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DEPARTMENT OF MECHANICAL ENGINEERING

SUBJECT:-DESIGN OF MECHINE ELEMENTS

(TH-2)

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SEMESTER – 5TH

CONTENTS:

Chapter 1
Introduction

Introduction to Machine Design and classify:

The subject Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time consuming one. From the study of existing ideas, a new idea has to be conceived. The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings. In the preparation of these drawings, care must be taken of the availability of resources in money, in men and in materials required for the successful completion of the new idea into an actual reality. In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing.

Classifications of Machine Design

1.Development design.

This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

2.New design.

This type of design needs lot of research, technical ability and creative thinking. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design. The designs, depending upon the methods used, may be classified as follows:

(a) Rational design. This type of design depends upon mathematical formulae of principle of mechanics.

(b) Empirical design. This type of design depends upon empirical formulae based on the practice and past experience.

(c) Industrial design. This type of design depends upon the production aspects to manufacture any machine component in the industry.

(d) Optimum design. It is the best design for the given objective function under the specified constraints. It may be achieved by minimizing the undesirable effects.

(e) System design. It is the design of any complex mechanical system like a motor car.

(f) Element design. It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.

(g) Computer aided design. This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimization of a design.

Factors considerations in designing a machine component:

1. Type of load and stresses caused by the load.

   The load, on a machine component, may act in several ways due to which the internal stresses are set up. The various types of load and stresses are discussed later.

2. Motion of the parts or kinematics of the machine. The successful operation of any machine depends largely upon the simplest arrangement of the parts which will give the motion required

3. Selection of materials. It is essential that a designer should have a thorough knowledge of the properties of the materials and their behaviour under working conditions. Some of the important characteristics of materials are: strength, durability, flexibility, weight, resistance to heat and corrosion, ability to cast, welded or hardened, machinability, electrical conductivity, etc

4. Form and size of the parts. The form and size are based on judgment. The smallest practicable cross-section may be used, but it may be checked that the stresses induced in the designed cross-section are reasonably safe. In order to design any machine part for form and size, it is necessary to know the forces which the part must sustain
5. Frictional resistance and lubrication. There is always a loss of power due to frictional resistance and it should be noted that the friction of starting is higher than that of running friction. It is, therefore, essential that a careful attention must be given to the matter of lubrication of all surfaces which move in contact with others, whether in rotating, sliding, or rolling bearings.
6. Use of standard parts. The use of standard parts is closely related to cost, because the cost of standard or stock parts is only a fraction of the cost of similar parts made to order. The standard or stock parts should be used whenever possible; parts for which patterns are already in existence such as gears, pulleys and bearings and parts which may be selected from regular shop stock such as screws, nuts and pins.
7. Safety of operation. Some machines are dangerous to operate, especially those which are speeded up to insure production at a maximum rate. Therefore, any

moving part of a machine which is within the zone of a worker is considered an accident hazard and may be the cause of an injury.

8. Number of machines to be manufactured. The number of articles or machines to be manufactured affects the design in a number of ways. The engineering and shop costs which are called fixed charges or overhead expenses are distributed over the number of articles to be manufactured.
9. Cost of construction. The cost of construction of an article is the most important consideration involved in design. In some cases, it is quite possible that the high cost of an article may immediately bar it from further considerations. If an article has been invented and tests of handmade samples have shown that it has commercial value.
10. Assembling. Every machine or structure must be assembled as a unit before it can function. Large units must often be assembled in the shop, tested and then taken to be transported to their place of service.

General Procedure in Machine Design

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways. However, the general procedure to solve a design problem is as follows:

1. Recognition of need. First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.

2. Synthesis {Mechanisms). Select the possible mechanism or group of mechanisms which will give the desired motion.

3. Analysis of forces. Find the forces acting on each member of the machine and the energy transmitted by each member.

4. Material selection. Select the material best suited for each member of the machine.

5. Design of elements {Size and Stresses). Find the size of each member of the machine by considering the force acting on the member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform than the permissible limit.

6. Modification. Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.

7. Detailed drawing. Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.

Production.

The component, as per the drawing, is manufactured in the workshop. The flow chart for the general procedure in machine design is shown in Fig.

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modification

Detailed drawanp*

. General Machine Design Procedure

Engineering materials and their properties:

The knowledge of materials and their properties is of great significance for a design engineer. The machine elements should be made of such a material which has properties suitable for the conditions of operation. In addition to this, a design engineer must be familiar with the effects which the manufacturing processes and heat treatment have on the properties of the materials. Now, we shall discuss the commonly used engineering materials and their properties in Machine Design.

Classification of Engineering Materials

The engineering materials are mainly classified as:

1. Metals and their alloys, such as iron, steel, copper, aluminum, etc.

2. Non-metals, such as glass, rubber, plastic, etc.

Physical Properties of Metals

The physical properties of the metals include luster, colour, size and shape, density, electric and thermal conductivity, and melting point. The following table shows the important physical properties of some pure metals.

Mechanical Properties of Metals

The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load. These mechanical properties of the metal include strength, stiffness, elasticity, plasticity, ductility, brittleness, malleability, toughness, resilience, creep and hardness. We shall now discuss these properties as follows:

1. Strength. It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is called stress.

Stiffness. It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.

3. Elasticity. It is the property of a material to regain its original shape after deformation when the external forces are removed. This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber.

4. Plasticity. It is property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.

5. Ductility. It is the property of a material enabling it to be drawn into wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper, aluminium, nickel, zinc, tin and lead.

6. Brittleness. It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Brittle materials when subjected to tensile loads snap off without giving any sensible elongation. Cast iron is a brittle material.

7. Malleability. It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it is not essential to be so strong. The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper and aluminium.

8. Toughness. It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed upto the point of fracture. This property is desirable in parts subjected to shock and impact loads.

9. Machinability. It is the property of a material which refers to a relative case with which a material can be cut. The machinability of a material can be measured in a number of ways such as comparing the tool life for cutting different materials

10. Resilience. It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume within

elastic limit. This property is essential for spring materials.

11. Creep. When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called creep. This property is considered in designing internal combustion engines, boilers and turbines.

12. Fatigue. When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as *fatigue. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. This property is considered in designing shafts, connecting rods, springs, gears, etc.

Hardness. It is a very important property of the metals and has a wide variety of meanings. It embraces many different properties such as resistance to wear, scratching, deformation and machinability etc. It also means the ability of a metal to cut another metal.

Stress-Strain Curves

Properties are quantitative measure of materials behavior and mechanical properties pertain to material behaviors under load. The load itself can be static or dynamic. A gradually applied load is regarded as static. Load applied by a universal testing machine upon a specimen is closet example of gradually applied load and the results of tension test from such machines are the basis of defining mechanical properties. The dynamic load is not a gradually applied load — then how is it applied. Let us consider a load P acting at the center of a beam, which is simply supported at its ends. The reader will feel happy to find the stress (its maximum value) or deflection or both by using a formula from Strength of Materials. But remember that when the formula was derived certain assumptions were made. One of them was that the load P is gradually applied. Such load means that whole of P does not act on the beam at a time but applied in instalments.

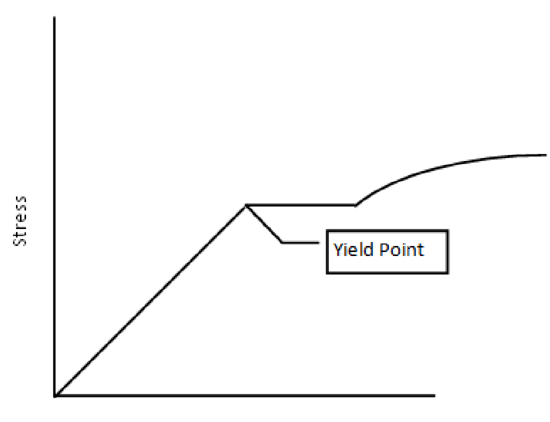
Working Stress:

Working Stress. When designing machine parts, it is desirable to keep the stress lower than the maximum or ultimate stress at which failure of the material takes place. This stress is known as the working stress or design stress. It is also known as safe or allowable stress.

Yield Stress:

Stress and strain are directly related to each other: as one increases, the other increases as well. So, the more stress that an object experiences, the more it deforms until the object fails.

All objects will begin experiencing elastic deformation at first, but once the stress on the object exceeds a certain amount, it will experience plastic deformation. When that switch happens, the object has reached its yield stress.



Typical stress-strain diagram for steel

Ultimate stress.

The stress, which attains its maximum value is known as
ultimate stress. It is defined as the largest stress obtained by dividing the largest value of the load reached in a test to the original cross-sectional area of the test piece.

Factor of Safety.

It is defined, in general, as the ratio of the maximum stress to
the working stress. Mathematically,

Factor of safety = Maximum stress/Working or design stress

In case of ductile materials e.g. mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress. In such cases,

Factor of safety = Yield point stress/Working or design stress

In case of brittle materials e.g. cast iron, the yield point is not well defined as for ductile materials. Therefore, the factor of safety for brittle materials is based on ultimate stress.

Factor of safety = Ultimate stress/Working or design stress

This relation may also be used for ductile materials.

Note: The above relations for factor of safety are for static loading

Chapter 2

Design of Fastening Element

Fasteners:

It is a Mechanical Joints which is used to become a fixed / attaches to something or holds something in place.

Types of Fasteners

Classification of Fasteners

- Permanent
  - Welding
  - Brazing
  - Stapling
  - Nailing
  - Gluing
  - riveting
- Temporary
  - Screws
  - Bolts
  - Keys
  - Pins

The Fastenings may be classified into the following two groups:

1. The Permanent Fastenings are those fastenings which cannot be disassembled without destroying the connecting components.
2. The Temporary or Detachable Fastenings are those fastenings which can be disassembled without destroying the connecting components.

The 5 basic welding joints are:

1. Butt joint
2. Corner joint
3. Lap joint
4. Tee joint and
5. Edge joint

ADVANTAGES OF WELDING JOINTS

- As no hole is required for welding, hence no reduction of area. So structural members are more effective in taking the load.

- In welding filler plates, gusseted plates, connecting angles etc, are not used, which leads to reduced overall weight of the structure.
- Welded joints are more economical as less labor and less material is required.
- The efficiency of welded joint is more than that of the riveted joint.
- The welded joints look better than the bulky riveted/butted joints.
- The speed of fabrication is faster in comparison with the riveted joints.
- Complete rigid joints can be provided with welding process.
- The alternation and addition to the existing structure is easy.
- No noise is produced during the welding process as in the case of riveting.

Design of Welded joints for eccentric loads:

In the design of welded structures, shape and length of the weld are defined by the contours of the structure parts and their sizes. The cross section of the fillet weld has the form of the right triangle. The destruction of weld occurs along the weld throat. From the strength calculation we find the throat thickness of the fillet weld and determine the legs of the triangle. When designing a new, eccentrically loaded joint, the stress at the critical point of the weld depends on the position of the centre of mass of the calculated cross-section of the weld. The coordinates of the centre of mass are defined by the shape and size of the cross-section including the unknown throat thickness of the weld. To solve the problem, the leg value is assigned based on the recommendations, experience or intuition, and a test calculation is performed.

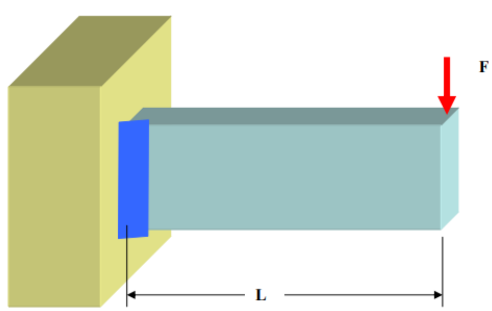
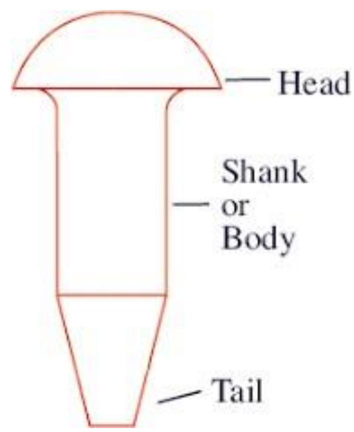


Figure 11.2.1: Eccentrically loaded welded joint

Riveted joint:

The riveted joint is a permanent joint cause rivet is a permanent mechanical fastener. A rivet is a cylindrical shaft that has a head on one end and the opposite end known as a tail.



Types of Riveted joints:

1. Lap Joint:
   In this type the ends carrying the drillings of the two members are positioned such that their surfaces slightly overlap. The riveting is then done through the coincident holes
2. Butt Joint:
   Here, the two members or the elements are linked edge to edge in one straight line. The clamping is produced using an external cover plate which is then riveted as above through the parallel drilled holes.
3. Single Riveted Joint:
   In this type more than one rivet are fixed along a single row typically in a lap joint, while in a butt joint the rows may appear from both the upper and the lower surfaces.
4. Double Riveted Joint:
   When two rows of rivets are included over a lap joint or when two rows of rivets are utilized from both top and bottom in a butt joint are referred to as double butt joint.

Types of Rivets:

Following are the different types of Rivets:

- Snap head or cup head rivets
- Pan head rivets
- Conical head rivets
- Countersunk head rivets
- Flathead rivets
- Buffercated head rivet
- Hollow head rivets.

Failure of Riveted joints:

A riveted joint may fail in the following ways

1. Tearing of the plate at an edge
2. Shear failure of the rivets
3. Tensile failure of rivet across the row of rivets
4. Crushing failure of plate
5. Shear failure of plate at the marginal area.

1. Tearing of the plate at an edge

   . A joint may fail due to tearing of the plate

   at an edge as shown in Fig. This can be avoided by keeping the margin, m = 1.5d,where “d” is the diameter of the rivet hole.

2. Tearing of the plate across a row of rivets.

   Due to the tensile stresses

   in the main plates, the main plate or cover plates may tear off across a row of rivets as shown in Fig. In such cases, we consider only one pitch length of the plate, since every rivet is responsible for that much length of the plate only

3. Shearing of the rivets.

   The plates which are connected by the rivets exert tensile stress on the rivets, and if the rivets are unable to resist the stress, they are sheared off.

4. Crushing of the plate or rivets.

Sometimes, the rivets do not actually shear off under the tensile stress, but are crushed as shown in Fig. Due to this, the rivet hole becomes of an oval shape and hence the joint becomes loose.

Strength of a Riveted Joint

The strength of a joint may be defined as the maximum force, which it can transmit, without causing it to fail. Pt , Ps and Pc are the pulls required to tear off the plate, shearing off the rivet and crushing off the rivet

Strength of the riveted joint = Least of Pt, Ps and Pc

Efficiency of a Riveted Joint

The efficiency of a riveted joint is defined as the ratio of the strength of riveted joint to the strength of the un-riveted or solid plate.

Strength of the riveted joint = Least of Pt, Ps and Pc

Strength of the un-riveted or solid plate per pitch length, $P = p \times t \times \sigma t$

∴ Efficiency of the riveted joint,

η =Least of pt, ps & pc / $p \times t \times \sigma t$

where, p = Pitch of the rivets, t = Thickness of the plate, and σt = Permissible tensile stress of the plate material.

Problems1.

Find the size and length of the fillet weld for the lap joint to transmit a factored load of 120 kN as shown in Fig. 41. Assume site welds, Fe 410 grade steel and E41 electrode. Assume width of plate as 75 mm and thickness as 8 mm

Solution

Minimum size of weld for 8 mm thick section = 3 mm (Table 5, Cl. 10.5.2.3)

Maximum size of weld = 8 – 1.5 = 6.5 mm (Cl. 10.5.8.1)

Choose the size of weld, a = 6 mm

Effective throat thickness = te= 0.70 a = 4.2 mm

Strength of 6 mm weld / mm length = 4.2 x 410 / (√3 x 1.5)Cl. 10.5.7.1.1= 662.7 N/mm

Assuming only two longitudinal welds along the sides Required length of weld = 120 x 103/662.7 = 181 mm Length to be provided on each side = 181/2 = 90.5 mm> 75 mm (width of plate)Hence, provide 90.5 mm weld on each side with an end return of 2x 6 = 12 mm Overall length of the weld provided = 2 x (90.5 + 2 x 6) = 205 mm

1. A double riveted lap joint with zig-zag riveting is to be designed for 13 mm thick plates. Assume σt = 80 MPa ; τ = 60 MPa ; and σc = 120 MPa. State how the joint will fail and find the efficiency of the joint.

Solution:Nature of Joint: double riveted lap joint with zig-zag riveting

Given data : t = 13 mm ; σt = 80 MPa = 80 N/mm2; τ = 60 MPa = 60 N/mm2; σc = 120 MPa =

120 N/mm2

1. Diameter of rivet

Since the thickness of plate is greater than 8 mm,

therefore diameter of rivet hole, d = 6√ t = 6 √ 13 = 21.6 mm

From Design Data Hand book, DDHB Table,

we find that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard size of the

rivet hole (d) is 23 mm and the corresponding diameter of the rivet is [Refer DDHB, K.

Mahadevan & K.Balveerareddy]

d-= 22 mm

Pitch of rivets

Let p = Pitch of the rivets.

Since the joint is a double riveted lap joint with zig-zag riveting [See Fig.], therefore

there are two rivets per pitch length, i.e. n = 2.

Also, in a lap joint, the rivets are in single shear.

We know that tearing resistance of the plate,

$P_t = (p - d)\, t \times \sigma_t = (p - 23)\, 13 \times 80 = (p - 23)\, 1040$ N ....(i)

and shearing resistance of the rivets, $P_s = n \times \pi/4\, d^2$

$\times \tau$

$= 2 \times \pi/4\, (23)^2$

60 = 49 864N ……………..(ii) ...

(There are two rivets in single shear)

From equations (i) and (ii),

$p - 23 = 49864/1040 = 48$ or

$p = 48 + 23 = 71$ mm

The maximum pitch is given by, $p_{max} = C \times t + 41.28$ mm

$\therefore p_{max} = 2.62 \times 13 + 41.28 = 75.28$ mm

Since p_{max} is more than p, therefore we shall adopt p = 71 mm Ans.

3. Distance between the rows of rivets

We know that the distance between the rows of rivets (for zig-zag riveting),

$p_b = 0.33\, p + 0.67\, d = 0.33 \times 71 + 0.67 \times 23$ mm = 38.8 say 40 mm Ans.

4. Margin We know that the margin,

$m = 1.5\, d = 1.5 \times 23 = 34.5$ say 35 mm Ans.

Failure of the joint

Now let us find the tearing resistance of the plate, shearing resistance and crushing resistance of

the rivets. WKT,

Tearing resistance of the plate,

$P_t = (p - d)\, t \times \sigma_t = (71 - 23)13 \times 80 = 49\,920$ N

Shearing resistance of the rivets,

$P_s = n \times \pi/4 \times d^2$

$\times \tau = 2 \times \pi/4\, (23)^2$

$60 = 49\,864$ N

and crushing resistance of the rivets,

$P_c = n \times d \times t \times \sigma_c = 2 \times 23 \times 13 \times 120 = 71\,760$ N

The least of P_t, P_s and P_c is $P_s = 49\,864$ N.

Hence the joint will fail due to shearing of the rivets.

Strength of the un riveted plate per pitch length

$P = p \times t \times \sigma_t = 71 \times 13 \times 80 = 73840$ N

∴ Efficiency of the joint, =Least of P t

Ps and Pc /p= 49 864 / 73840

Efficiency of the joint = 0.675 or 67.5%

Chapter 3

Design of Shaft and Keys

Shaft

A shaft is defined as a rotating machine element, usually circular in cross-section, which is used to transmit power from one part to another, or from a machine that produces power to a machine that absorbs power.

Material for Shaft

The material used for ordinary shafts is mild steel. When high strength is required, alloy steel such as nickel, nickel-chromium, or chromium-vanadium steel is used. Shafts are generally formed by hot rolling and finished to size by cold drawing or turning and grinding.

The material used for the shafts must have the following properties:

- It should have high strength.
- It should have good mechanization.
- It should have a low-notch sensitivity factor.
- It should have good heat treatment properties.
- It should have high wear-resistant properties.

The materials used for regular shafts are carbon steel of grade 40 C8, 45 C8, 50 C4, and 50 C12.

design of shaft

The shafts may be designed on the basis of

1. Strength, and 2. Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered:

(a) Shafts subjected to twisting moment or torque only,

(b) Shafts subjected to bending moment only,

(c) Shafts subjected to combined twisting and bending moments, and

(d) Shafts subjected to axial loads in addition to combined torsional and bending loads.

Case (a) Shafts subjected to twisting moment or torque only

- When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft 4 may be obtained by using the torsion equation. We know that -

(i)

where T' - Twisting moment (or torques acting upon the shaft,

J = Polar moment of inertia of the shaft about axis of rotation,

i = Torsional shear seess, and

r — Distance from neutral axis to the outer most fibre d12, where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inenia,

J u x //4

32

Ftom this equation, we may determine the diameter of round solid shaft [d .

We also know that for hollow shaft, polar moment of inertia,

$$J = \pi/32[(d_0)^4 - (d_i)^*]$$

where

d. and d; = Outside and inside diameter of the shaft, and r - d / 2.

where d_o and d_i = Outside and inside diameter of the shaft, and r - d / 2.

Substituting these values in equation (i), we have

$$\frac{T}{(d_o)^4 - d_i^4} \quad \frac{d_o}{2} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right]$$

Let k -- Ratio of inside diameter and outside diameter of the :

$= d_i / d_o$

Now the equation (iii) may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4)$$

From the equations (iii) or (iv), the outside and inside diameter of a hollow shaft may be determined.

It may be noted that 5

1. The hollow shafts are usually used iii marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft.

When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same, In other words, for the same material of both the shafts,

π $\left[(d_o)^4-(d_i)^4\right]$ π

16 fig' 16x x d'

($d0$ $(d)'$

—-é or $(dp)(1-£^4)=d'$

dq

2. The twisting mcimerit {T1 may be obtained by using the following relation :

We know that the power transmitted (in watts) by the shaft,

P ———— or F' 2 N

where $T\ N \times 2$ / 60

T —Twisting moment in N-m, and

N -- Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment *(T)* is given by

wheve $T>$ T_i T $)R$

pi •fld *Th* —— Tensions in the tight side and slack side of the belt respectively, and

fi = Radius of the pulley.

Case (b) Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$\cdot M$ σ_b

M Bending moment, =Moment of inertia of cross-sectional area of the shaft about the axis of rotation,

£fb —— Bending stress, and

y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation *(i)*, we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft *(d)* may be obtained.

We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4(1 - k^4) \quad \text{...(where } k = d_i / d_o\text{)}$$

$$y = d_o / 2$$

Again substituting these values in equation (i), we have

$$\frac{M}{\frac{\pi}{64}(d_o)^4(1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

From this equation, the outside diameter of the shaft (d_o) may be obtained.

Case (c) Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view :

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.
2. Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let z = Shear stress induced due to twisting moment, and

ab — Bending stress (tensile or compressive) induced due to bending moment.

Problem on shafts

1.A line shaft rotating at 200 rpm is to transmit 20 kW power. The allowable shear stress for the shaft material is 42 N/mm2. Determine the diameter of the shaft.

Solution:

Given : N = 200 r.p.m. ;P = 20 kW = 20 × 103 W; τ= 42 MPa = 42 N/mm2

Let d = Diameter of the shaft.

We know that torque transmitted by the shaft,

= P∗ 60

2πN 3

= 20 ∗ 10 ∗60 2π∗ 200

= 5 N-m

= 955 × 103 N-mm

We also know that torque transmitted by the shaft (T),

955 × 103 = π *

τ*d3

16

d3 = 955 × 103 / 8.25

= 115 733 or d

= 48.7 say 50 mm

2.A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and an ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.

Solution.

Given :

M = 3000 N-m = 3 × 106 N-mm ;

T = 10 000 N-m = 10 × 106 N-mm ;

σtu = 700 MPa = 700 N/mm2 ;

τu = 500 MPa = 500 N/mm2

We know that the allowable tensile stress,

σt or σb = σtu

FOS = 700/6

= 116.7 N/mm2

and allowable shear stress,

τ = τ u

FOS

500/6

= 83.3 N/mm2

Let d = Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

Te = √M^2 + T^2

= √(3 × 106)2 + (10 × 106)2

= 10.44 × 106 N-mm

We also know that equivalent twisting moment (Te),

10.44 × 106 = π * τ*d3=16

d3 = 10.44 × 106 / 16.36 = 0.636 × 106 or

= 86 mm ans

Key

A key is a machine element which is used to connect a shaft to other machine elements like Gears, Pullies, Couplings, Sprockets or Flywheels. Its primary function is to prevent relative motion between connected parts.

Functions of the a key

It has two basic functions

- Transmit torque from the shaft to hub and from hub to the shaft
- Prevent relative motion of the shaft and the joined machine element

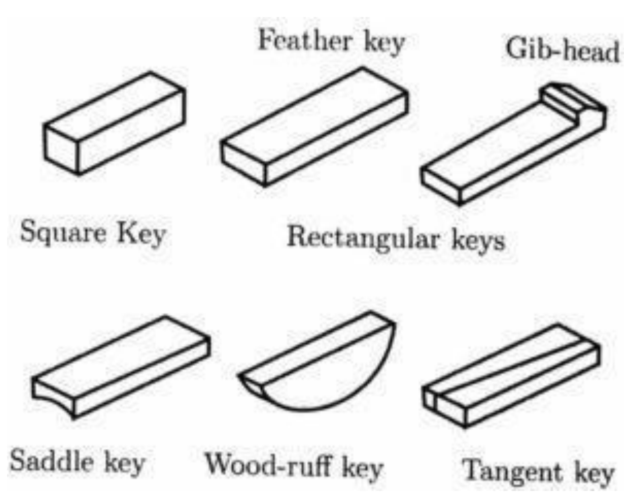
Types of keys

They can be classified on the basis of following factors.

- Cost
- Rigidity of connection
- Tightness of fit
- Power to be transmitted

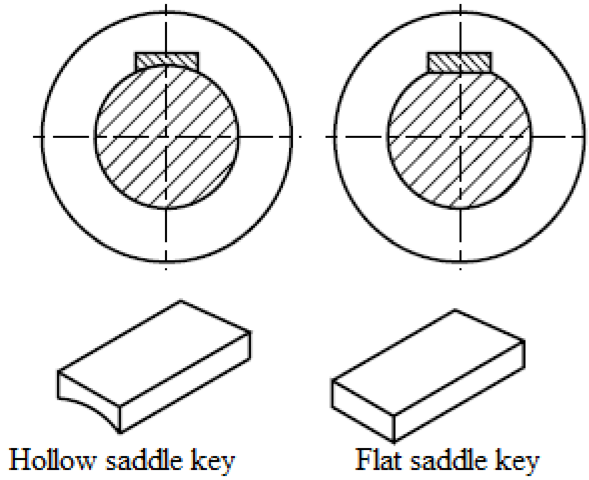
On the basis of above factors, we have following types of keys

- Key with and without Gib head
- Parallel and taper key
- Flat and square key
- Sunk and saddle key



1. Saddle keys

A saddle is a key which fits in the keyway of the hub only.



Advantage of saddle key

- Since keyway is to be made only in the hub hence its cost is low.

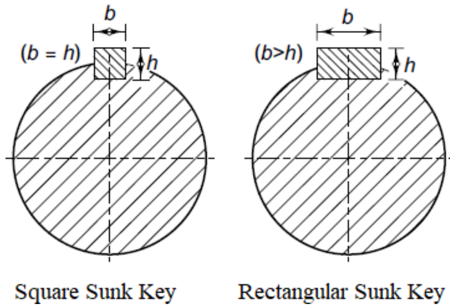
Disadvantage of saddle key

- Power transmission capacity is low

2. Sunk key

A sunk key is a key in which half the thickness of key fits into the keyway of hub and remaining half remains in the keyway of shaft.

- A square key has square cross section.
- A rectangular key has rectangular cross section.



Advantage of sunk key

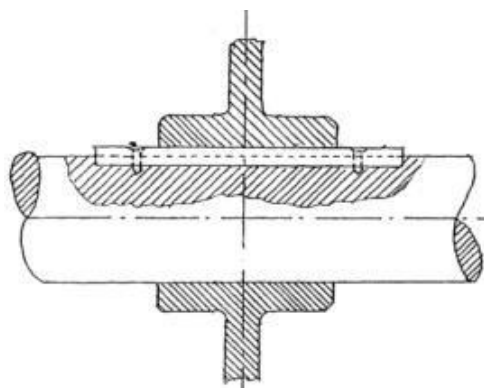
- Power transmission capacity is high compared to saddle key.

Disadvantage of sunk key

- Since keyway is to be made in both shaft and hub hence its cost is high.

3. Feather key

A feather key is a parallel key which allows relative axial movement between shaft and hub. These can be used with gears or clutches.



4. Woodruff key

Woodruff key is a sunk key in the form of almost semi circular disc of uniform thickness.

Advantage of woodruff key

- It can tilt and align itself according to the shaft
- Extra depth of key in the shaft prevents slip of shaft and hub

Disadvantage of woodruff key

- Due to extra depth of keyway, shaft has low strength
- No axial movement is permitted between shaft and hub

5. Tangent Key

A tangent key is composed of two tapered keys to ensure tight fit. They are used in heavy duty equipment.

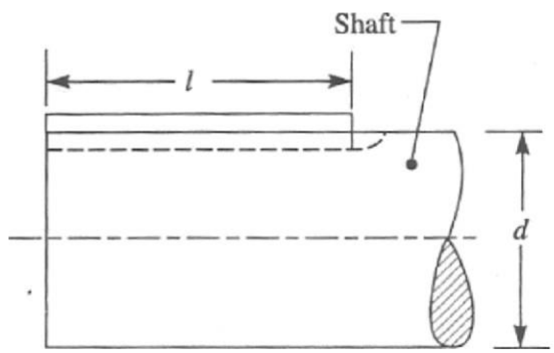
Desiqn of Sunk Key

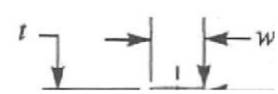
- When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of 14 forces act on the key :

1. Forces (F1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.
2. Forces (Fj due to the torque transmitted by the shaft. These forces produce shearing and

compressive (or crushing) stresses in the key.

In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.





Let

T — Torque transmitted by the shaft,

F — Tangential force acting at the circumference of the shaft,

d — Diameter of shaft,

I — Length of key, yr = Width of key.

I = Thickness of key, and

oc — Shear and crushing stresses for the material of key.

The usual proportions of this key are

Width of key, w = d/4 thickness of key, I = d/6 Where

and

d = Diameter of the shaft or diameter of the hole in the hub.

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

In ordel' to find the length of the key to transmit t'ull power of the shat't, the shearing strength of the key is equal to the torsi¿inal shear strength ‹it the shaft.

We kn‹ u thai the shearing strength of key,

T — / x i˜ i 2 ...(iv)

and torsional sheai stren ¿u th of the shaft,

$T = \frac{\pi}{16} \times \tau_1 \times d^3$ ...(v)

(Tak ing 1 - Shear stress for the shaft material)

From c°quatinns (i›') anal (t'), e havc

/ x ›+' X 2 — — 16 — X I x & '

× w × τ 2 x $\frac{*i}{\tau}$ = 1.5 71 d x (Taking w = l/4)...(vi)

When the ke ' material is same as that ‹)f the shaft, then T = 1, .

1.571 d I equation (vi)

Effect of keywayQfi shaft

- A little consideration will show that the keyway cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft.
- It other words, the torsional strength of the shaft is reduced.
- The following relation for the weakening effect of the keyway is based on the experimental results by H.P. Moore.

$$e \quad 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right)$$

where e = Shaft strength factir. It is the ratio of the strength of the shaft 'ith keyway to the strength of lhe same shaft without keyway,

ii Width of keyway,

d —— Diameter' of shaft, and

li — Depth tit keyw'ay $\frac{\text{ThickneSS Of key fi)}}{2}$

- It is usually assumed that the strength of the keyed shaft is 75% of the solid shaft, which

Case 1 Failure of key due to shearing

- Considering shearing of the key, the relation between tangential shearing force ,area resisting shearing and shear stress is 15

shear stress , == I——

Therefore , tangential force is

F = I x w x

Torque transmitted by the shaft,

d d

2 2

Case 2 Failure of key due to crushing

- Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

— Area resisting crushing x Crushing stress = *l* r 2 X a,

Torque transmitted by the shaft,

d t d

T——FX2 !x2 x o, x ()

Key is equally strong in shearing and crushing

The key is equally strong in shearing and crushing if

$l \times w \times \tau \times \frac{}{2} \times \frac{}{2} \times \sigma_c \times \frac{}{2}$ (Equating equations (i) and (ii))

r 2a

The permissible crushing stress for the usual key material is at least twice the permissible shearing stress.

Therefore from equation (iii), we have w = t.

In other words, a square key is equally strong in shearing and crushing.

Chapter 4

Design of Coupling

Shaft Coupling

Shafts are usually available up to 7 metres length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

Shaft couplings are used in machinery for several purposes, the most common of which are the following :

To provide for the connection of shafts of units that are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.

- To provide for misalignment of the shafts or to introduce mechanical flexibility.
- To reduce the transmission of shock loads from one shaft to another.
- To introduce protection against overloads.
- It should have no projecting parts.

Requirements Of A Good Shaft Coupling

- It should be easy to connect or disconnect.
- It should transmit the full power from one shaft to other shaft without losses.
- It should hold the shaft in perfect alignment.
- It should reduce transmission of shock loads.
- It should have no projecting parts.

Types of shaft coupling:

1. Rigid coupling.

   It is used to connect two shafts which are perfectly aligned. Following types of rigid coupling are important from the subject point of view:

   - Sleeve or muff coupling.
   - Clamp or split-muff or compression coupling, and
   - Flange coupling.

2. Flexible coupling.

   It is used to connect two shafts having both lateral and angular misalignment. Following types of flexible coupling are important from the subject point of view

- Bushed pin type coupling,
- Universal coupling, and
- Oldham coupling.

S/eeve or muff coupling.

• It is the simplest type of rigid coupling, made of cast iron.

• It consists of a hollow cylinder whose inner diameter is the same as that of the shaft.

• It is fitted over the ends of the two shafts by means of a gib head key, as shown in following Fig. The power is transmitted from one shaft to the other shaft by means of a key and a sleeve. It is, therefore, necessary that all the elements must be strong enough to transmit the torque.

Design Procedure

Step 1. Design of Shaft .

•Generally power transmitted by shaft is given , hence first of all find torque transmitted by shaft as

P=2 TiNT/6s

p — 6s

T = ... N-m

T =........ x 103 N-mm

Now as per torsion equation ,

T ?!- « d3

From above equation find

(d)

Step 2. Proportions of sleeve

- The usual proportions of a cast iron sleeve coupling are as follows Outer diameter of the sleeve, D = 2 d + 13 mm

  and length of the sleeve, L = 3.5 d

Where,

d is the diameter of the shaft.

Step 3. Design of Key

- The usual proportions for rectangular key are

  Width of key, w = d/4 and

  thickness of key, t = d/6

Where , d = Diameter of the shaft or diameter of the hole in the hub.

- And for square key proportions are

Width of key, w = d/4and 19

thickness of key, t = d/4

Where , d = Diameter of the shaft or diameter of the hole in the hub.

The length of the coupling key is atleast equal to the length of the syeve (i.e. , 3.5 d).

The coupling key is usually made into two parts so that the length of the key in each shaft,

L/2=3.5d/2

After fi xing the length of kcy in each shaft, the inciuced shearing and crushing stresses may be checked. We know that torque transmitted,d

T—/ H w H 1 X 2 (Considering shearing of the key)

2 x a, x d

(Considering crushing of the key)

Step 4. Design of sleeve

•The sleeve is designed by considering it as a hollow shaft.

Let T - Torque to be transmitted by the coupling, and

i = Permissiblc shear stress for the material of the sleeve which is castrion. The safe value of shear stress for cast iron may be taken as 14 MPa.

We know' that torquc transmitted by a hollow section,

— d" n

T--16 D 16 X D (-I k4) . ("." I —— d I D)

From this expression, the induced shear stres5 in the sleeve may be checked.

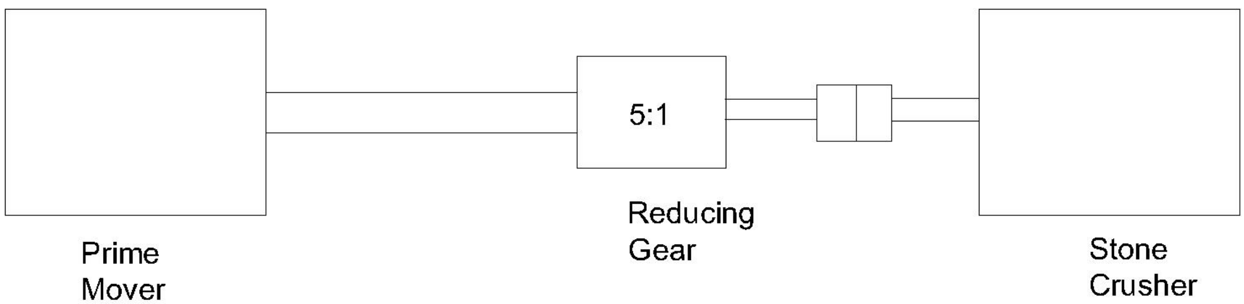
Design of the compression coupling:

- The number of bolts may be two, four or six. The nuts are recessed into the bodies of the muff castings.
- This coupling may be used for heavy duty and moderate speeds.
- The advantage of this coupling is that the position of the shafts need not be changed for assembling or disassembling of the coupling.
- The usual proportions of the muff for the clamp or compression coupling are :
  Diameter of the muff or sleeve, D = 2d 13 mm
  Length of the muff or sleeve, L = 3.5 d

where d = Diameter of the shaft.

In the clamp or compression coupling, the power is transmitted from one shaft to the other by means of key and the friction between the muff and shaft. In designing this type of coupling, the following procedure may be adopted.

1.Design a flexible coupling of pin-bush types construction for connecting reduction gear shaft to a stone crushes shaft. The unit is driven by 30kw & 720 rpm motor through 5:1 reduction. Choose suitable materials and the design stresses for the parts of coupling.



i=n1/n2

n2=144rpm

Selection of standard coupling:from PSG 7.108

Rated power = KW power application×servicefactor×100RPM of application

In this problem, consider motor as a prime mover to drive stone crusher

service factors for considered driven and driven combination is selected from PSG 7.109

Rated power at 100 rpm= 30×2.5×100/144=52.08KW

from PSG 7.108

Question :Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.

Solution.

1. Design for shaft

Let d = Diameter of the shaft.

We know that the torque transmitted by the shaft, key and muff,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{40 \times 10^3 \times 60}{2\,\pi \times 350} = 1100 \text{ N-m}$$

$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore \quad d^3 = 1100 \times 10^3 / 7.86 = 140 \times 10^3$ or $d = 52$ say 55 mm **Ans.**

2. Design for sleeve

We know that outer diameter of the muff,

D = 2d + 13 mm = 2 × 55 + 13 = 123 say 125 mm Ans.

and length of the muff,

L = 3.5 d = 3.5 × 55 = 192.5 say 195 mm Ans.

Let us now check the induced shear stress in the muff. Let τ_c be the induced shear stress in the muff which is made of cast iron. Since the muff is considered to be a hollow shaft, therefore the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125} \right]$$

$$= 370 \times 10^3 \, \tau_c$$

$$\therefore \quad \tau_c = 1100 \times 10^3 / 370 \times 10^3 = 2.97 \text{ N/mm}^2$$

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm 2, therefore the design of muff is safe.

3. Design for key

From Table 13.1, we find that for a shaft of 55 mm diameter,

Width of key, w = 18 mm Ans.

Since the crushing stress for the key material is twice the shearing stress, therefore a square key may be used.

Thickness of key, t = w = 18 mm Ans.

We know that length of key in each shaft,

l = L / 2 = 195 / 2 = 97.5 mm Ans.

Let us now check the induced shear and crushing stresses in the key. First of all, let us consider shearing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3 \, \tau_s$$

$$\therefore \quad \tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$$

Now considering crushing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3 \, \sigma_{cs}$$

$$\therefore \quad \sigma_{cs} = 1100 \times 10^3 / 24.1 \times 10^3 = 45.6 \text{ N/mm}^2$$

Since the induced shear and crushing stresses are less than the permissible stresses, therefore the design of key is safe.

Question:Design a clamp coupling to transmit 30 kW at 100 r.p.m. The allowable shear stress for the shaft and key is 40 MPa and the number of bolts connecting the two halves are six. The permissible tensile stress for the bolts is 70 MPa. The coefficient of friction between the muff and the shaft surface may be taken as 0.3.

Solution.

1. *Design for shaft*

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{30 \times 10^3 \times 60}{2\pi \times 100} = 2865 \text{ N-m} = 2865 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted by the shaft (T),

$$2865 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 2865 \times 10^3 / 7.86 = 365 \times 10^3$ or $d = 71.4$ say 75 mm **Ans.**

2. *Design for muff*

We know that diameter of muff,

$$D = 2d + 13 \text{ mm} = 2 \times 75 + 13 = 163 \text{ say } 165 \text{ mm } \textbf{Ans.}$$

and total length of the muff,

$$L = 3.5\, d = 3.5 \times 75 = 262.5 \text{ mm } \textbf{Ans.}$$

3. *Design for key*

The width and thickness of the key for a shaft diameter of 75 mm (from Table 13.1) are as follows :

Width of key, w = 22 mm **Ans.**

Thickness of key, t = 14 mm **Ans.**

and length of key = Total length of muff = 262.5 mm **Ans.**

4. *Design for bolts*

Let d_b = Root or core diameter of bolt.

We know that the torque transmitted (T),

$$2865 \times 10^3 = \frac{\pi^2}{16} \times \mu (d_b)^2 \sigma_t \times n \times d = \frac{\pi^2}{16} \times 0.3\, (d_b)^2\ 70 \times 6 \times 75 = 5830 (d_b)^2$$

$\therefore$ $(d_b)^2 = 2865 \times 10^3 / 5830 = 492$ or $d_b = 22.2$ mm

From Table 11.1, we find that the standard core diameter of the bolt for coarse series is 23.32 mm and the nominal diameter of the bolt is 27 mm (M 27). **Ans.**

Chapter 5

Design a closed coil helical spring

Spring

Springs are mechanical devices that can store potential energy because of their elasticity. The term elasticity refers to a property of materials that reflects their tendency to return to their original shape and size after having been subjected to a force that causes deformation after that force has been removed. The basic notion underlying the operation of springs is that they will always attempt to return to their initial size or position whenever a force is applied which changes their size, whether that be forces which are from compression, extension, or torsion.

Helical spring:

A helical spring is a coiled mechanical device which stores and releases energy to absorb impacts or shock and to resist either compression or pulling forces between objects. It is typically cylindrically shaped and features varying numbers of coils according to its intended use. The wires used to manufacture helical springs are generally specially tempered after their construction to give the spring its compression characteristics.

Terms used in Compression Springs

1. Solid length. When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be solid. The solid length of a spring is the product of total number of coils and the diameter of the wire. Mathematically,

Solid length of the spring,

$L_S = n'.d$

where n' = Total number of coils, and

d = Diameter of the wire.

2. Free length. The free length of a compression spring, as shown in Fig., is the length of the spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils (when fully compressed). Mathematically,

Free length of the spring,

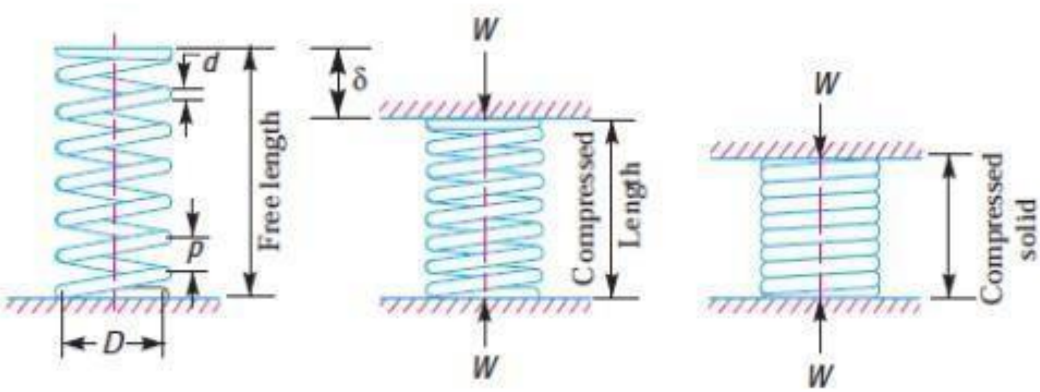
$$L_F = \text{Solid length} + \text{Maximum compression} + {}^*\text{Clearance between adjacent coils (or clash allowance)}$$

$$= n'.d + \delta_{max} + 0.15\,\delta_{max}$$

The following relation may also be used to find the free length of the spring, *i.e.*

$$L_F = n'.d + \delta_{max} + (n' - 1) \times 1 \text{ mm}$$

In this expression, the clearance between the two adjacent coils is taken as 1 mm.



3. Spring index. The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Mathematically,

Spring index, C = D / d

where D = Mean diameter of the coil, and

d = Diameter of the wire.

4. Spring rate. The spring rate (or stiffness or spring constant) is defined as the load required per unit

deflection of the spring. Mathematically, Spring rate, $k = W / \delta$

where W = Load, and

δ = Deflection of the spring.

5. Pitch. The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically,

$$\text{Pitch of the coil,} \quad p = \frac{\text{Free length}}{n' - 1}$$

The pitch of the coil may also be obtained by using the following relation, *i.e.*

$$\text{Pitch of the coil,} \quad p = \frac{L_F - L_S}{n'} + d$$

where LF = Free length of the spring,

LS = Solid length of the spring,

n' = Total number of coils, and

d = Diameter of the wire.

Stress in helical spring of circular wire

Consider a helical compression spring made of circular wire and subjected to an axial load W, as shown in Fig.

Let D = Mean diameter of the spring coil,

d = Diameter of the spring wire,

n = Number of active coils,

G = Modulus of rigidity for the spring material,

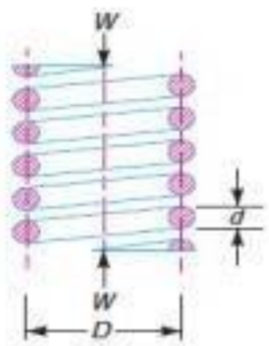
W = Axial load on the spring,

τ = Maximum shear stress induced in the wire,

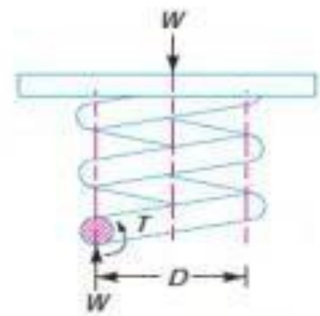
C = Spring index = D/d,

p = Pitch of the coils, and

δ = Deflection of the spring, as a result of an axial load W.



(a) Axially loaded helical spring.



(b) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

Now consider a part of the compression spring as shown in Fig (b). The load W tends to rotate the wire due to the twisting moment (T) set up in the wire. Thus torsional shear stress is induced in the wire. A little consideration will show that part of the spring, as shown in Fig (b), is in equilibrium under the action of two forces W and the twisting moment T. We know that the twisting moment,

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\tau_1 = \frac{8 W.D}{\pi\, d^3}$$

The torsional shear stress diagram is shown in Fig (a).

In addition to the torsional shear stress ($\tau 1$) induced in the wire, the following stresses also act on the wire :

1. Direct shear stress due to the load W, and
2. Stress due to curvature of wire

We know that direct shear stress due to the load W,

$$\tau_2 = \frac{\text{Load}}{\text{Cross-sectional area of the wire}}$$

$$= \frac{W}{\frac{\pi}{4} \times d^2} = \frac{4W}{\pi\, d^2}$$

We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \pm \tau_2 = \frac{8W.D}{\pi\, d^3} \pm \frac{4W}{\pi\, d^2}$$

The *positive* sign is used for the inner edge of the wire and *negative* sign is used for the outer edge of the wire. Since the stress is maximum at the inner edge of the wire, therefore

Maximum shear stress induced in the wire,

= Torsional shear stress + Direct shear stress

$$= \frac{8W.D}{\pi\, d^3} + \frac{4W}{\pi\, d^2} = \frac{8W.D}{\pi\, d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8\,W.D}{\pi\,d^3}\left(1 + \frac{1}{2C}\right) = K_S \times \frac{8\,W.D}{\pi\,d^3} \qquad \text{...}(iii)$$

... (Substituting $D/d = C$)

where K_S = Shear stress factor $= 1 + \dfrac{1}{2C}$

From the above equation, it can be observed that the effect of direct shear $\left(\dfrac{8\,WD}{\pi\,d^3} \times \dfrac{1}{2C}\right)$ is appreciable for springs of small spring index C. Also we have neglected the effect of wire curvature in equation (*iii*). It may be noted that when the springs are subjected to static loads, the effect of wire curvature may be neglected, because yielding of the material will relieve the stresses.

In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl's stress factor (K) introduced by A.M. Wahl may be used. The resultant diagram of torsional shear, direct shear and curvature shear stress is shown in Fig. 23.11 (d).

∴ Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} = K \times \frac{8\,W.C}{\pi\,d^2} \qquad \text{...}(iv)$$

where $K = \dfrac{4C - 1}{4C - 4} + \dfrac{0.615}{C}$

Deflection of Helical Springs of Circular Wire

Total active length of the wire,

$$l = \text{Length of one coil} \times \text{No. of active coils} = \pi D \times n$$

Let θ = Angular deflection of the wire when acted upon by the torque T.

$\therefore$ Axial deflection of the spring,

$$\delta = \theta \times D/2 \qquad \ldots(i)$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

$$\therefore \quad \theta = \frac{T.l}{J.G} \qquad \ldots\left(\text{considering } \frac{T}{J} = \frac{G.\theta}{l}\right)$$

where J = Polar moment of inertia of the spring wire

$= \dfrac{\pi}{32} \times d^4$, d being the diameter of spring wire.

and G = Modulus of rigidity for the material of the spring wire.

Now substituting the values of l and J in the above equation, we have

$$\theta = \frac{T.l}{J.G} = \frac{\left(W \times \dfrac{D}{2}\right)\pi D.n}{\dfrac{\pi}{32} \times d^4 G} = \frac{16 W.D^2.n}{G.d^4} \qquad \ldots(ii)$$

Substituting this value of θ in equation (i), we have

$$\delta = \frac{16 W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8\,W.D^3.n}{G.d^4} = \frac{8\,W.C^3.n}{G.d} \qquad \ldots(\because C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8\,D^3.n} = \frac{G.d}{8\,C^3.n} = \text{constant}$$

Surge in Springs

When one end of a helical spring is resting on a rigid support and the other end isloaded suddenly, then all the coils of the spring will not suddenly deflect equally,because some time is required for the propagation of stress along the spring wire.A little consideration will show that in the beginning, the end coils of the spring incontact with the applied load takes up whole of the deflection and then ittransmits a large part of its deflection to the adjacent coils. In this way, a wave ofcompression propagates through the coils to the supported end from where it isreflected back to the deflected end. This wave of compression travels along thespring indefinitely. If the applied load is of fluctuating type as in the case of valvespring in internal combustion engines and if the time intervalbetween the loadapplications is equal to the time required for the wave to travelfrom one end tothe other end, then resonance will occur. This results in verylarge deflections ofthe coils and correspondingly very high stresses. Under these conditions, it is justpossible that the spring may fail. This phenomenon is called surge. It has been found that the natural frequency of spring should be atleasttwenty times the frequency of application of a periodic load in order to avoidresonance with all harmonic frequencies upto twentieth order.

The natural frequency for springs clamped between two plates is given by:

$$f_n = \frac{d}{2\pi D^2 . n} \sqrt{\frac{6\,G.g}{\rho}} \text{ cycles/s}$$

where d = *Diameter of the wire,*

D = *Mean diameter of the spring,*

n = *Number of active turns,*

G = *Modulus of rigidity,*

g = *Acceleration due to gravity, and*

ρ = Density of the material of the spring.

The surge in springs may be eliminated by using the following methods:

1. By using friction dampers on the centre coils so that the wave propagation dies out.
2. By using springs of high natural frequency.
3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

Question:

A closely-coiled helical spring is made of 10 mm diameter steel wire, with the coil consisting of 10 turns with a mean diameter 120 mm. The spring carries an axial pull of 200 N. What is the value of shear stress-induced in the spring neglecting the effect of stress concentration and of deflection in the spring, when the modulus of rigidity is 80 kN/mm^2?

Solution:

Concept:

Shear stress developed in the spring is:

$$\tau_{max} = \frac{8PD}{\pi d^3}$$

Deflection of spring

$$\delta = \frac{8PD^3N}{Gd^4}$$

here d is wire diameter of spring, D is mean coil diameter, P is axial spring force, N is the number of active coils.

Calculation:

Given,

d = 10 mm, N = 10, D =120 mm, P =200 N, G = 80 GPa = 80 × 10^3 MPa

$$\tau_{max} = \frac{8PD}{\pi d^3} = \frac{8\times 200\times 120}{\pi 10^3} = 61.11\ Mpa$$

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8\times 200\times 120^3\times 10}{80\times 10^3\times 10^4} = 34.56\ mm$$