respectively

The mass of water striking a series of vanes per second $= \rho a V_1$

where a is the area of jet or flow and V_1 is the velocity of flow at inlet . The momentum of water striking a series of vanes per second at inlet is given by the product of mass of water striking per second and the component of velocity of flow at inlet

$= \rho a V_1 \times V_{w1}$ (V_{w1} is the velocity component of flow at inlet along tangential direction)

Similarly momentum of water striking a series of vanes per second at outlet is given by

$= \rho a V_1 \times (-V_{w2})$ (V_{w2} is the velocity component of flow at outlet along

component is acting in the opposite direction)

Now angular momentum per second at inlet is given by the product of momentum of water at inlet and its radial distance $= \rho a V_1 \times V_{w1} \times R_1$

And angular momentum per second at outlet is given by $= -\rho a V_1 \times V_{w2} \times R_2$

Torque exerted by water on the wheel is given by impulse momentum theorem as the rate of change of angular momentum

$$T = \rho a V_1 \times V_{w1} \times R_1 - (-\rho a V_1 \times V_{w2} \times R_2)$$

$$T = \rho a V_1 (V_{w1} R_1 + V_{w2} R_2)$$

Workdone per second on the wheel = Torque \times Angular velocity $= T \times \omega$

$$\text{WD/s} = \rho a V_1 (V_{w1} R_1 + V_{w2} R_2) \times \omega$$

$$= \rho a V_1 (V_{w1} R_1 \times \omega + V_{w2} R_2 \times \omega)$$

As $u_1 = \omega R_1$ and $u_2 = \omega R_2$, we can simplify the above equation as

$$\text{WD/s} = \rho a V_1 (V_{w1} u_1 + V_{w2} u_2)$$

In the above case, always the velocity of whirl at outlet is given by both magnitude and direction as $V_{w2} = (V_{r2} \cos \phi - u_2)$

If the discharge is radial at outlet, then $V_{w2} = 0$ and hence the equation reduces to

$$\text{WD/s} = \rho a u_1 V_1 V_{w1}$$

$$\text{KE/s} = \frac{1}{2} \rho a V_1^3$$

Efficiency of the reaction turbine is given by $\frac{\rho a V_1 (V_{w1} + V_{w2})}{\rho a V_1^3}$

$$(iii) = \frac{\text{Workdone/second}}{\text{Kinetic Energy/second}} = \frac{V_{w1} + V_{w2}}{\frac{1}{2} V_1^2}$$

$$(iv) = \frac{2 V_{w1} u_1}{V_1^2}$$

velocity of whirl at outlet is to be substituted as along with its sign.

Note: The value of the

$$V_{w2} = (V_{r2} \cos \phi - u_2)$$

Summary

$$(i) \text{ Speed ratio} = \frac{u_1}{\sqrt{2 g H}} \text{ where } H \text{ is the Head on turbine}$$

$$(ii) \text{ Flow ratio} = \frac{V_{f1}}{\sqrt{2 g H}} \text{ where } V_{f1} \text{ is the velocity of flow at inlet}$$

(iii) Discharge flowing through the reaction turbine is given by

$$Q = \pi D_1 B_1 V_{f1} = \pi D_2 B_2 V_{f2}$$

Where D_1 and D_2 are the diameters of runner at inlet and exit

B_1 and B_2 are the widths of runner at inlet and exit

V_{f1} and V_{f2} are the Velocity of flow at inlet and exit

If the thickness (t) of the vane is to be considered, then the area through which flow takes place is given by $(\pi D_1 - nt)$ where n is the number of vanes mounted on the runner.

Discharge flowing through the reaction turbine is given by

$$Q = (\pi D_1 - nt) B_1 V_{f1} = (\pi D_2 - nt) B_2 V_{f2}$$

$$(iv) \text{ The head (} H \text{) on the turbine is given by } H = \frac{p}{\rho g} + \frac{V^2}{2g}$$

Where p_1 is the pressure at inlet .

$$(v) \text{ Work done per second on the runner} = \rho a V_1 (V_{w1} u_1 \pm V_{w2} u_2) = \rho Q (V_{w1} u_1 \pm V_{w2} u_2)$$

$$(vi) u_1 = \frac{\pi D N}{60}$$

$$(vii) u_2 = \frac{\pi D N}{60}$$

$$(viii) \text{ Work done per unit weight} = \frac{\text{Work done per second}}{\text{Weight of water striking per second}} = \frac{Q (V_{w1} u_1 \pm V_{w2} u_2)}{Q g} = \frac{1}{g} (V_{w1} u_1 \pm V_{w2} u_2)$$

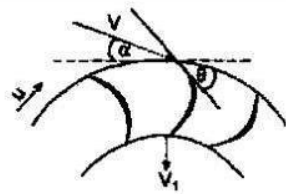
If the discharge at the exit is radial, then $V_{w2} = 0$ and hence

$$\text{Work done per unit weight} = \frac{1}{g} (V_{w1} u_1)$$

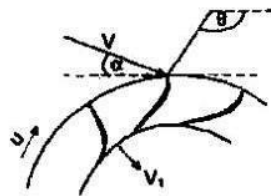
$$(ix) \text{ Hydraulic efficiency} = \frac{R.P.}{W.P.} = \frac{Q (V_{w1} u_1 \pm V_{w2} u_2)}{g Q H} = \frac{1}{g H} (V_{w1} u_1 \pm V_{w2} u_2)$$

If the discharge at the exit is radial, then $V_{w2} = 0$ and hence

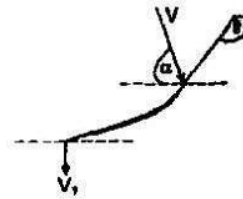
$$\text{Hydraulic efficiency} = \frac{1}{g H} (V_{w1} u_1)$$



(a) Slow Francis Runner

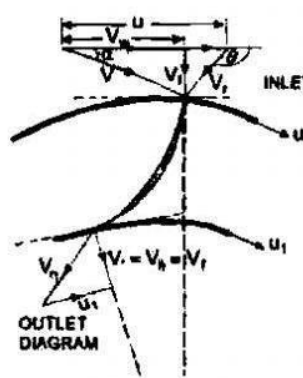


(b) Fast Francis Runner

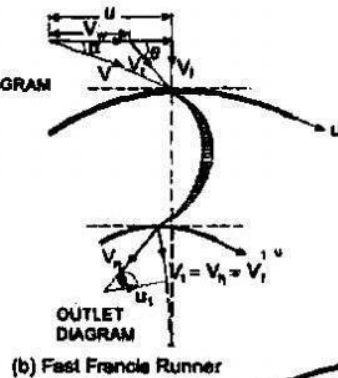


(c) Kaplan Runner

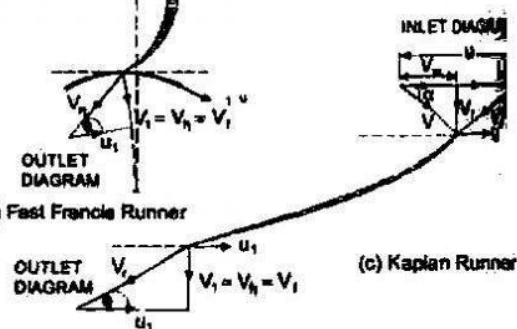
Blade



(a) Slow Francis Runner

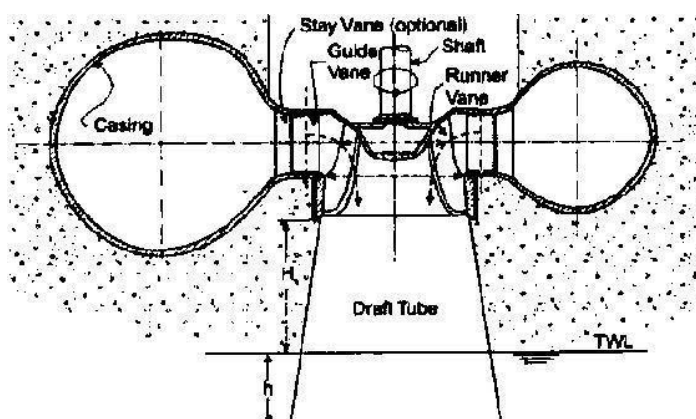


(b) Fast Francis Runner



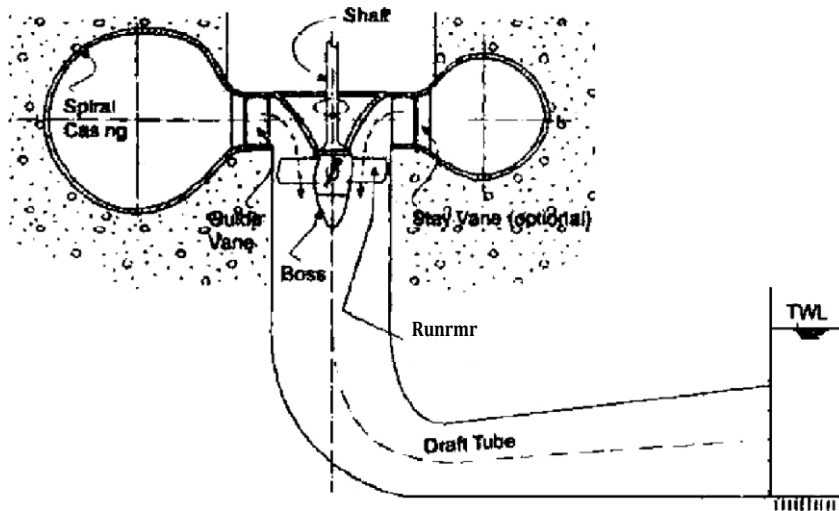
(c) Kaplan Runner

Velocity



Francis Turbine installation with straight

WORKING OF A KAPLAN TURBINE



Kaplan Turbine installation with an Elbow

The reaction turbine developed by Victor Kaplan (1815 - 1892) is an improved version of the older propeller turbine. It is particularly suitable for generating hydropower in locations where large quantities of water are available under a relatively low head. Consequently the specific speed of these turbines is high, viz., 300 to 1000. As in the case of a Francis turbine, the Kaplan turbine is provided with a spiral casing, guide vane assembly and a draft tube. The blades of a Kaplan turbine, three to eight in number are pivoted around the central hub or boss, thus permitting adjustment of their orientation for changes in load and head. This arrangement is generally carried out by the governor which also moves the guide vane suitably. For this reason, while a fixed blade propeller turbine gives the best performance under the design load conditions, a Kaplan turbine gives a consistently high efficiency over a larger range of heads, discharges and loads. The facility for adjustment of blade angles ensures shock-less flow even under non-design conditions of operation.

Water entering radially from the spiral casing is imparted a substantial whirl component by the wicket gates. Subsequently, the curvature of the housing makes the flow become axial to some extent and finally then relative flow as it enters the runner, is tangential to the leading edge of

the blade as shown in Fig 1(c), Energy transfer from fluid to runner depends essentially on the extent to which the blade is capable of extinguishing the whirl component of fluid. In most Kaplan runners as in Francis runners, water leaves the wheel axially with almost zero whirl or tangential component. The velocity triangles shown in Fig 1(c) are at the inlet and outlet tips of the runner vane at mid radius, i.e., midway between boss periphery and runner periphery.

Comparison between Reaction and Impulse Turbines

SN	<i>Reaction turbine</i>	<i>Impulse turbine</i>
1	Only a fraction of the available hydraulic energy is converted into kinetic energy before the fluid enters the runner.	All the available hydraulic energy is converted into kinetic energy by a nozzle and it is the jet so produced which strikes the runner blades.
2.	Both pressure and velocity change as the fluid passes through the runner. Pressure at inlet is much higher than at the outlet.	It is the velocity of jet which changes, the pressure throughout remaining atmospheric.
3	The runner must be enclosed within a watertight casing (scroll casing).	Water-tight casing is not necessary. Casing has no hydraulic function to perform. It only serves to prevent splashing and guide water to the tail race
4.	Water is admitted over the entire circumference of the runner	Water is admitted only in the form of jets. There may be one or more jets striking equal number of buckets simultaneously.
5.	Water completely fills at the passages between the blades and while flowing between inlet and outlet sections does work on the blades	The turbine does not run full and air has a free access to the buckets
6.	The turbine is connected to the tail race through a draft tube which is a gradually expanding passage. It may be installed above or below the tail race	The turbine is always installed above the tail race and there is no draft tube used
7.	The flow regulation is carried out by means of a guide-vane assembly. Other component parts are scroll casing, stay ring, runner and the draft tube	Flow regulation is done by means of a needle valve fitted into the nozzle.

KAPLAN TURBINE - SUMMARY

- 1 . Peripheral velocities at inlet and outlet are same and given by

$$u_1 = u_2 = \frac{D_o N}{60}$$

where D_o is the outer diameter of the runner

- 2 . Flow velocities at inlet and outlet are same. i . e. $V_f = U$ 3
Area of flow at inlet is same as area of flow at outlet

$$Q = \frac{\pi}{4} (D_o^2 - D_b^2) V_f$$

where D_b is the diameter of the boss .

wheel at inlet is 150 mm and the velocity of flow at inlet is 1.5 m/s. Find the rate of flow passing through the turbine.

Solution:

$$D_1 = 0.5 \text{ m}, B_1 = 0.15 \text{ m}, V_{f1} = 1.5 \text{ m/s}, Q = ?$$

$$\text{Discharge through the turbine} = Q = \pi D_1 B_1 V_{f1} = \pi \times 0.5 \times 0.15 \times 1.5$$

$$Q = 0.353 \text{ m}^3/\text{s} \text{ (Ans)}$$

The external and internal diameters of an inward flow reaction turbine are 600 mm and 200 mm respectively and the breadth at inlet is 150 mm. If the velocity of flow through the runner is constant at 1.35 m/s, find the discharge through turbine and the width of wheel at outlet.

Solution:

$$D_1 = 0.6 \text{ m}, D_2 = 0.2 \text{ m}, B_1 = 0.15 \text{ m}, V_{f1} = V_{f2} = 1.35 \text{ m/s}, Q = ?, B_2 = ?$$

$$\text{Discharge through the turbine} = Q = \pi D_1 B_1 V_{f1} = \pi \times 0.6 \times 0.15 \times 1.35$$

$$Q = 0.382 \text{ m}^3/\text{s} \text{ (Ans)}$$

$$\text{Also discharge is given by } Q = \pi D_2 B_2 V_{f2} = \pi \times 0.2 \times B_2 \times 1.35 \Rightarrow 0.382$$

$$B_2 = 0.45 \text{ m/s (Ans)}$$

An inward flow reaction turbine running at 500 rpm has an external diameter is 700 mm and a width of 180 mm. If the guide vanes are at 20° to the wheel tangent and the absolute velocity of water at inlet is 25 m/s, find (a) discharge through the turbine (b) inlet vane angle.

Solution:

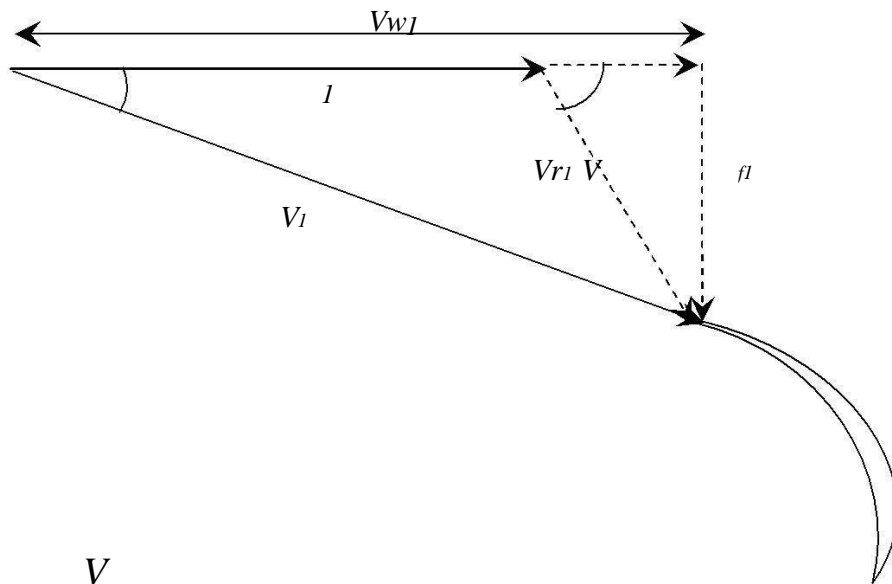
$N = 500 \text{ rpm}, D_1 = 0.7 \text{ m}, B_1 = 0.18 \text{ m}, \alpha = 20^\circ, V_1 = 25 \text{ m/s}, Q = ?, \theta = ?$ We know that the peripheral velocity is given by

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.7 \times 500}{60} = 18.33 \text{ m/s}$$

From inlet velocity triangle, we have

$$V_{f1} = V_1 \sin \alpha = 25 \times \sin 20 = 8.55 \text{ m/s}$$

$$V_{w1} = V_1 \cos \alpha = 25 \times \cos 20 = 23.49 \text{ m/s}$$



$$\tan \theta = \frac{V_{f1}}{V_{w1} - u_1} = \frac{8.55}{23.49 - 18.33} = 1.657$$

$$\theta = 58.89^\circ \text{ (Ans)}$$

$$Q = \pi D_1 B_1 V_{f1} = \pi \times 0.7 \times 0.18 \times 8.55 = 3.384 \text{ m}^3/\text{s} \text{ (Ans)}$$

A reaction turbine works at 450 rpm under a head of 120 m. Its diameter at inlet is 1.2 m and the flow area is 0.4 m². The angle made by the absolute and relative velocities at inlet are 20° and 60° respectively with the tangential velocity. Determine (i) the discharge through the turbine (ii) power developed (iii) efficiency. Assume radial discharge at outlet.

Solution:

$$N = 450 \text{ rpm}, H = 120 \text{ m}, D_1 = 1.2 \text{ m}, a_1 = 0.4 \text{ m}^2, \alpha = 20^\circ \text{ and } \theta = 60^\circ$$

$$Q = ?, \eta = ?, V_{w2} = 0$$

We know that the peripheral velocity is given by

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.2 \times 450}{60} = 28.27 \text{ m/s}$$

$$\tan \theta = \frac{V_{f1}}{V_{w1} - u_1}$$

$$\tan 60 = \frac{V_{f1}}{V_{w1} - 28.27}$$

$$\text{Hence } V_{f1} = (V_{w1} - 28.27) \tan 60 \quad (01)$$

$$\text{Further } \tan \alpha = \frac{V_{f1}}{V_{w1}} = \tan 20$$

$$\text{Hence } V_{f1} = (V_{w1}) \tan 20 \quad (02)$$

From equations 1 and 2, we get

$$(V_{w1} - 28.27) \tan 60 = V_{w1} \tan 20$$

$$\text{Hence } V_{w1} = 35.79 \text{ m/s}$$

$$V_{f1} = 35.79 \times \tan 20 = 13.03 \text{ m/s}$$

$$\text{Discharge } Q = \pi D_1 B_1 V_{f1} = a_1 V_{f1} = 0.4 \times 13.03 = 5.212 \text{ m}^3/\text{s} \text{ (Ans)}$$

Work done per unit weight of water =

$$\frac{1}{g} (V_{u1} - u_1) \frac{1}{10} (35.79 \times 28.27) = 101.178 \text{ kN-m/N}$$

$$\text{Water Power or input per unit weight} = H = 120 \text{ kN-m/N}$$

$$\eta = \frac{101.178}{120} =$$

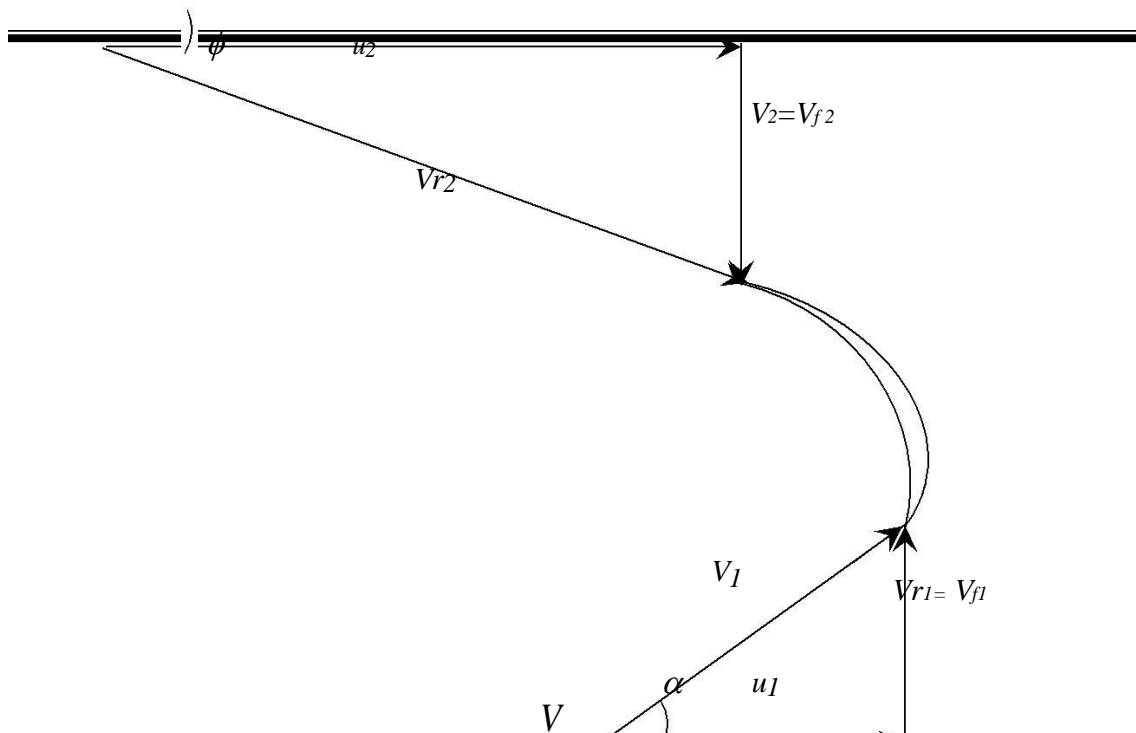
$$\text{Hydraulic efficiency} = 84.31\%$$

The peripheral velocity at inlet of an outward flow reaction turbine is 12 m/s. The internal diameter is 0.8 times the external diameter. The vanes are radial at entrance and the vane angle at outlet is 20°. The velocity of flow through the runner at inlet is 4 m/s. If the final discharge is radial and the turbine is situated 1 m below tail water level, determine:

1. The guide blade angle
2. The absolute velocity of water leaving the guides
3. The head on the turbine
4. The hydraulic efficiency

Solution:

$$u_1 = 12 \text{ m/s}, D_1 = 0.8 D_2, \theta = 90^\circ, \phi = 20^\circ, V_{f1} = 4 \text{ m/s}, V_{w2} = 0, \text{ Pressure head at outlet} = 1 \text{ m}, \alpha = ?, V_1 = ?, H = ?, \eta_h = ?$$



From inlet velocity triangle, $\tan \alpha =$

$$\frac{V_{f1}}{u_1} = \frac{4}{12}, \text{ Hence } \alpha = 18.44^\circ$$

Absolute velocity of water leaving guide vanes is

$$V_1 = \sqrt{u_1^2 + V_{f1}^2} = \sqrt{12^2 + 4^2} = 12.65 \text{ m/s}$$

$$u_1 = \frac{\pi D_1 N}{60} \text{ and } u_2 = \frac{\pi D_2 N}{60}$$

Comparing the above 2 equations, we have

$$\frac{60 u_1}{\pi D_1} = \frac{60 u_2}{\pi D_2} \text{ and hence } \frac{u_1}{D_1} = \frac{u_2}{D_2}$$

$$\text{Hence } u_2 = \frac{D_2}{D_1} u_1 = \frac{12}{0.8} = 15 \text{ m/s}$$

From outlet velocity triangle, $V_2 = V_{f2} = u_2 \tan 20 = 15 \tan 20 = 5.46 \text{ m/s}$ As $V_{w2} = 0$

$$\text{Work done per unit weight of water} = \frac{V_{w1} u_1}{g} = \frac{12 \times 12}{10} = 14.4 \text{ kN-m/N}$$

Head on turbine H

Energy Head at outlet = WD per unit weight + losses

$$H = 1 + \frac{(V^2)}{2g} + \frac{(V_w)^2}{2g} \quad \text{and hence}$$

$$10 = \frac{1}{2} \frac{1}{g} \left(5.46^2 \right) + 14.4 = 16.89 \text{ m}$$

$$\text{Hydraulic efficiency} = \eta_h = \frac{V_{w1} u_1}{g H} = \frac{12 \times 12}{10 \times 16.89} \times 100 = 85.26 \%$$

Jan/Feb 2006

An inward flow water turbine has blades the inner and outer radii of which are 300 mm and 50 mm respectively. Water enters the blades at the outer periphery with a velocity of 45 m/s making an angle of 25° with the tangent to the wheel at the inlet tip. Water leaves the blade with a flow velocity of 8 m/s. If the blade angles at inlet and outlet are 35° and 25° respectively, determine

(i) Speed of the turbine wheel

(ii) Work done per N of water (08)

Solution:

$D_1 = 0.6 \text{ m}; D_2 = 0.1 \text{ m}, V_1 = 45 \text{ m/s}, \alpha = 25^\circ, V_2 = 8 \text{ m/s}, \theta = 35^\circ, \phi = 25^\circ$
 $N = ?, WD/N = ?$

$$\sin \alpha = \frac{V_{f1}}{V_1} = \sin 25 = 0.423$$

$$\text{Hence } V_{f1} = 0.423 \times 45 = 19.035 \text{ m/s}$$

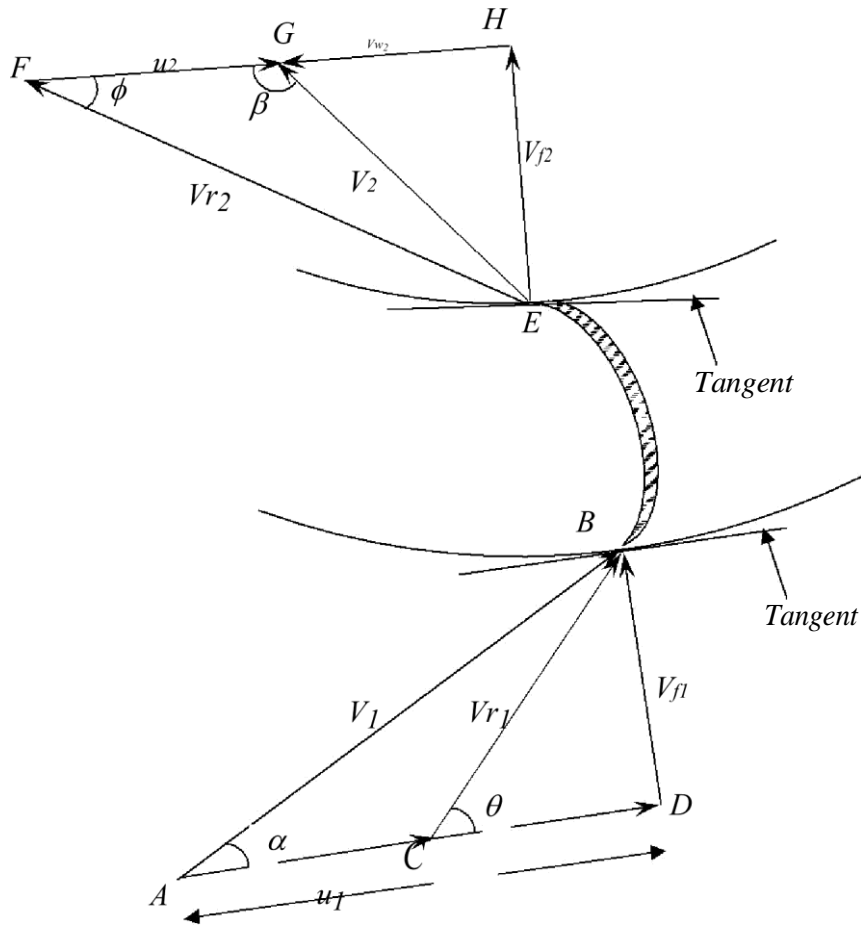
$$\tan \alpha = \frac{V_{f1}}{V_{w1}} = \tan 25 = 0.466$$

$$\text{Hence } V_{w1} = 40.848 \text{ m/s}$$

$$\tan \theta = \frac{V_{f2}}{V_{w2}} = \tan 35 \quad 0.7 = \frac{10.035}{40.848 - u_1}$$

$$u_1 = 13.655 \text{ m/s}$$

$$u_1 = \frac{\pi D_1 N}{60} \quad \text{and hence } N = \frac{60 u_1}{\pi D_1} = \frac{60 \times 13.655}{\pi \times 0.6} = 434.65 \text{ RPM (Ans)}$$



$$u_2 = \frac{wD}{60} = \frac{zN}{60} = \frac{0.1 \times 869.3}{60} = 4.552 \text{ m/s}$$

Ignoring shock losses, $r_2 = r_1 \frac{\sin \theta_1}{\sin \theta_2} = \frac{19.035}{\sin 35} = 33.187 \text{ m/s}$

$$u_2 = r_2 \cos \theta_2 = 33.187 \cos 25 = 4.552 = 25.526 \text{ m/s}$$

Work done per unit weight of water = $\frac{1}{g} (u_1 V_{w1} + u_2 V_{w2})$

$$WD / N = 40.848 \times 13.655 + 25.526 \times 4.552 = 67.4 \text{ m/s (Ans)}$$

July/Aug 2005

A reaction turbine 0.5 m dia develops 200 kW while running at 650 rpm and requires a discharge of 2700 m³/hour; The pressure head at entrance to the turbine is 28 m, the elevation of the turbine casing above the tail

water level is 1 . 8 m and the water enters the turbine with a velocity of 3 . 5 m/s . Calculate (a) The effective head and efficiency, (b) The speed, discharge and power if the same machine is made to operate under a head of 65 m

Solution:

$$D = 0 . 5 \text{ m}, P = 200 \text{ kW}, N = 650 \text{ rpm}, Q = 2700/60 = 0 . 75 \text{ m}^3/\text{s},$$

$$V_1 = 3 . 5 \text{ m/s}, \frac{P}{\rho g Q H} = 28 \text{ m}$$

The effective head = H = Head at entry to runner – Kinetic energy in tail race + elevation of turbine above tailrace

$$H = \frac{P_1}{\rho g} - \frac{V_1^2}{2g} = 28 - \frac{3.5^2}{2 \times 10} + 1.8 = 29.1875 \text{ m (Ans)}$$

$$\text{Hydraulic efficiency} = \frac{P}{\rho g Q H} = \frac{200 \times 10^3}{1000 \times 10 \times 0.75 \times 29.1875} \times 100 = 91.36 \%$$

Further unit quantities are given by

$$\text{Unit speed} = N_u = \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$$\text{Unit Discharge} = Q_u = \frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}$$

$$\text{Unit Power} = P_u = \frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$$

$$N_u = \frac{650}{\sqrt{29.1875}} = \frac{N_2}{\sqrt{65}} \quad 120.31$$

$$N_2 = 969 . 97 \text{ rpm (Ans)}$$

$$Q_u = \frac{0.75}{\sqrt{29.1875}} = \frac{Q_2}{\sqrt{65}} \quad 0.1388$$

$$Q_2 = 1 . 119 \text{ m}^3/\text{s (Ans)}$$

$$P_u = \frac{200}{29.1875^{3/2}} = \frac{P_2}{65^{3/2}} \quad 1.268$$

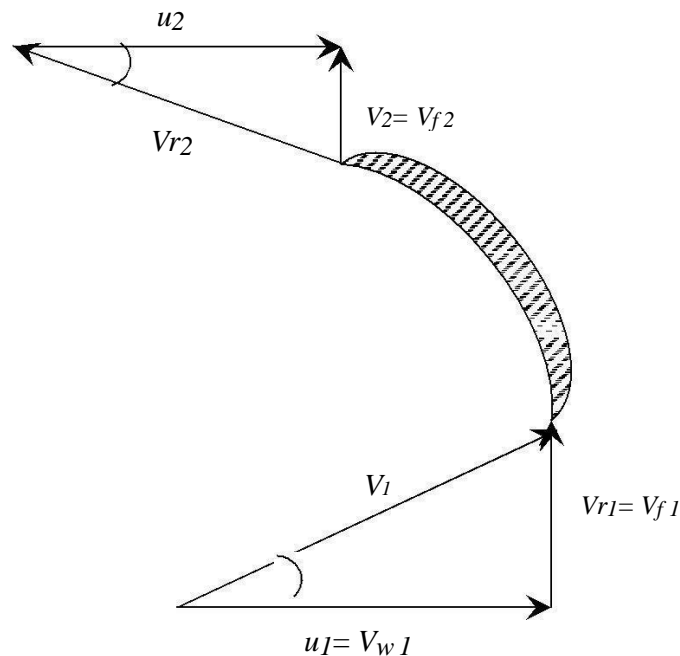
$$P_2 = 664 . 49 \text{ kW (Ans)}$$

July/Aug 2005

A Francis turbine has inlet wheel diameter of 2 m and outlet diameter of 1.2 m. The runner runs at 250 rpm and water flows at 8 cumecs. The blades have a constant width of 200 mm. If the vanes are radial at inlet and the discharge is radially outwards at exit, make calculations for the angle of guide vane at inlet and blade angle at outlet (10)

Solution:

$D_1 = 2 \text{ m}, D_2 = 1.2 \text{ m}, N = 250 \text{ rpm}, Q = 8 \text{ m}^3/\text{s}, b = 0.2 \text{ m}, V_{w1} = u_1, V_{w2} = 0, \alpha = ?, \phi = ?$



$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 2 \times 250}{60} = 26.18 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1.2 \times 250}{60} = 15.71 \text{ m/s}$$

$$Q = \pi D_1 b V_{f1} = \pi D_2 b V_{f2}$$

$$8 = \pi \times 2 \times 0.2 \times V_{f1}$$

$$\text{Hence } V_{f1} = 6.366 \text{ m/s}$$

$$\text{Similarly } 8 = \pi \times 1.2 \times 0.2 \times V_{f2}$$

$$V_{f2} = 10.61 \text{ m/s}$$

$$\alpha = \frac{V_{f1}}{u_1} = 6.366$$

$$\tan \alpha = \frac{V_{f1}}{u_1} = \frac{16.18}{26.18}$$

$$\alpha = 31.67^\circ (\text{Ans})$$

$$\tan \phi = \frac{V_{f2}}{u_2} = \frac{10.61}{15.71}$$

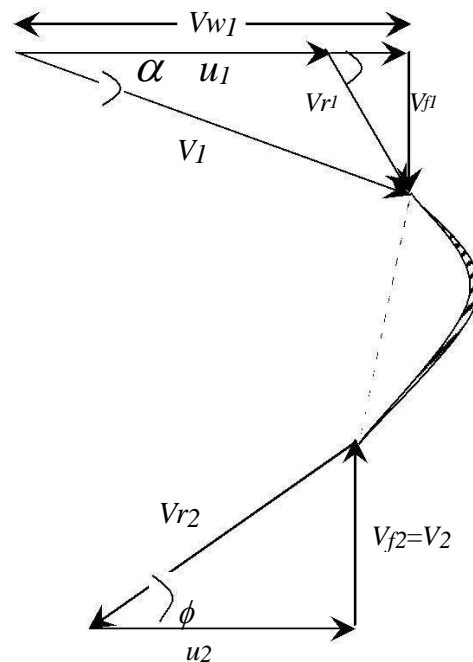
$$\phi = 34.03^\circ (\text{Ans})$$

Determine the overall and hydraulic efficiencies of an inward flow reaction turbine using the following data. Output Power = 2500 kW, effective head = 45 m, diameter of runner = 1.5 m, width of runner = 200 mm, guide vane angle = 20° , runner vane angle at inlet = 60° and specific speed = 100.

Solution:

$$P = 2500 \text{ kW}, H = 45 \text{ m}, D_1 = 1.5 \text{ m}, b_1 = 0.2 \text{ m}, \alpha = 20^\circ, \theta = 60^\circ,$$

$$N_s = 110, \eta_o = ?, \eta_h = ?$$



We know that specific speed is given by

$$N_s = \frac{N \sqrt{P}}{H^{5/4}} \quad \text{and hence } N = \frac{H^{5/4}}{\sqrt{P}} = \frac{100 \times 45^{5/4}}{\sqrt{2500}} = 233 \text{ rpm}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.5 \times 233}{60} = 18.3 \text{ m/s}$$

But from inlet velocity triangle, we have

$$u_1 = \frac{V_{f1}}{\tan 20^\circ} = \frac{V_{f1}}{\tan \theta}$$

$$18.3 = \frac{V_{f1}}{\tan 20^\circ} = \frac{V_{f1}}{\tan 60^\circ} \quad \text{and hence } V_{f1} = 8.43 \text{ m/s}$$

$$V_{w1} = \frac{V_{f1}}{\tan 20^\circ} = \frac{8.43}{\tan 20^\circ} = 23.16 \text{ m/s}$$

$V_{w2} = 0$ and hence

$$\eta_h = \frac{V_{w1} u_1}{g H} = \frac{23.16 \times 18.3}{10 \times 45} \times 100 = 94.18 \% \text{ (Ans)}$$

$$Q = \pi D_1 b_1 V_{f1} = \pi \times 1.5 \times 0.2 \times 8.43 = 7.945 \text{ m}^3/\text{s}$$

$$\eta_o = \frac{P}{g Q H} = \frac{2500 \times 10^3}{1000 \times 10 \times 7.945 \times 45} \times 100 = 69.93 \% \text{ (Ans)}$$

Determine the output Power, speed, specific speed and vane angle at exit of a Francis runner using the following data . Head = 75 m, Hydraulic efficiency = 92%, overall efficiency = 86 %, runner diameters = 1 m and 0.5 m, width = 150 mm and guide blade angle = 18 °. Assume that the runner vanes are set normal to the periphery at inlet .

Solution:

Data: $H = 75 \text{ m}$, $\eta_h = 0.92$, $\eta_o = 0.86$, $D_1 = 1 \text{ m}$, $D_2 = 0.5 \text{ m}$, $\alpha = 18^\circ$,

$V_{w1} = u_1$, $P = ?$, $N = ?$, $\phi = ?$

$$\eta_h = \frac{V_{w1} u_1}{g H} = \frac{u_1^2}{g H}$$

$$u_1^2 = 0.92 \times 10 \times 75 =$$

$$690 \quad u_1 = 26.27 \text{ m/s}$$

$$u_1 = \frac{\pi \times 60}{60} = 26.27 \text{ m/s}$$

$$N = 501.7 \text{ RPM}$$

$$V_{f1} = u_1 \tan \alpha = 26.27 \times \tan 18 = 8.54 \text{ m/s}$$

$$Q = \pi D_1 b_1 V_{f1} = 1.0 \times 0.15 \times 8.54 = 4.02 \text{ m}^3/\text{s}$$

$$\frac{u_1}{D_1} = \frac{u_2}{D_2} \text{ and hence } u_2 = 0.5 \times u_1 = 13.135 \text{ m/s}$$

$$\text{Assuming } V_{f1} = V_{f2}$$

From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f2}}{u_2} = \frac{8.54}{13.135} = 0.65$$

$$\text{Hence } \phi = 33^\circ$$

$$\frac{P}{\rho g Q H} = \frac{P}{1000 \times 10 \times 4.02 \times 75} = 0.86$$

$$\text{Hence } P = 2592.9 \text{ kW (Ans)}$$

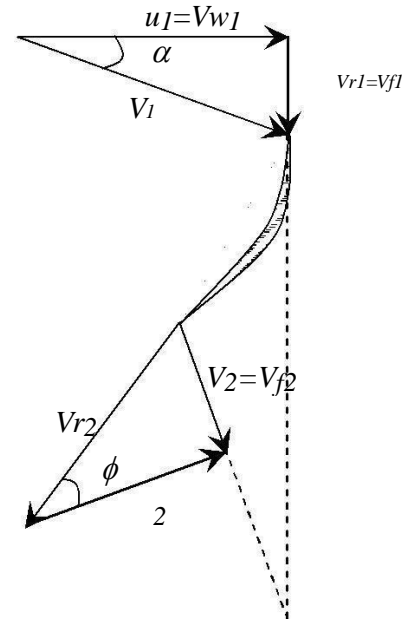
$$\text{Specific speed} = N_{s5} = \frac{N \sqrt{P}}{H^{5/4}} = \frac{501.7 \sqrt{2592.9}}{75^{5/4}} = 115.75 \text{ RPM}$$

The following data is given for a Francis turbine . Net Head = 60 m; speed N = 700 rpm; Shaft power = 294.3 kW; $\eta_o = 84\%$; $\eta_h = 93\%$; flow ratio = 0.2; breadth ratio n = 0.1; Outer diameter of the runner = 2 x inner diameter of the runner . The thickness of the vanes occupies 5% circumferential area of the runner, velocity of flow is constant at inlet and outlet and discharge is radial at outlet . Determine:

- (i) Guide blade angle
- (ii) Runner vane angles at inlet and outlet
- (iii) Diameters of runner at inlet and outlet
- (iv) Width of wheel at inlet

Solution

$$H = 60 \text{ m}; N = 700 \text{ rpm}; P = 294.3 \text{ kW}; \eta_o = 84\%; \eta_h = 93\%;$$



$$\text{flow ratio} = \frac{V_{f1}}{2gH} = 0.2$$

$$V_{f1} = 0.2 \times 2 \times 10 \times 60 = 6.928 \text{ m/s}$$

$$\text{Breadth ratio} = \frac{B_1}{D_1} = 0.1$$

$$D_1 = 2 \times D_2$$

$$V_{f1} = V_{f2} = 6.928 \text{ m/s}$$

Thickness of vanes =

5% of circumferential area of runner

$$\therefore \text{Actual area of flow} = 0.95 \pi D_1 B_1$$

Discharge at outlet = Radial and hence

$$V_{w2} = 0 \text{ and } V_{f2} = V_2$$

We know that the overall efficiency is

given by

$$\eta_o = \frac{P}{gQH} = 0.84 = \frac{294.3 \times 10^3}{1000 \times 10 \times Q \times 60}$$

$$Q = 0.584 \text{ m}^3/\text{s}$$

$$Q = 0.95 \pi D_1 B_1 V_{f1} = 0.95 \pi D_1 \times (0.1 D_1) \times 6.928 = 0.584$$

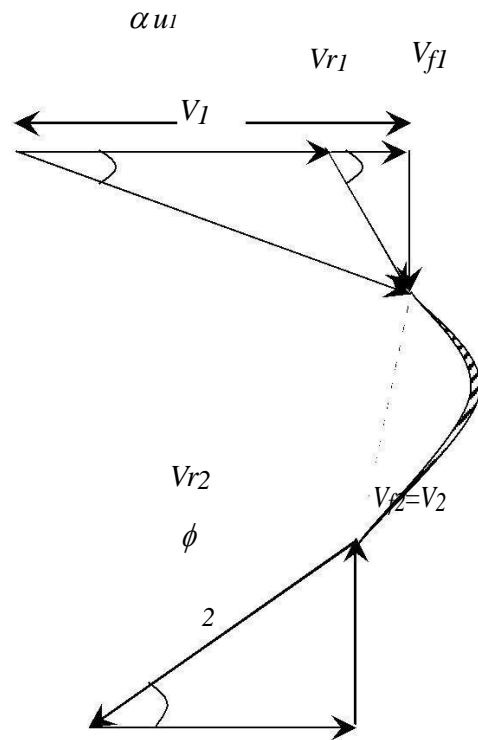
$$\text{Hence } D_1 = 0.531 \text{ m (Ans)}$$

$$\frac{B_1}{D_1} = 0.1 \text{ and } B_1 = 53.1 \text{ mm (Ans)}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.531 \times 700}{60} = 19.46 \text{ m/s}$$

$$\text{Hydraulic efficiency } \eta_h = \frac{V_{w1} u_1}{gH} = 0.93 = \frac{V_{w1} \times 19.46}{10 \times 60}$$

$$V_{w1} = 28.67 \text{ m/s}$$



$$\text{From Inlet velocity triangle } \tan \alpha = \frac{V_{f1}}{V_{w1}} = \frac{6.928}{28.67} = 0.242$$

$$\text{Hence Guide blade angle } = \alpha = 13.58^\circ \text{ (Ans)}$$

$$\tan \theta = \frac{V_{f1}}{w_1 - u_1} = \frac{6.928}{28.67 - 19.46} = 0.752$$

Vane angle at inlet $\theta = 37^\circ$ (Ans)

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times (2) \times 700}{60} = 9.73 \text{ m/s}$$

From outlet velocity triangle, we have

$$\frac{6.928}{9.73} = \frac{\tan \phi}{1}$$

$$\phi =$$

$$35.45^\circ$$

$$(Ans)$$

$$\phi = 35.45^\circ \text{ (Ans)}$$

Diameters at inlet and outlet are $D_1 = 0.531 \text{ m}$ and $D_2 = 0.2655 \text{ m}$

A Kaplan turbine develops 9000 kW under a net head of 7.5 m. Overall efficiency of the wheel is 86%. The speed ratio based on outer diameter is 2.2 and the flow ratio is 0.66. Diameter of the boss is 0.35 times the external diameter of the wheel. Determine the diameter of the runner and the specific speed of the runner.

2.2 Solution:

$$P = 9000 \text{ kW}; H = 7.5 \text{ m}; \eta_o = 0.86; \text{Speed ratio} = 2.2; \text{flow ratio} = 0.66;$$

$$D_b = 0.35 D_o ;$$

$$u_1 = 2.2 \sqrt{2 \times 10 \times 7.5} = 26.94 \text{ m/s}$$

$$V_{f1} = \frac{0.66}{1} \sqrt{2 \times 10 \times 7.5} = 8.08 \text{ m/s}$$

$$V_{f1} = 0.66$$

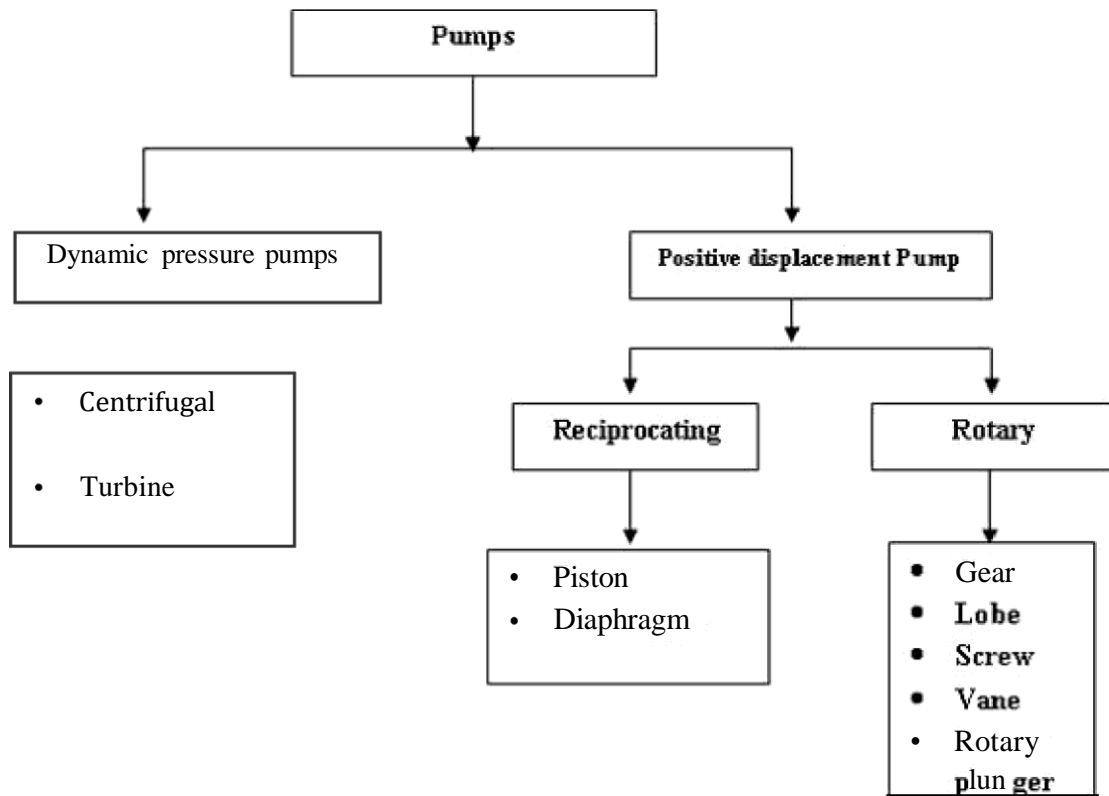
$$\eta_o = \frac{P}{\rho g Q H} = 0.86 = \frac{9000 \times 10^3}{1000 \times 10 \times Q \times 7.5}$$

CENTRIFUGAL PUMP

Purpose: To lift the liquid to the required height.

Pump: A hydraulic machine which converts mechanical energy of prime mover (Motors, I.C. Engine) into pressure energy

Classification of Pumps



Application:

1. Agriculture & Irrigation
2. Petroleum
3. Steam and diesel Power plant
4. Hydraulic control system
5. Pumping water in buildings
6. Fire Fighting

Positive Displacement:

Amount of liquid taken on suction side is equal to amount of liquid transferred to deliver side. Hence discharge pipe should be opened before starting the pump to avoid the bursting of casing.

Rotodynamic Pump:

Increase in energy level is due to a combination of centrifugal energy, Pressure energy and kinetic energy. i.e. fluid is not displaced positively from suction side to delivery side. Pumps can run safely even the delivery valve is closed.

Centrifugal Pump: Mechanical energy of motor is converted into pressure energy by means of centrifugal force acting on the fluid.

Sr. No.	Centrifugal Pump	Inward Flow Turbine
1	It consumes power	It produces power
2	Water flows radially outward	Water flows radially inward from periphery
3	Flow from low pressure to high pressure	Flow from high pressure to low pressure
4	Flow is decelerated	Flow is accelerated

Construction and working of centrifugal Pump

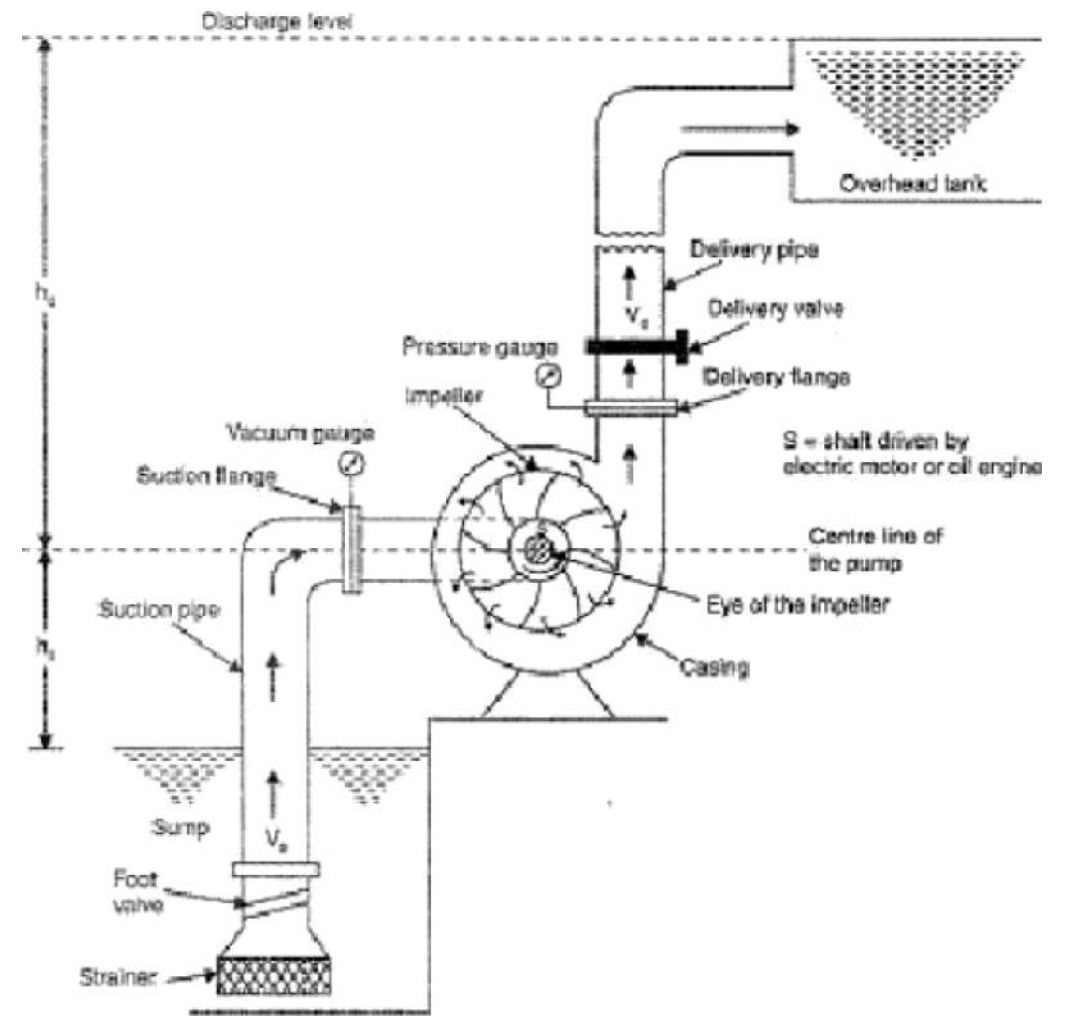
Components:

1. Impeller: A wheel with series of backward curved vanes.
2. Casing: Air tight chamber surrounding the impeller.
3. Suction Pipe: One end is connected in eye and other is dipped in a liquid.
4. Delivery pipe: One end is connected to eye, other to overhead tank.
5. Foot valve: Allow water only in upward direction.
6. Strainer: Prevent the entry of foreign particle/material to the pump

Working of Centrifugal Pump:

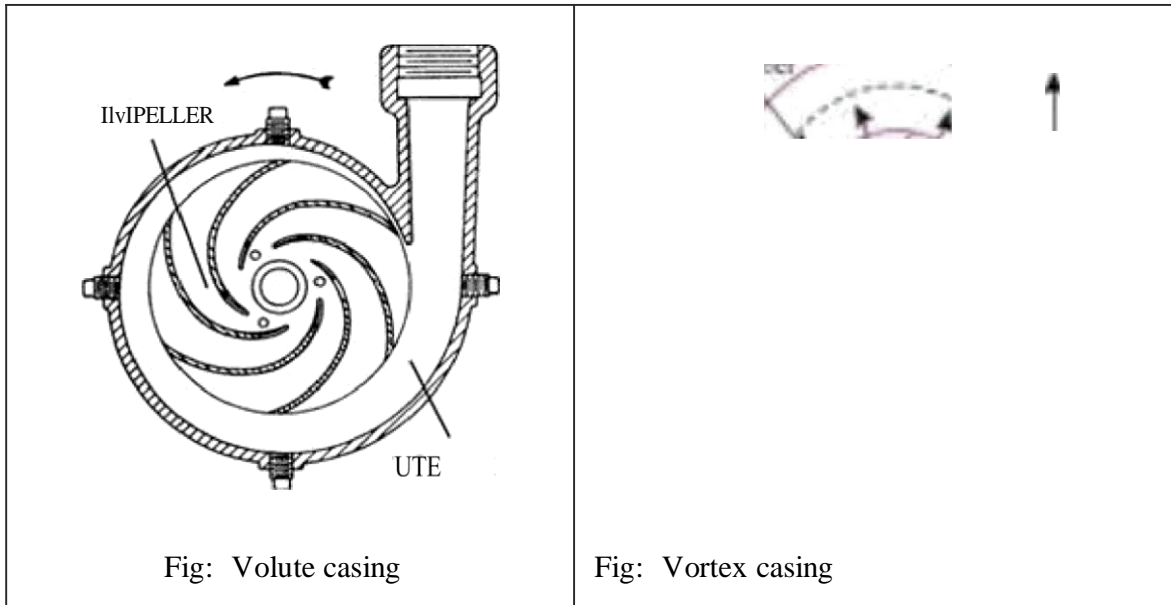
When a certain mass of fluid is rotated by an external source, it is thrown away from the central axis of rotation and centrifugal head is impressed which enables it to rise to a higher level.

1. The delivery valve is closed and pump is primed i.e. suction pipe, casing and portion of delivery pipe up to the delivery valve are completely filled with water so that no air pocket is left.
2. Keeping the delivery valve is closed the impeller is rotated by motor, strong suction is created at the eye.
3. Speed enough to pump a liquid when is attained delivery valve is opened. Liquid enter the impeller vane from the eye, come out to casing.
4. Impeller action develops pressure energy as well as velocity energy.
5. Water is lifted through delivery pipe upto required height.
6. When pump is stopped, delivery valve should be closed to prevent back flow from reservoir.



Types of casing

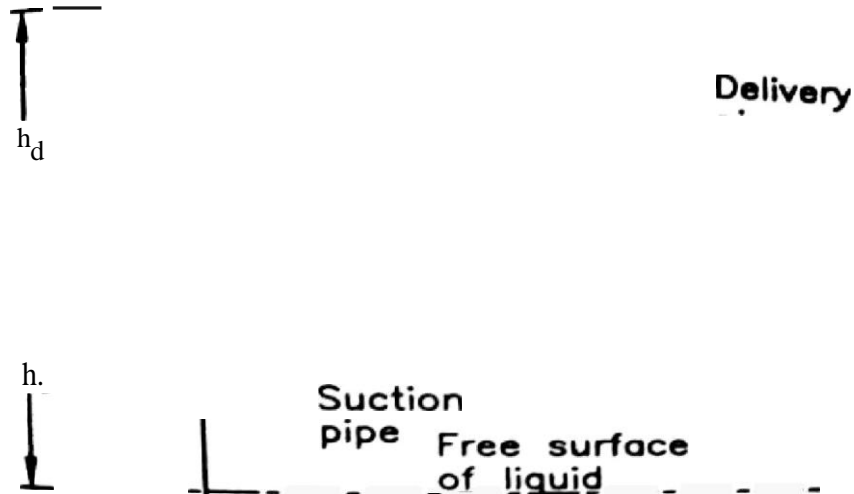
1. Volute Casing: Area of flow gradually increases from the eye of impeller to the delivery pipe. Same as shown in fig of components. Formation of eddies.



2. Vortex casing: Circular chamber provided between the impeller and volute chamber.
Loss of energy due to formation of eddies is reduced.
3. Casing with guide blades: Casing impeller is surrounded by a series of guide vanes mounted on a ring which is known as diffuser. Water enters the impeller without shock.



Various head of centrifugal Pump



The heads of a centrifugal pump are as follows:

- (1) Suction head (2) delivery head
- (3) Static head (4) Monometric head

1. Suction head (h_s) : It is vertical distance between level of sump and eye of an impeller. It is also called suction lift.
2. Delivery head (h_d): It is the vertical distance between between eye of an impeller and the level at which water is delivered.
3. Static head (H): It is sum of suction head and delivery head. It is given by

$$H = (h_s + h_d)$$
4. Manometric head (H_m): The head against which the centrifugal Pump has to work.

It is given by following equations:

- (i) $H_m = (\text{Head imparted by the impeller to the water})$
 $(\text{Loss of head in the pump impeller and casing})$

$$H_m = \frac{v_2^2 - v_1^2}{2g} - (h_{Li} + h_{Lj})$$

Where, h_{Li} =Loss in impeller

h_{Lj} = Loss in casing

$$H = \frac{F Q u^2}{g} \quad (\text{if losses are neglected})$$

(ii) $H = \text{Static head} + \text{losses in pipes} + \text{Kinetic head at delivery}$

$$H = (h_s + h_f + h_d) + \frac{V_d^2}{2g} \quad \text{------(8)}$$

Where,

h_s and $A = \text{Suction and delivery head}$

h_f and $h_d = \text{Loss of head due to friction in suction and delivery pipe.}$

$V_d = \text{Velocity pipe in delivery pipe.}$

(iii) $H = (\text{Total head at outlet of pump}) - (\text{Total head at inlet of Pump})$

$$\text{Total head at outlet} = \frac{P_d}{\rho g} + \frac{V_d^2}{2g} + Z_d = h_d + \frac{V_d^2}{2g} + Z_d$$

$$\text{Total head at inlet} = \frac{P_s}{\rho g} + \frac{V_s^2}{2g} + Z_s = h_s + \frac{V_s^2}{2g} + Z_s$$

$$H_m = (h_d + \frac{V_d^2}{2g}) - (h_s + \frac{V_s^2}{2g}) \quad \text{------(9)}$$

Inlet and outlet velocity

triangles for Centrifugal

Pump Work done By

Impeller on liquid

1. Liquid enters eye of impeller in radial direction i.e. $\alpha = 90^\circ$, $S_{ri} = 0$, $V_i = V_{f1}$
2. No energy loss in impeller due to eddy formation

S_e

. The velocity distribution in the impeller is uniform.

e

$N = \text{Speed of impeller (rpm)}$

$\omega = \text{Angular velocity} = \frac{2\pi N}{60} \quad (\text{rad/s})$

Tangential velocity of impeller

$$u_1 = \omega R_1 = \frac{\pi D_1 N}{60} \quad \text{m/s}$$

$$u_2 = \omega R_2 = \frac{\pi D_2 N}{60} \quad \text{m/s}$$

$H_i = \text{Absolute velocity of water at inlet}$

K_m = Velocity whirl *at inlet*

$P_r = \text{Relative Velocity at inlet}$

$*f_1 = \text{velocity of flow at inlet}$

$\alpha = \text{angle made by } P_r \text{ at inlet with direction of motion of vane}$

$\beta = \text{Angle made by } P_r \text{ at inlet with direction of motion of vane}$

$1, w_1, 1, f_1, f_1, \dots$ ----- Corresponding values at outlet

A Centrifugal pump is the reverse of radially inward flow reaction turbine.

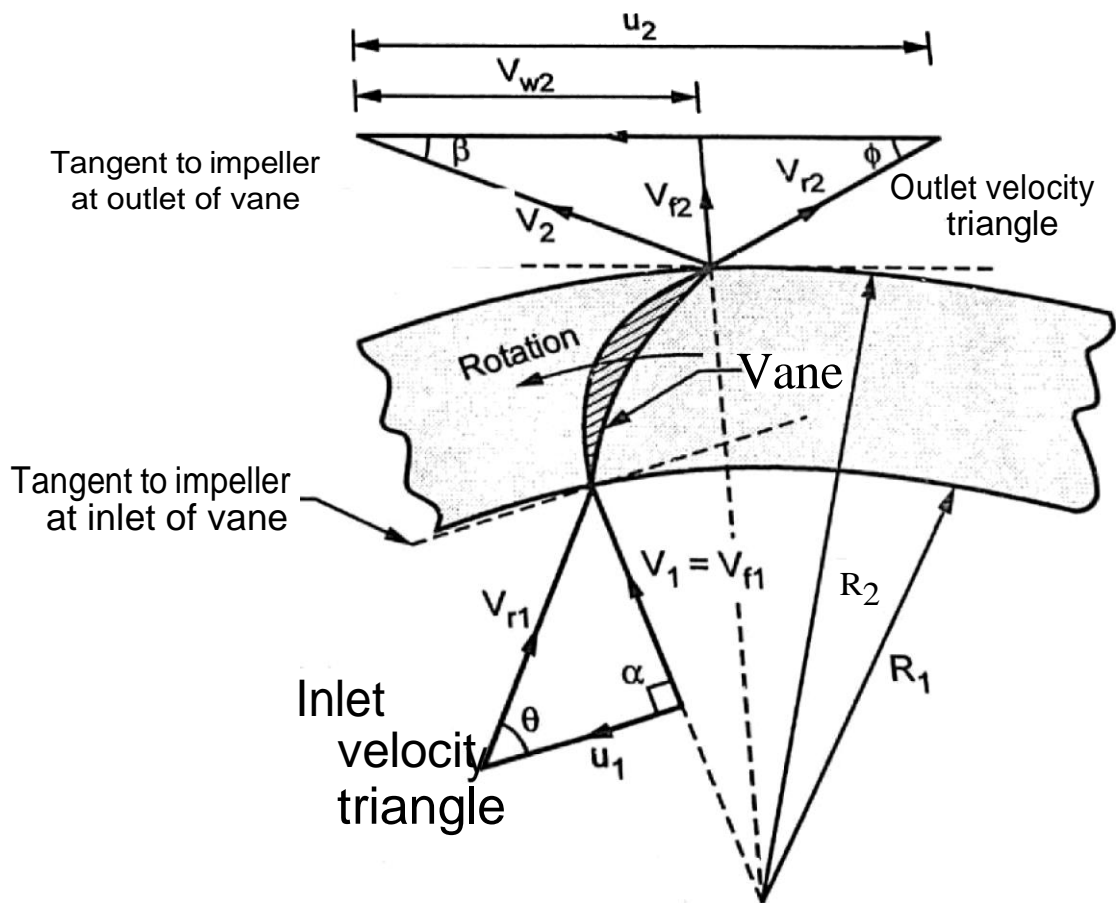


Fig. Velocity triangles for an impeller

Work done by water on runner of turbine per sec per unit weight of water $\frac{1}{g} (w_1 u_1 - w_2 u_2)$

W.D. by impeller on water per sec per unit weight of water = - (WD in case of turbine)

$$W.D = (P_p u_2 - P_1 u_1) \dots \dots \dots (1)$$

Eqn. (1) is known as Euler momentum equation for pump or Euler head.

Since radial entry $K_{pt} = 0$ and K_{fj}

$$W.D. \text{ per unit weight} = \frac{1}{g} (P_2 u_2 - P_1 u_1) \dots \dots \dots (2)$$

$Q = \text{Area} \times \text{velocity of flow}$

$$Q = \pi r_1^2 B_1 \times V_1$$

$$\text{Continuity equation } Q = \pi r_1^2 B_1 V_1 = \pi r_2^2 B_2 V_2$$

From the outlet velocity triangle

$$V_2^2 = u_2^2 + V_{w2}^2 - 2u_2 V_{w2} \cos \alpha_2 \dots \dots \dots (3)$$

Also,

$$V_{f2}^2 = V_2^2 - V_{w2}^2 \dots \dots \dots (4)$$

From equation 3 and 4

$$V_{f2}^2 = V_2^2 - V_{w2}^2 = u_2^2 + V_{w2}^2 - 2u_2 V_{w2} \cos \alpha_2 - V_{w2}^2$$

$$2u_2 V_{w2} = V_2^2 + u_2^2 - V_{f2}^2$$

$$u_2 V_{w2} = \frac{V_2^2 + u_2^2 - V_{f2}^2}{2}$$

Similarly from inlet velocity triangle

$$u_1 V_{w1} = \frac{V_1^2 + u_1^2 - V_{f1}^2}{2}$$

Putting in equation 1

$$W.D = \frac{1}{g} (u_2 V_{w2} - u_1 V_{w1}) = \frac{1}{g} \left(\frac{V_2^2 + u_2^2 - V_{f2}^2}{2} - \frac{V_1^2 + u_1^2 - V_{f1}^2}{2} \right) \dots \dots \dots (5)$$

W.D per sec/unit weight = Increase in K.E head + Increase in static pressure + Change in K.E due to retardation

Equation 5 is known as fundamental equation centrifugal Pump.

Losses in Centrifugal Pump

1. Hydraulic losses : Friction loss shock , eddy losses
2. Mechanical losses: Bearing friction, impeller
3. Leakage losses: leakage of liquid

Efficiencies of a Centrifugal Pump

1. Manometric efficiency (η_{mano})

$$\eta_{mano} = \frac{\text{Head added to liquid}}{\text{Head at impeller}} = \frac{H}{\frac{u^2}{2g}}$$

Francis Turbine,

2. Volumetric Efficiency (η_v)

$$\eta_v = \frac{\text{Liquid discharged per second from the Pump}}{\text{Quantity of liquid passing per second through the impeller}}$$

0+

Where,

Q = Actual liquid discharged at the pump outlet per second

q = Leakage of liquid per second from impeller

3. Mechanical efficiency (η_{mech})

$$\eta_{mech} = \frac{\text{Power at Impeller}}{\text{Power at shaft}} = \frac{\frac{WV^2}{2}}{P}$$

4. Overall efficiency (η_o)

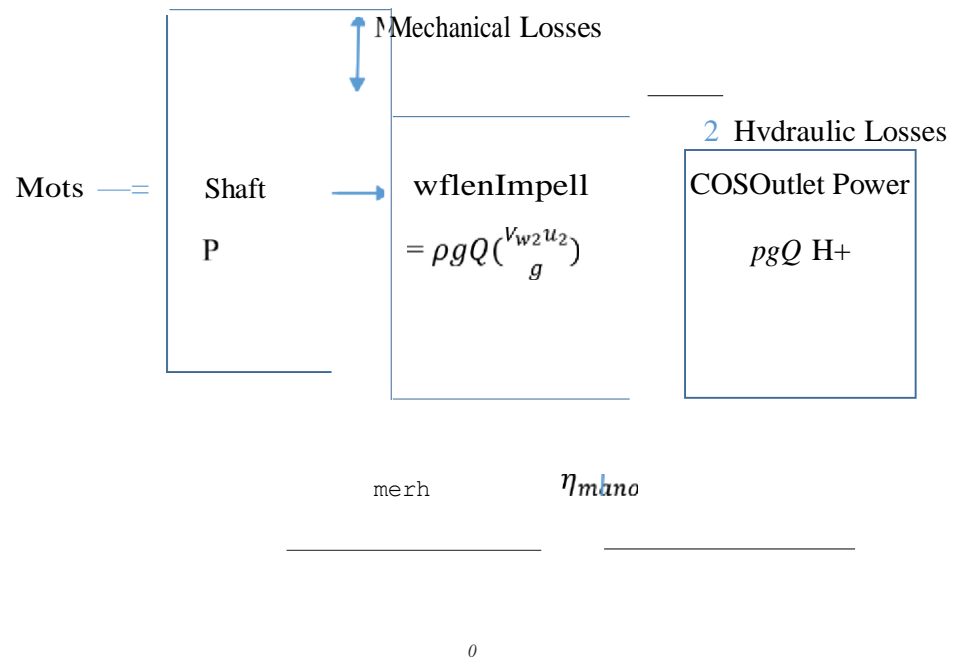
$$\eta_o = \frac{\text{Output Power of Pump}}{\text{Input Power of pump}} = \frac{\rho g Q H_m}{P}$$

$$\eta_o = \eta_{mech} \times \eta_{mano}$$

Turbine,

$$\eta_o = \frac{\rho g Q H_m}{P}$$

The various losses and corresponding efficiencies of a centrifugal Pump are tabulated as follow



Effect of Outlet blade angle on Manometric Efficiency η_{mano}

At outlet of an impeller the energy available in liquid has the pressure energy equal to the sum of manometric head (HP) and velocity head ($\frac{V^2}{2g}$)

Neglecting the losses in pump we have

$$\frac{V_w^2}{g} = H_m + \left(\frac{V_2^2}{2g} \right)$$

$$H_m = \frac{V_w^2}{g} - \frac{V_2^2}{2g}$$

From outlet velocity triangle,

$$\frac{V_w^2}{g} = \frac{V_2^2}{g} \sin^2 \phi \quad \text{and}$$

$$\frac{V_2^2}{g} = \frac{V_w^2}{g \sin^2 \phi}$$

$$= \frac{V_w^2}{g} \cot^2 \phi$$

Substituting the above values in H_m equation

$$H_m = \frac{V_w^2}{g} - \frac{V_w^2 \cot^2 \phi}{g} = \frac{V_w^2 (1 - \cot^2 \phi)}{g}$$

After simplification we get,

$$H_m = \frac{u_2^2 - V_f^2 \cot^2 Q}{2g}$$

Substituting this value in Manometric Efficiency η_{man}

$$\eta_{man} = \frac{u_2^2 - V_f^2 \cot^2 Q}{2u_2(u_2 - V_f \cot Q)} \quad (A)$$

- In the equation (A) if we vary the value of Q from 20° to 90° by keeping the other parameters constant then η_{man} is between 0.73 to 0.47
- If we further reduce the value of Q (below 20°), it increases the efficiency but also results in long size of blades and increased friction losses.
- Therefore the discharge vane angle(Q) is kept more than 20° for a centrifugal pump.

Cavitation

Whenever the pressure in the pipe falls below the vapour pressure corresponding to the existing temperature of the liquid, the liquid will vaporize and bubbles are formed collapse and this process is continued rapidly and creates high pressure which can damage the impeller very easily. This phenomenon is known as cavitation which is highly undesirable. The cavitation is generally occurs in centrifugal pumps near the inlet of the impeller.

Thomas cavitation factor

The cavitation factor is used to indicate whether it will occur or not. The cavitation factor σ for pump is given by

$$\sigma = \frac{NPSH}{2gH_m}$$

If the value of σ is less than the critical value of σ_c , then the cavitation occurs in the pump.

$$\sigma_c = 1.03 \times 10^{-3} \frac{N^2}{\omega_s^3}$$

Where N_s = Specific speed of the pump

The cavitation in the pump can be avoided by

- reducing the velocity in the suction pipe, and avoiding the bends
- reducing h_f in suction pipe by using smooth pipe,
- reducing the suction head and
- Selecting the pump whose specific speed is low.

Effects of cavitation

It is undesirable as it has following disadvantages.

1. The large number of vapour bubbles formed are carried with liquid a high pressure region is reached, where these bubbles suddenly collapse. This includes the rush of surrounding liquid and produces shock and noise. This phenomenon is known as **water hammer**.
2. The surface of blades and impeller are worn out because of bursting of bubbles.
3. The water hammer phenomenon is fatigue for the metal parts and it reduces the life by blow action.

Net Positive suction head (NPSH)

The term is very commonly used in pump industry because the minimum suction conditions are specified in terms of NPSH

Let,

P_i = Absolute pressure at inlet of the Pump

P_a = Absolute atmospheric pressure

P_v = Vapour pressure of the liquid

V_e - Velocity suction pipe.

h_r , = losses in suction pipe

H_a Atmospheric pressure head

H_v = vapour pressure head

NPSH = Absolute pressure head at inlet - Vapour pressure head + Inlet (suction) velocity head

$$\text{NPSH} = \frac{P_i}{\rho g} - \frac{P_v}{\rho g} + \frac{V_e^2}{2g} \quad \text{-----(i)}$$

But absolute pressure head at inlet of pump is given by

$$\frac{P_i}{\rho g} = \frac{P_a}{\rho g} - \frac{P_v}{\rho g} + \frac{V_e^2}{2g} + h_r$$

Substituting the above value in equation (i)

$$\text{NPSH} = \frac{P_a}{\rho g} - \frac{P_v}{\rho g} + \frac{V_e^2}{2g} + h_r - \frac{P_v}{\rho g} + \frac{V_e^2}{2g}$$

$$\text{NPSH} = \frac{P_a}{\rho g} - \frac{P_v}{\rho g} - h_r - h_{fs}$$

$$\text{NPSH} = H_a - H_v - h_r - h_{fs} \quad \text{-----(24)}$$

- The NPSH is also defined as the net head required to make the liquid flow through suction pipe from sump to impeller.
- NPSH term is also used to check cavitation in pump

Required NPSH

- It is value given by pump manufacturer
- This value can be determined experimentally and it varies with pump design, speed of the pump and capacity of the pump.

Available NPSH

- When pump is installed the value of available NPSH is calculated from equation 24
- The available NPSH should be greater than required NPSH for cavitation free operation of Pump.

Priming of centrifugal Pumps:

- The priming of centrifugal pump is the process of filling the suction pipe, casing of the pump and portion of the delivery pipe from outside source of the fluid to be raised.
- This removes the air, gas or vapour from these parts.
- Priming is done before the starting the pump
- It is necessary to avoid discontinuity of flow or dry running of pump
- The dry running of pump may result in rubbing and seizing of the wearing rings and cause severe damage.
- Also when the pump is running with air instead of water, the head generated is in terms of meters of air. But as the density of air very low, the generated head of air in terms of equivalent meter of water head is negligible and hence water may not be sucked from the pump.

For all above reason priming is necessary.

The following are the some of the methods for priming the centrifugal pump.

- i. Priming of small pumps: It is done by pouring the fluid into the funnel provided for priming. During this the air vent valve is kept open and priming is continued till all the air is removed.

- ii. Priming of large Pumps: It is done by removing the air from casing and suction pipe with the help of vacuum pump or by an ejector. This helps in drawing the liquid from sump and fill the pump with liquid.

There are some pumps having internal constructions for supply of liquid in suction pipe known as *self-priming pumps*

Installation of Centrifugal Pump

The following steps are used for efficient installation of the centrifugal Pump.

- i. Location of Pump
 - ii. Suction piping
 - iii. Delivery piping
 - iv. Foundation
 - v. Grouting
 - vi. Alignment
- i. Location
 - The pump unit should be located close to the water surface to minimize the vertical suction lift. The suction lift of length more than 5 m must be avoided.
 - ii. Suction piping:
 - Suction pipe must be continuously flooded have length of 3 times diameter for straight run and it can accommodate a strainer.
 - Entire suction piping should be inclined slightly and all the flanged joints should be fitted with gasket and be airtight.
 - iii. Deliver piping
 - The discharge valve must be of butterfly or ball or globe type if it is used as flow or pressure throttling device.
 - The maximum flow velocity in the discharge line should not exceed 2 m/s
 - iv. Foundation and grouting
 - The pump must be installed on a base plate. The base plate is attached to a foundation and grouting is placed between it.
 - The foundation and grouting will help to damp out the vibrations.

v. Alignment

- The pump alignment is extremely important.
- The suction and discharge piping should be naturally aligned with pump.
- The alignment should be done prior to grouting it and it is checked after grouting and during startup.

Specific Speed of a centrifugal pump

It is defined as the speed of geometrically similar pump which would deliver one cubic meter of liquid per second against a unit head (one meter)

The discharge through impeller of a centrifugal pump is given by

$Q = \text{Area} \times \text{velocity of flow}$

$$= \pi D B \times v_f$$

$$Q = \pi D B \times v_f \quad \text{-----(i)}$$

$$Q = \pi D B \times \left(\frac{\pi B D N}{60} \right) \quad \text{-----(ii)}$$

Now tangential velocity is given by

$$u = \frac{\pi D N}{60}$$

$$u = \frac{\pi D N}{60} \quad \text{-----(iii)}$$

Also from the relation of tangential velocity (u) and flow velocity (U) to the manometric head (H_m),

$$u = U \sin \alpha \quad \text{-----(iv)}$$

Now substituting the value of u from eqn. (iv) in equation (iii) we get

$$\frac{\pi D N}{60} = U \sin \alpha \quad \text{-----(v)}$$

Substitute iv and v in equation (ii) we get

$$Q = \pi D B \times \left(\frac{\pi B D N}{60} \right) \sin \alpha \quad \text{-----(vi)} \quad K = \text{constant of proportionality}$$

From the definition of specific speed of if H_m = 1, Q = 1 m³/s then N = N_s

$$1 = K \frac{Q^{3/2}}{N_s^2} \quad K = N_s^2$$

Substituting the value of k in equation (vi) we get

$$Q = N^2 \frac{H_m^{3/2}}{s N^2}$$

$$N_s = \frac{N \sqrt{Q}}{H_m^{3/4}}$$

Minimum speed for starting of Centrifugal Pump

For minimum speed to start the pump

$$\frac{u_2^2 - u_1^2}{2g} \geq H_m \quad \text{-----(a)}$$

$$\text{and} \quad \eta_{mano} = \frac{g H_m}{V_{w2} u_2}$$

$$H_m = \eta_{mano} \times \frac{V_{w2} u_2}{g}$$

$$\text{Also } u_1 = \frac{\pi D_1 N}{60} \quad \text{and } u_2 = \frac{\pi D_2 N}{60}$$

Substituting above value in equation a

$$\frac{u_2^2 - u_1^2}{2g} = \eta_{mano} \times \frac{V_{w2} u_2}{g}$$

$$\begin{aligned} u_2^2 - u_1^2 &= 2 \eta_{mano} V_{w2} u_2 \\ \left(\frac{\pi D_2 N}{60} \right)^2 - \left(\frac{\pi D_1 N}{60} \right)^2 &= 2 \eta_{mano} V_{w2} u_2 \\ \left(\frac{\pi N}{60} \right)^2 (D_2^2 - D_1^2) &= 2 \eta_{mano} V_{w2} u_2 \\ \left(\frac{\pi N}{60} \right)^2 (D_2^2 - D_1^2) &= 2 \eta_{mano} V_{w2} \times \frac{\pi D_2 N}{60} \\ \left(\frac{\pi N}{60} \right) (D_2^2 - D_1^2) &= 2 \eta_{mano} V_{w2} D_2 \\ N &= \frac{2 \eta_{mano} V_{w2} D_2}{(D_2^2 - D_1^2)} \times \frac{60}{\pi} \end{aligned}$$

$$N_{min} = \frac{120 \eta_{mano} V_{w2} D_2}{\pi (D_2^2 - D_1^2)} \quad \text{-----(18)}$$

Performance characteristics of Centrifugal Pump

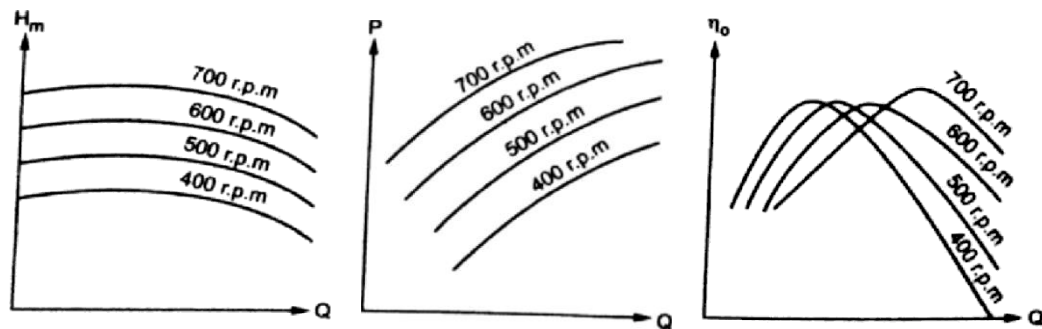
The following are the important characteristics curves of centrifugal pump.

- i. Main characteristics curves
- ii. Operating characteristics curves
- iii. Constant efficiency curves or Muschel curves
- iv. Constant head and constant discharge curves.

1. Main characteristics curves

The main characteristics curves are obtained by keeping the pump at constant speed and varying the discharge over desired range.

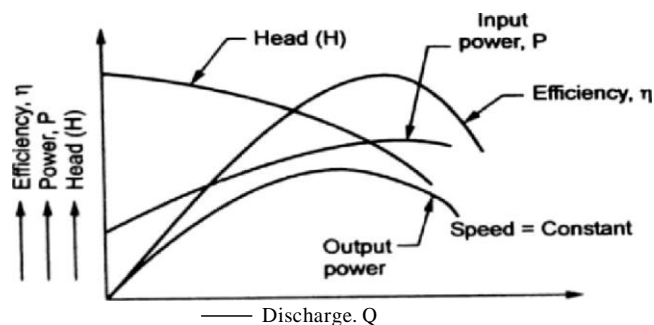
The discharge is varied by means of deliver valve. For different values of discharge the measurements are taken or calculated for manometric head, shaft power and efficiency. These curve are useful in evaluating the performance of pump at different speeds.



2. Operating characteristics curve

The maximum efficiency occurs when centrifugal pump operates at the constant designed speed.

If the speed is kept constant, the variation in manometric head power and efficiency with respect to discharge gives the operating characteristic curves for pump.



3. Constant efficiency curve

The constant efficiency or iso efficiency curve gives the performance of pump over its entire range of operations.

With the help of data obtained in main characteristic curves the constant efficiency curves are plotted.

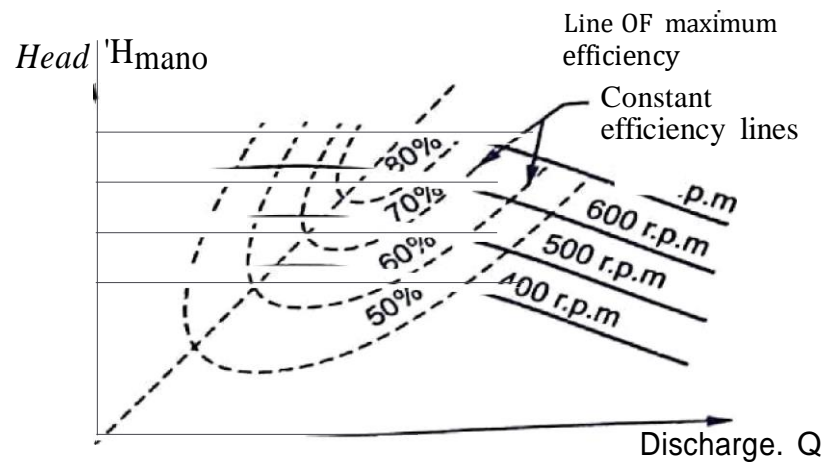


Fig. **Constant efficiency or Muschel curve**

4. Constant head and constant discharge curves

These curves are helpful in determining the performance of variable speed pump.

These curves are plotted as follows.

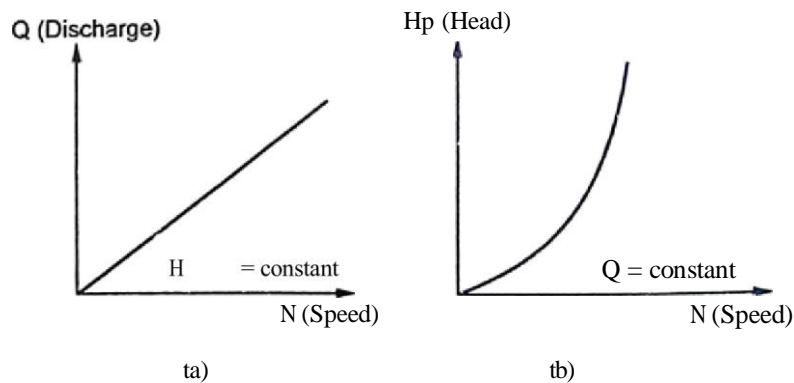


Fig. : (a) Q v/s N and (b) H_q v/s N curves of a centrifugal $P > P$

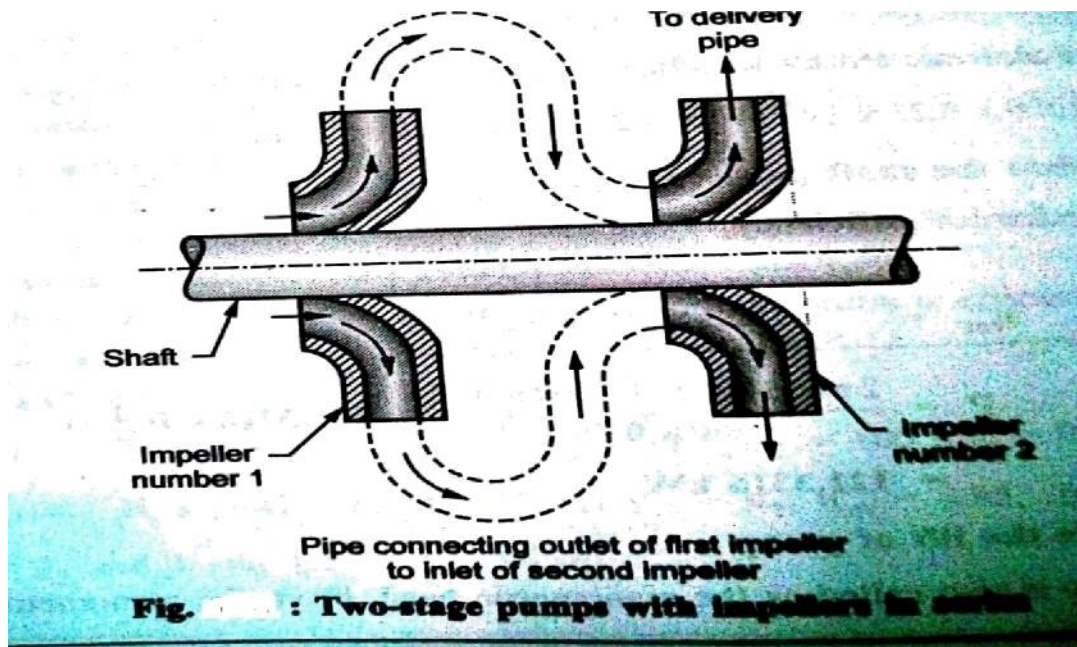
Multistage Centrifugal Pump

A multistage centrifugal pump consist of two or more identical impellers monted on the same shaft or on different shafts.

To produce the heads higher than that of using single impeller keeping the discharge constant. This is achieved by *Series arrangement of pumps*

To discharge the large quantity of fluid keeping the head constant. This is achieved by *parallel arrangement of piunps*.

Series Arrangement of Pumps



The discharge from first impeller having high pressure is fed to second impeller through guided passage.

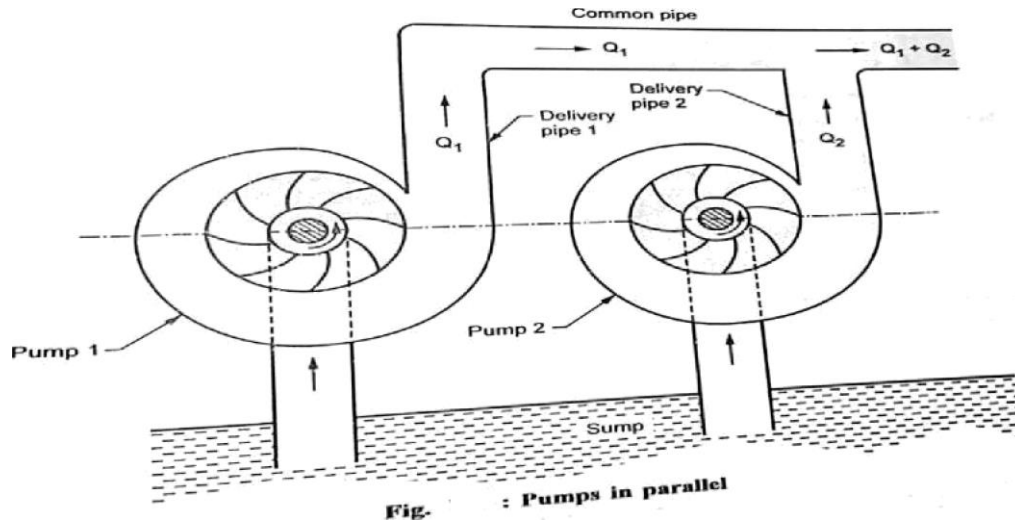
The pressure at the outlet of second impeller will be more than the pressure at the outlet of first impeller.

If the more number of impellers are mounted on the same shaft in series arrangement then the pressure will be increases further.

For each stage, the head developed will be H hence for number of stages (n) total head developed will be given by

$$H_{\text{total}} = n \times H_P$$

Parallel Arrangement of Pumps



To obtain a high discharge at relatively small head number of impellers are mounted in parallel arrangement.

The pumps are arranged such that each of these pump is working separately to lift the liquid from common sump and deliver it to the common delivery pipe

In this arrangement the head remains constant and the discharge of each pump gets added to give large quantity of liquid at the outlet

$$Q_{\text{total}} = Q_1 + Q_2 + \dots + Q_n$$

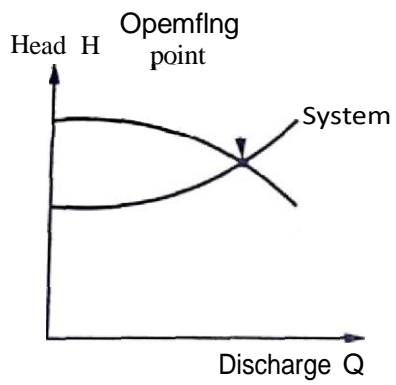
Selection pump based on system resistance curve

The pump manufacture always gives the head discharge characteristic curve for their manufactured pump and operated under different test conditions.

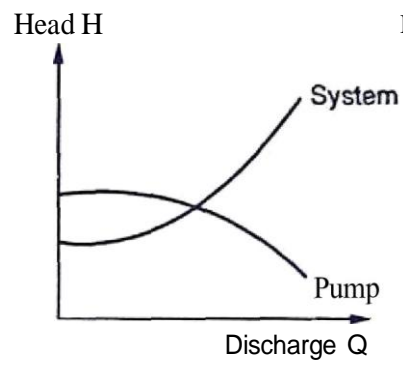
But in actual application this pump is required to operate under different conditions with respect to suction and discharge pipelines elbows and number of valves.

The user of the pump find out his system requirement and a head discharge curve is drawn.

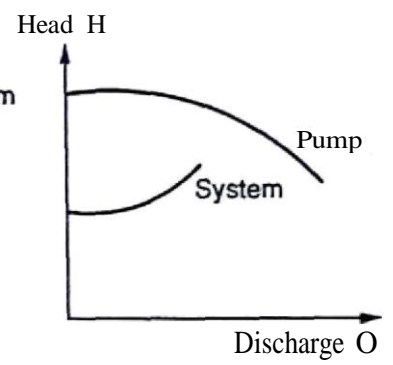
This curve is called as system resistance curve or system characteristic curve. As shown in following figure.



(a) System resistance curve



(b) Undersized pump



(c) Oversize
d pump

RECIPROCATING PUMP

- Reciprocating pump generally operates at low speeds and it is coupled to an electric motor with V-belts.
- The reciprocating pump is best suited for relatively small flow rate and high heads. In oil drilling operation this type of pump is very common.

MAIN COMPONENTS OF THE RECIPROCATING PUMP-

- Cylinder with a piston, piston rod, connecting rod, crank.
- Suction pipe.
- Delivery pipe
- Suction, delivery valve.

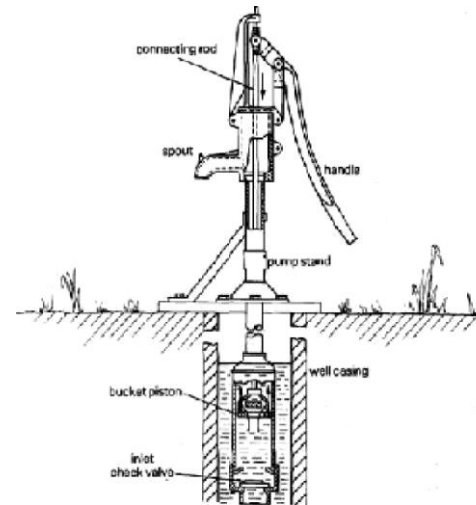
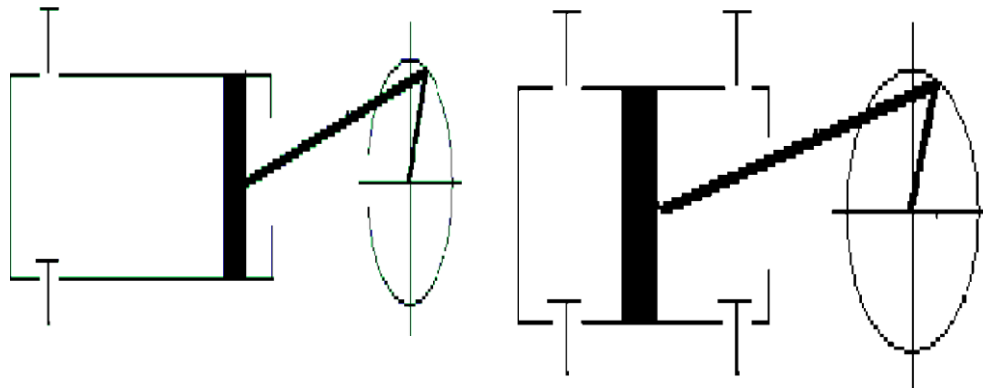
CLASSIFICATION OF RECIPROCATION PUMP

On the basis of water being in contact with one side or both sides of the piston.

(i) Single acting cylinder

(ii) Double acting cylinder.

If the water is in contact with one side of the piston, the pump is called single acting pump and if the water is in contact with both sides of the piston, the pump is called double acting pump.



CLASSIFICATION OF RECIPROCATION PUMP

According to the number of cylinders provided-

- Single cylinder pump
- Double cylinder pump
- Multi cylinder pump



Single cylinder pump:- A reciprocating pump having only one cylinder is known as single cylinder pump. It may be either single action or double action pump.

Double (Two) cylinder pump: - Pumps having more than one cylinders are known as multi cylinder pumps. Two pumps, three pumps, three throw pumps etc. having two or three single acting cylinders driven from cranks set at 180° , as shown in Fig a. or 120° , as shown in Fig b, their main advantage is more uniform discharge as compared with a single cylinder pump.

WORK DONE BY RECIPROCATING PUMP

Single acting pump:- In the single acting pump, as explained earlier, it has only one suction stroke and one delivery stroke for one revolution of the crank. It delivers the liquid only during the delivery stroke.

Hence, the flow rate of the liquid delivered per second.
$$\frac{LAN}{60} \quad \dots(1)$$

L = length of stroke = 2r

r = radius of stroke

A = Cross-section of cylinder

N = revolutions of crank per minute

The theoretical work done by the pump

$$W_{net} = \rho g Q (H_s + H_d) \quad \dots(2)$$

H_s = suction head

H_d = delivery head

$$W_{net} = \rho g \frac{LAN}{60} \left(H_s + \frac{H_d}{2} \right)$$

Double acting pump:- It has two suction and two delivery pipes connected to one cylinder.

$$Q = \frac{2LAN}{60} \text{ m}^3/\text{sec}$$

cross-section area of piston.

Force acting on piston in forward stroke

$$F_{Head} = \rho g H_s A + \rho g H_d A$$

Force acting on piston in backward stroke

$$F = \rho g (H_s + H_d) A$$

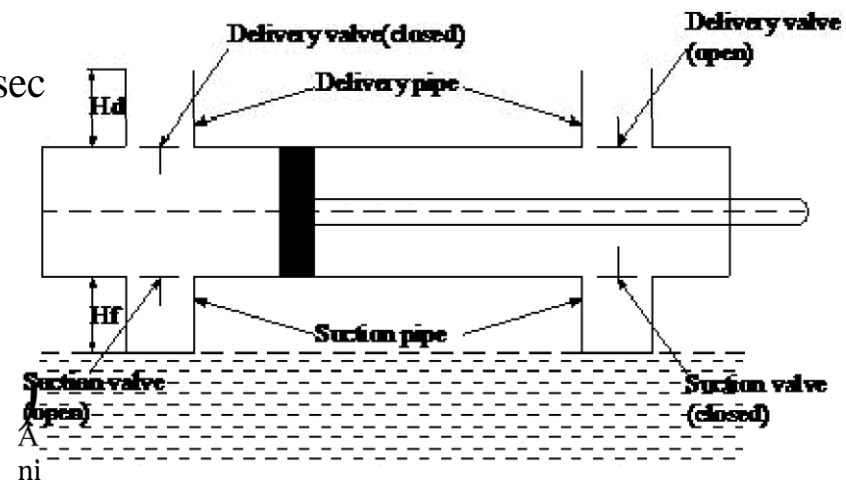


Fig. Double acting pump

$$F = \rho g (H_s + H_d) A$$

Power required to drive the pump

$$P = \rho g Q (H_s + H_d + H_f)$$

$$= \frac{\rho g 2LAN}{60} \frac{H_s + H_d}{d}$$

Two-throw pump:- In two-throw pump there are two cylinders with one suction and one delivery pipe.

The rate of flow through two throw pumps is $Q = \frac{2LAN}{60} \text{ sec}$

Three-throw pump:- It has three cylinders and three pistons working with three connecting rods fitted with one suction pipe and delivery pipe. The rate of flow through three-throw pump is

$$Q = \frac{3LAN}{60} \text{ /sec}$$

SLIP OF RECIPROCATING PUMP: Slip of pump is defined as the difference between the theoretical discharge (Q_{th}) and actual discharge

$$\text{Slip} = Q_{th} - Q_{act} \quad \text{Percentage Slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100$$

where C_d = Co-efficient of discharge

$$\text{Percentage slip} = (1 - C_d) \times 100$$

$$Q_{th} \times 100$$

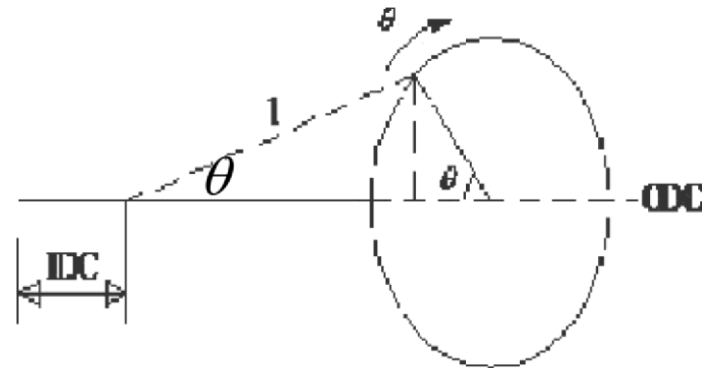
Negative Slip:- In most of the cases, the slip is positive. But in some cases, the actual discharge of the pump may be more than the theoretical discharge in which case C_d will be more than one and the slip will be negative, which is known as negative slip. This occurs in pumps having long suction pipe and low delivery head, especially when these are running at high speed.

This is due to the reason that the inertia pressure in the suction pipe becomes so large that it causes the delivery valve to open before the suction stroke is completed. Thus, some liquid is pushed directly into the delivery pipe even before the delivery stroke is commenced. This results in making the actual discharge more than the theoretical discharge.

CHANGE OF ACCELERATION AND VELOCITY IN RECIPROCATING PUMP

During the reciprocating motion of the piston, velocity of the piston is not uniform at all points. It is zero at ends and maximum at the centre. If the motion of piston is assumed simple harmonic, this assumption is only true when the connecting rod is very long as compared with the length of the crank. Suppose the crank is rotating with an angular velocity radians in its inner dead centre. Then

$$\theta = w.t. = \frac{2\pi Nt}{60}$$



If the X is the linear movement of the piston from the end of the stroke in t seconds, From Fig ,

If the velocity (v) and acceleration

Velocity of liquid in pipe

$$v = \frac{dx}{dt} = \omega r \sin \theta$$

Where, area of pipe = A

$$V = \left(\frac{A}{\alpha_p} \right) v = \frac{A}{\alpha_p} \omega r \sin \theta$$

cross-section area of piston

rate of delivery (v), proportional to \sin

Mass of water to be accelerated

Where L = length of the pipe

$$\alpha_c = \frac{dv}{dt} = r \omega^2 \cos \theta$$

$$= \left(\frac{A}{\alpha_p} \right) \omega r \sin \theta$$

$$\alpha_c = \frac{dv}{dt} = \left(\frac{A}{\alpha_p} \right) r \omega^2 \cos \theta$$

L

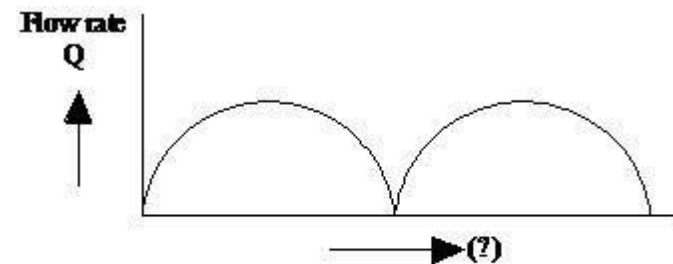


Fig. Flow rate

Intensity of pressure due to acceleration of liquid in the

pipe- $h_{as} = \frac{P_\alpha}{\rho g} = \rho \alpha_p L \times \frac{A}{\alpha_p} r \omega^2 \cos \theta$ (From Newton's law of motion)

$$P_\alpha = \rho L \frac{A}{\alpha_p} r \omega^2 \cos \theta$$

$$h_{as} = \frac{P_\alpha}{\rho g} = \rho L \times \frac{A}{\alpha_p / \rho g} r \omega^2 \cos \theta$$

$$h_{as} = \left(\frac{L}{g} \right) \left(\frac{A}{\alpha_p} \right) r \omega^2 \cos \theta$$

For suction side

$$h_i = \left(\frac{L_s}{g} \right) \left(\frac{A}{\alpha_{p \text{ suction pipe}}} \right) r \omega^2 \cos \theta$$

For delivery side

$$h_{\text{delivery}} = \left(\frac{L_d}{g} \right) \left(\frac{A}{\alpha_d} \right) r \omega^2 \cos \theta$$

At the beginning of each stroke

0 and

1

h_{as}

$$\left(\frac{L}{g} \right) \left(\frac{A}{\alpha_p} \right) r \omega^2$$

At middle of each stroke when $h = 0$

$$\theta = 180^\circ, \cos \theta = -1$$

$$h_{as} = \left(\frac{L}{g} \right) \left(\frac{A}{\alpha_p} \right) r \omega^2$$

when ratio L/r is not very large, simple harmonic motion cannot be assumed for the piston,

$$h_{as} = \left(\frac{L}{g} \right) \left(\frac{A}{\alpha_p} \right) r \omega^2 \cos \theta \left(\cos \theta \pm \frac{\cos 2\theta}{n} \right)$$

at the beginning of the stroke

$$h_{as} = \left(\frac{L}{g} \right) \left(\frac{A}{\alpha_p} \right) r \omega^2 \left(1 + \frac{1}{n} \right)$$

at the end of the stroke

$$h_{as} = \left(\frac{L}{g} \right) \left(\frac{A}{\alpha_p} \right) r \omega^2 \left(1 - \frac{1}{n} \right)$$

EFFECT OF CHANGE OF VELOCITY ON FRICTION IN THE SUCTION AND DELIVERY PIPES

Velocity of water in suction or delivery pipe

$$v = \frac{A}{\alpha_p} \omega \sin r \theta$$

Loss of head due to friction in pipes

$$h_f = \frac{4fLv^2}{2dg}$$

$$h_{f \text{ suction}} = \frac{4fL_s}{2dg} \left(\frac{A}{\alpha_p} r \omega \sin \theta \right)^2$$

$$h_{f \text{ delivery}} = \frac{4fL_d}{2dg} \left(\frac{A}{\alpha_p} r \omega \sin \theta \right)^2$$

Case I. At beginning
at middle

$$0, \sin 0 = 0$$

$$h_f = \frac{4fL}{d \times 2g} \times 0 = 0$$

Maximum value of loss of head due to friction

$$\theta = 90^\circ, \sin 90^\circ = 1$$

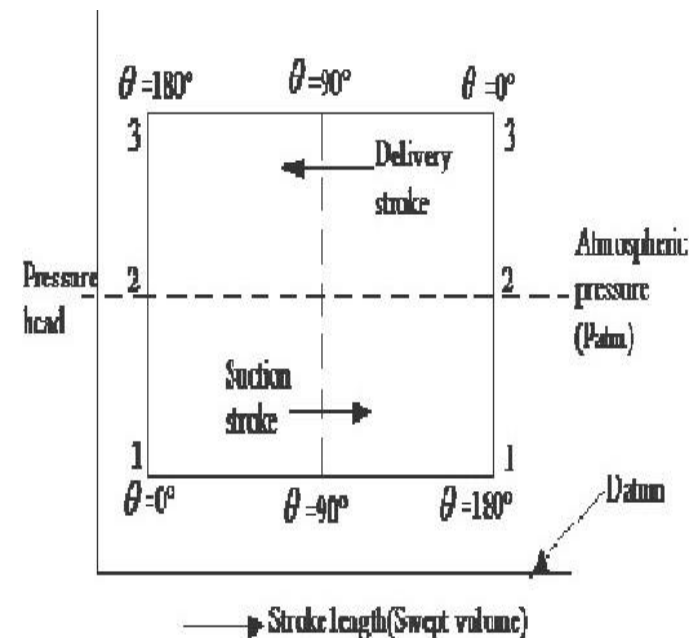
$$h_f = \frac{4fL}{2dg} \left[\frac{A}{\alpha_p} r\omega \right]^2$$

$$(h_{f \max}) = \frac{4fL}{2dg} \left[\frac{A}{\alpha_p} r\omega \right]^2$$

$$\theta = 180^\circ, \therefore h_f = 0$$

INDICATOR DIAGRAM

It is a graph which indicates the pressure head on the piston plotted along the vertical ordinate and the length of the stroke (swept volume is proportional to stroke length) along the abscissa for one complete revolution of crank.



EFFECT OF ACCELERATION ON INDICATOR DIAGRAM

The area represents total work done by the piston during one revolution of the crank.

The pressure head due to acceleration in the pipe is

$$h_{a\theta} = \frac{L}{g} \left(\frac{A}{\alpha_p} \right) r \omega^2 \cos \theta \quad \text{and} \quad h_{a\theta} = \frac{L}{g} \left(\frac{A}{\alpha_p} \right) r \omega^2 \cos \theta$$

At $\theta = 90^\circ$, $\cos \theta = 0$ and $h_a = 0$

$$\text{At } \theta = 180^\circ, \cos \theta = -1 \text{ and } h_{as} = -\frac{L}{g} \left(\frac{A}{\alpha_p} \right) r \omega^2$$

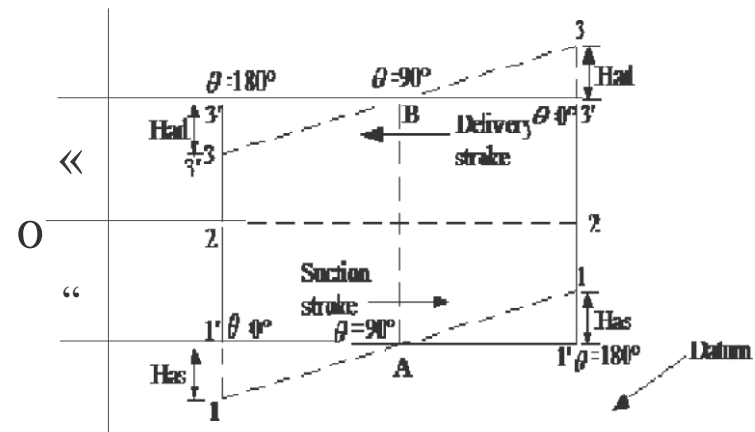
Pressure head inside the cylinder during suction stroke will not be equal to h_s , as was case for ideal indicator diagram, but it will be equal to the sum of h_e and h_{as}

At the beginning of suction stroke h_{as} is positive and hence total pressure head in cylinder will be $h_s + h_{as}$ below the atmospheric pressure head.

At the middle $\alpha = 90^\circ$ Of suction stroke and hence $h_{as} = 0$ and hence pressure head in the cylinder will be h_s below the atmospheric head.

At the end of the suction stroke $\alpha = 180^\circ$ and h_{as} is negative and hence the pressure head in the cylinder will be $h_s - h_{as}$ below the atmospheric pressure head. For suction stroke; the indicator diagram will be shown by 1A1, also the area of 1A1 = Area of 1'A1'.

Similarly, the indicator diagram for the delivery stroke can be drawn, at the beginning of delivery stroke, h_{ad} is +ve and hence the pressure head in the cylinder will be $(h_d + h_{ad})$ above the atmospheric pressure head.



At the middle of delivery stroke $h_{ad} = 0$ and hence at the middle pressure head in the cylinder is equal to h_d above the atmospheric pressure head. At the end of the delivery stroke h_{ad} is (-)ve and hence pressure in the cylinder will be $(h_d - h_{ad})$ above the atmospheric pressure head and hence indicator diagram for delivery stroke is represented by 3P—3' and also area of 3P—3' is $\frac{P}{2} \times \frac{h_d}{2}$.

Now due to acceleration in suction and delivery pipe, the indicator diagram has changed for 1'-1'-3'-3' to 1-1-3-3. But area of indicator diagram is same; hence work done by the pump remains same.

Effect of Pipe Friction on indicator Diagram

The indicator diagram may be further modified considering the effect of friction losses in the suction and delivery pipes. The loss of head due to friction in pipe is given by the equation.

$$h_{fp} = \frac{4fL_s}{2gd} \left(\frac{A}{\alpha_p} \right)^2 \sin^2 \theta + \frac{4fL_d}{2gd} \left(\frac{A}{\alpha_p} \right)^2 \sin^2 \theta$$

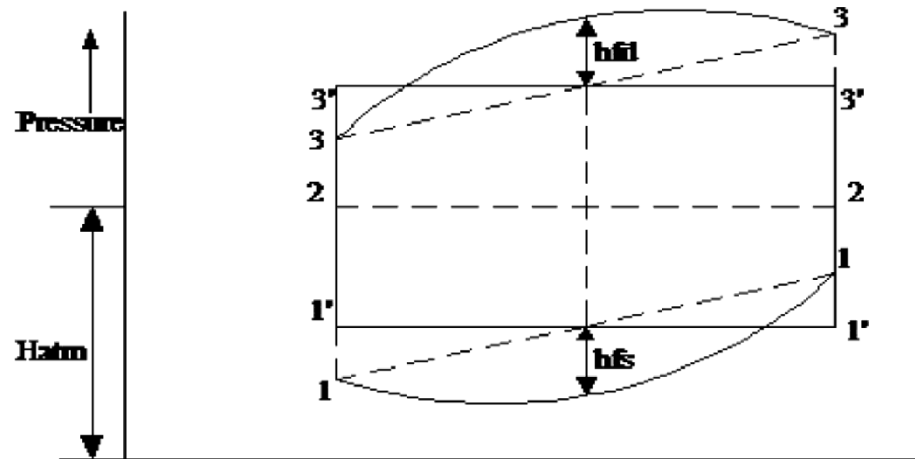
From the above equations it is clear that the variation of h_s with is parabolic. At the beginning and the end of each stroke when the head loss due to friction h_{p_s} and h_{i_d} will be equal to zero. At the middle of the stroke when $\sin B=1$ the head loss due to friction h_{p_s} will be maximum. The maximum values of h_e and

h_{i_d} are given as from the equation

$$h_d = \frac{4 f L_d \frac{A}{\rho} r m}{2 g d}$$

$$h_s; \frac{4 f L \frac{A}{\rho} r m^2}{2 g d}$$

The new indicator diagram developed considering the effect of friction along with the effect of piston acceleration is given below



CAVITATION

Cavitation is likely to occur at a point where the pressure of liquid is minimum, falling to the value at which dissolved gasses are liberated from the liquid. For the water the value of this limiting pressure is about 2.5m of water absolute below which the cavitation will start. This means that at any point during the suction stroke the head (//, 'la.) must not be greater than (10.3-2.5) = 7.8 m of water.

$$10.3 < H + H_p + 2.5 \quad 7.8 < H + H$$

$$H_a = zH, \quad H_y + H_{gp}$$

$$H_{sep} \quad s-l- 'OS$$

where H_{sep} is the pressure head below atmospheric pressure at which the separation and cavitation may occur

Cavitation is not likely to occur during delivery stroke, but if the length of delivery pipe is too long and the delivery head is small, we may get a net positive head at point 3 less than 2.6 m of water, cavitation will occur at the end of delivery stroke. The limiting condition is:

$$10.3 + h_d - i_d = 2.5$$

$$h_{ad} = 7.8 + h_d$$

The piping arrangement can be done by either of the two ways,
 In case (a) of above the delivery head h_d will become zero at the bend after which there is still a long horizontal pipe which will have a considerable value of accelerating head h_{ad} . Therefore, separation may take place at the bend if

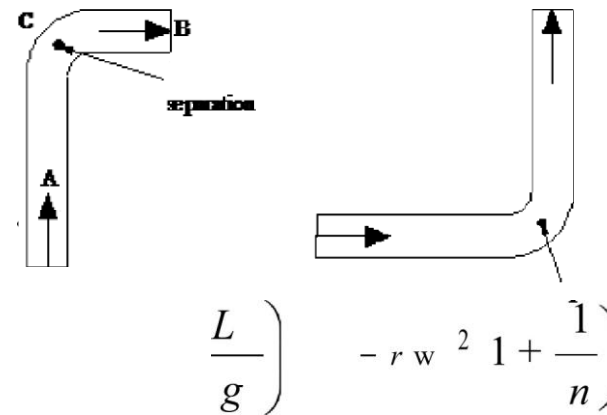
$$h_{ad} \text{ — } 7.7 \text{ m of water}$$

On the other hand, in the arrangement (b) the pipe is horizontal first and then it is vertical. As such is still considerable (h_d) available at the bend due to which there is

no possibility of separation to occur.

Maximum Speed-

$$H_{atm} < H_s + H_{as} + H_{sep} \quad h_a.$$



$$7.8 < \left(\frac{L}{g} \right) \left(\frac{v^2}{r} \right) \left(1 + \frac{1}{n} \right)$$

$$7.8 < (H + H_{SE})$$

$H_a + H_{as}$ should be less than 7.8m of water when the $H_a + H_{as}$ is more than 7.8m of water. Vacuum, the vapors are formed and separation occurs.

- H_s - suction head = constant for a particular pump installation means H_{as} is the dependent factor for avoiding

the cavitations.

$$DUTI_{aX} = \frac{L}{g} + \frac{A}{p} + 2 \quad 0 \text{ and } i \text{ — } i$$

flow

D = diameter of the cylinder

d_p = diameter of the suction pipe L_s

l = length of suction stroke

r = crank radius

N = speed

$$(H_{as}) = \left(\frac{L}{g} + \frac{A}{p} + 2 \right) \left(\frac{\pi}{4} d_p^2 \right) r (2\pi N)^2$$

From the equation H_{is} depends upon Lay r, m . In order to

d
 limit L_s the pump should not be installed away from the sump from
 which the water has to be drawn D will be seen that
 the cylinder bore is not much bigger than the suction pipe diameter.
 Considering all the L_s , d , D and L constant for a particular pump, the
 only variable will be its speed N . Since the
 value of H_{as} is limited, the speed of reciprocating pump is also
 restricted. Thus the maximum permissible speed can be found if H_{as} is
 known.

AIR VESSELS

Air vessel is a closed cast iron chamber having an opening at the bottom which is connected to suction or delivery pipe. The top portion of the vessel contains compressed air.

Functions:-

- Reduce the possibility of separation in suction pipe.
- Length of suction pipe below the air vessel can be increased.
- Pump can run at higher speed.
- On the delivery side constant rate of flow.
- Large amount of power can be saved in supplying accelerating head

Working: An air vessel in a reciprocating pump acts like a fly wheel of an I.C. Engine. The top of the vessel contains compressed air which can contract or expand to absorb most of the pressure fluctuations. When the pressure increases, water in excess of mean discharge is forced into the air vessel, thereby compressing the air therein. When the water pressure in pipe falls, the compressed air ejects the excess water out which means air vessel acts like an intermediate reservoir on suction side, the water first accumulates here and is then transferred to cylinder of the pump. On delivery side, water first goes to air vessel and then goes to delivery pipe. Water flows in the pipe continuously.

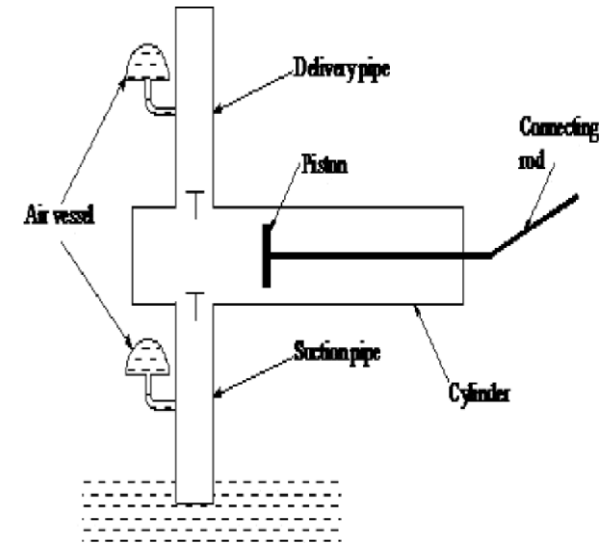


Fig. Air vessel

RATE OF FLOW OF LIQUID WITH AIR VESSEL:

Consider a single cylinder single-acting pump. The mean discharge from the pump is given by $(Q)_d = \frac{LAN}{60} - \frac{Ax}{2r} \frac{m}{r} \frac{Arm}{r}$

The instantaneous flow rate to or from the cylinder of the pump is given as (considering the flow in the suction or delivery pipe) $Q_d = \frac{A}{2r} \frac{m}{r} \frac{Arm}{r} \sin \theta$

Hence, the net discharge at any time into or from the air vessel will be the difference of the above two flow rates. Therefore, the rate of flow of

liquid into the air vessel, $Q_{Si} = \frac{Arm}{r} \sin \theta$

If equation is positive, it means that liquid is flowing into the air vessel and if it is negative, the liquid is flowing from the air vessel and when there is no flow of liquid into or from the air vessel, the above equation equals to zero.

That is

$$\frac{Arm}{r} \sin \theta = 0$$

$$\sin \theta = 0.3185.$$

This gives two values of B , i.e. $B = 18^\circ 34'$ or $161^\circ 26'$

At these positions of crank angles the discharge is equal to mean discharge.

Considering a double-acting pump, the mean discharge from the pump is

instantaneous discharge to or from the cylinder of the pump is

$$Q = \frac{2zlr}{\pi} (\omega \sin \theta) \quad (Qd)_{mean} = \frac{2As}{\pi}$$

Power of liquid into or from the air vessel

$$zlr \omega \sin \theta = \frac{2zlr \omega}{\pi} \left(\sin \theta - \frac{\omega^2}{2} \right)$$

WORK DONE SAVED AGAINST FRICTION WITH AIR VESSEL

By providing air vessels, the fluctuations in the velocity of flow in suction and delivery pipes are eliminated with results in reducing the head frictional losses in the pipes and thus certain amount of energy is saved. It is assumed that air vessel is fitted very near to pump cylinder and loss of head due to friction in the small portion of pipe between the pump and air vessel is negligible. The velocity of flow in the pipe beyond air vessel is uniform and equal to the mean velocity therefore

the power lost in friction per second is given by

$$m.A.(2r) \left(\frac{4,^{\circ}F}{2gd} \right) \frac{A}{a} \frac{rni}{z}$$

Power lost in friction per stroke when there is no air vessel

$$ni.A.\$2r\left[\begin{array}{cc}2 & 4j\epsilon \text{ K}^2 \\ 3 & 2gd\end{array}\right] - (mA/2r) \left[\begin{array}{c}2 \\ -x \\ 3\end{array}\right] \left(\begin{array}{c}4jC \\ 2gd\end{array}\right) \left(\begin{array}{c}A g \\ a^r\end{array}\right)^2 \left[\begin{array}{c}4 j'' L \text{ K}^2\end{array}\right]$$

Power saved by fitting air vessel

$$\omega A (2r) \left[\left(\frac{4fL}{2gd} \right) \left(\frac{A}{r} \right)^2 \left(\frac{2}{3} - \frac{1}{\pi^2} \right) \right]$$

The percentage of the power saved due to fitting of air vessel

$$= \left[\frac{\left(\frac{2}{3} - \frac{1}{\pi^2} \right)}{\frac{2}{3}} \right] \times 100 = 84.8$$

For a double-action pump, the discharge becomes double, and

p

$$\omega A (2r) \left[\left(\frac{4fL}{2gd} \right) \left(\frac{A}{r} \right)^2 \left(\frac{2}{3} - \frac{4}{\pi^2} \right) \right]$$

Percentage of the power saved

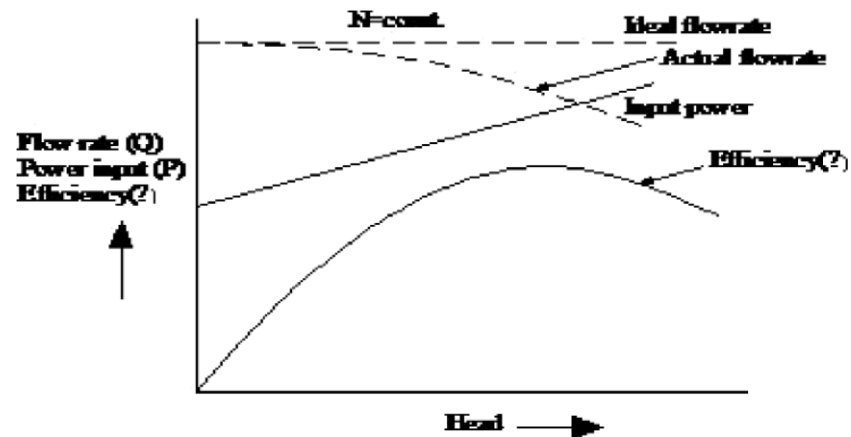
$$= \left[\frac{\left(\frac{2}{3} - \frac{4}{\pi^2} \right)}{\frac{2}{3}} \right] \times 100 = 39.2$$

CHARACTERISTIC CURVES OF RECIPROCATING PUMP

Curves are obtained by plotting discharge, power input and overall efficiency against the head developed by the pump when it is operating at a constant speed.

Under the ideal condition flow rate of reciprocating pump operating at constant speed is independent of the head developed by the pump, but in actual practice observed that the flow rate of reciprocating pump slightly decreases as the head developed by the pump increases.

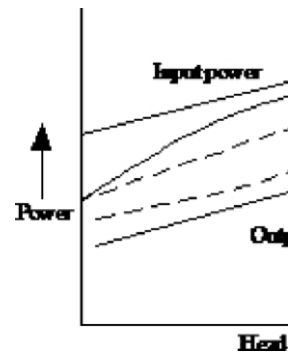
Input power for a reciprocating pump increases almost linearly beyond a certain minimum value with increase in the head developed by the pump. The overall efficiency of a reciprocating pump also increases with the increase in the head.



These losses comprise of (i) mechanical losses (ii) leakage losses

(iii) hydraulic losses, when the pump runs at constant speed and supplier discharge at uniform rate under varying heads, the hydraulic losses remain. Substantially unchanged but the mechanical and leakage losses, both increase as the head on the pump increases.

Sometimes the reciprocating pumps are required to run at variable speed. Seed, discharge curve for a reciprocating pump is shown in Fig (c). It is observed that discharge varies linearly with the



speed as given in equation.

$$Q \propto \frac{LAN}{QaN}$$



1.60

d for deep well or submersible pumps.