DEPARTMENT OF ELECTRICAL ENGINEERING

LECTURE NOTES ON

ENERGY CONVERSION-I

(4th Semester)

Department of Electrical Engineering

NILASAILA INSTITUTE OF SCIENCE AND TECHNOLOGY
SERGARH, BALASORE, 756060

ENERGY CONVERSION – I

Sl. No.	Topic
1.	DC GENERATORS
2.	DC MOTORS
3.	SINGLE PHASE TRANSFORMER
4.	AUTO TRANSFORMER
5.	INSTRUMENT TRANSFORMERS

D. COURSE CONTENT IN TERMS OF SPECIFIC OBJECTIVES

1. D.C GENERATOR

- 1.1. Operating principle of generator
- 1.2. Constructional features of DC machine.
  - 1.2.1. Yoke, Pole & field winding, Armature, Commutator.
  - 1.2.2. Armature winding, back pitch, Front pitch, Resultant pitch and commutator- pitch.
  - 1.2.3. Simple Lap and wave winding, Dummy coils.
- 1.3. Different types of D.C. machines (Shunt, Series and Compound)
- 1.4. Derivation of EMF equation of DC generators. (Solve problems)
- 1.5. Losses and efficiency of DC generator. Condition for maximum efficiency and numerical problems.
- 1.6. Armature reaction in D.C. machine
- 1.7. Commutation and methods of improving commutation.
  - 1.7.1. Role of inter poles and compensating winding in commutation.
- 1.8. Characteristics of D.C. Generators
- 1.9. Application of different types of D.C. Generators.
- 1.10. Concept of critical resistance and critical speed of DC shunt generator
- 1.11. Conditions of Build-up of emf of DC generator.
- 1.12. Parallel operation of D.C. Generators.
- 1.13. Uses of D.C generators.

2. D. C. MOTORS

- 2.1. Basic working principle of DC motor
- 2.2. Significance of back emf in D.C. Motor.
- 2.3. Voltage equation of D.C. Motor and condition for maximum power output(simple problems)
- 2.4. Derive torque equation (solve problems)
- 2.5. Characteristics of shunt, series and compound motors and their application.
- 2.6. Starting method of shunt, series and compound motors.
- 2.7. Speed control of D.C shunt motors by Flux control method. Armature voltage Control method. Solve problems
- 2.8. Speed control of D.C. series motors by Field Flux control method, Tapped field method and series-parallel method
- 2.9. Determination of efficiency of D.C. Machine by Brake test method(solve numerical problems)
- 2.10. Determination of efficiency of D.C. Machine by Swinburne’s Test method(solve numerical problems)

2.11. Losses, efficiency and power stages of D.C. motor(solve numerical problems)
2.12. Uses of D.C. motors

3. SINGLE PHASE TRANSFORMER

3.1 Working principle of transformer.
3.2 Constructional feature of Transformer.
  3.2.1 Arrangement of core & winding in different types of transformer.
  3.2.2 Brief ideas about transformer accessories such as conservator, tank, breather, and explosion vent etc.
  3.2.3 Explain types of cooling methods
3.3 State the procedures for Care and maintenance.
3.4 EMF equation of transformer.
3.5 Ideal transformer voltage transformation ratio
3.6 Operation of Transformer at no load, on load with phasor diagrams.
3.7 Equivalent Resistance, Leakage Reactance and Impedance of transformer.
3.8 To draw phasor diagram of transformer on load, with winding Resistance and Magnetic leakage with using upf, leading pf and lagging pf load.
3.9 To explain Equivalent circuit and solve numerical problems.
3.10 Approximate & exact voltage drop calculation of a Transformer.
3.11 Regulation of transformer.
3.12 Different types of losses in a Transformer. Explain Open circuit and Short Circuit test.(Solve numerical problems)
3.13 Explain Efficiency, efficiency at different loads and power factors, condition for maximum efficiency (solve problems)
3.14 Explain All Day Efficiency (solve problems)
3.15 Determination of load corresponding to Maximum efficiency.
3.16 Parallel operation of single phase transformer.

4. AUTO TRANSFORMER

4.1. Constructional features of Auto transformer.
4.2. Working principle of single phase Auto Transformer.
4.3. Comparison of Auto transformer with an two winding transformer (saving of Copper).
4.4. Uses of Auto transformer.
4.5. Explain Tap changer with transformer (on load and off load condition)

5. INSTRUMENT TRANSFORMERS

1.1 Explain Current Transformer and Potential Transformer

1.2 Define Ratio error, Phase angle error, Burden.

1.3 Uses of C.T. and P.T

UNIT – I
D.C GENERATORS

CONTENTS:

- **Principle of operation- Constructional features**
- **Action of Commutator**
- **Armature windings – lap and wave windings – simplex and multiplex windings**
- **Use of laminated core –**
- **E. M.F Equation – Problems,**
- **Armature Reaction – Cross magnetizing and demagnetizing AT/pole – compensating winding**
- **Commutation – reactance voltage – methods of improving commutation.**
- **Methods of Excitation – separately excited and self- excited generators – build-up of E.M.F - critical field resistance and critical speed - causes for failure to self-excite and remedial measures.**
- **Load characteristics of shunt, series and compound generators**
    - **Important concepts and Formulae:**
    - Illustrative examples

Introduction:

A DC generator is a rotating machine which converts mechanical energy into DC electrical energy. It requires a prime mover like a Diesel engine, wind turbine or a steam turbine to rotate the DC generator. An EMF is induced in a DC Generator when there is a relative motion between a Magnetic field and a set of electrical conductors. The EMF induced is called a dynamically induced EMF or motional EMF . Normally the magnetic field is stationary and is obtained from stationary field coils placed on the Stator poles and the conductors are placed on a rotating shaft called Rotor. The basic constructional features of a DC generator and a DC Motor are same, and the same DC machine can work either as a DC generator or a DC motor.

The conversion of Mechanical energy into Electrical energy in DC generator is based on the principle of electromagnetic Induction. According to Faradays laws of Electromagnetic induction, whenever a conductor moves in a magnetic field a dynamically induced EMF is produced across the conductor. When the terminals of the conductor are connected to an electrical load the induced EMF enables a current flow through the load. Thus a mechanical energy in the form of a rotational motion given to a conductor is converted into Electrical energy. The EMF induced in a single conductor is very small. Hence a large set of conductors are used in practical generators and such a set of conductors placed on a rotating round shaft is called an armature.

Principle of operation of DC Machines:

Let us consider a single turn of coil **ABCD** mounted on a cylindrical shaft and rotated in an anticlockwise direction at constant angular velocity of **'ω'** rad/sec within a uniform magnetic field of flux density **B webers/mtrs^2** as shown in the figure below .

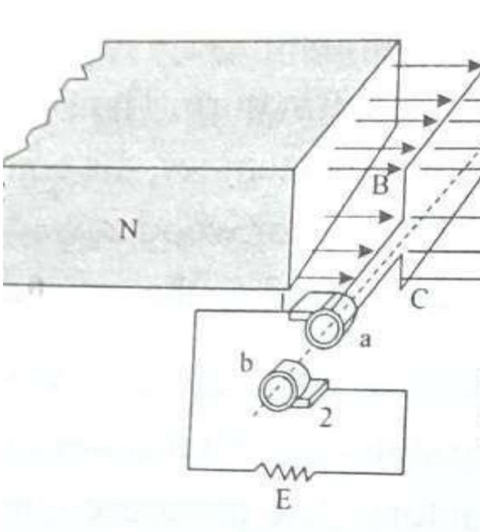


Fig.7.6 Single Turn Generator

Let **l** be the length and **b** be the breadth of the rectangular coil in meters. According to Faradays law the emf induced in a conductor is given by **e = — N.dØ/dt** where **e** is the induced emf , **N** is the number of conductors **, Ø** is the flux linkage and **t** is the time. The flux linkage **Ø** is given by **: Ø = B.area of the coil.cos ωt = B.l.b.Cos ωt**

Since we are considering only one conductor the induced emf in the conductor is given by:

e = −dØ/dt = -d(B.l.b.Cos ωt)/dt = B.l.b.ω .Sin ωt = E_m **Sin ωt where** E_m **= B.l.b.ω**

As can be seen from the above equation for induced emf the voltage in a given generator can be increased by either increasing the flux density **'B'** or the rotational speed **'ω' .**

The induced emf '**e'**at any position of the coil as a function of time '**t'** as derived above is then given by : **e =** E_m **Sin ωt** where E_m **= B.l.b.ω**. As can be seen **dØ/dt** i.e rate of change of flux linkage is minimum (=0) when the coil is at perpendicular position to the flux lines and hence the induced voltage **e** is also minimum (=0) . We will call this as position **Θ = 0^0** at the instant of say **t = 0 sec.** And **dØ/dt** is maximum when the coil is at parallel position to the flux lines and hence the induced voltage **e** is also maximum(**= E_m = B.l.b.ω**) and this position will then be **Θ = 90^0** .When **Θ = 180^0** the induced emf is again zero and when 0 = **270^0** the emf induced is again maximum but now it would be negative. When **Θ = 360^0** the coil is back to the original position and the induced emf is again equal to zero. For the two pole generator shown in the figure one complete cycle of change takes place in one rotation of the coil. A plot of the induced emf 'e' **as** function of coil position **Θ** is an alternating voltage as shown in the figure below.

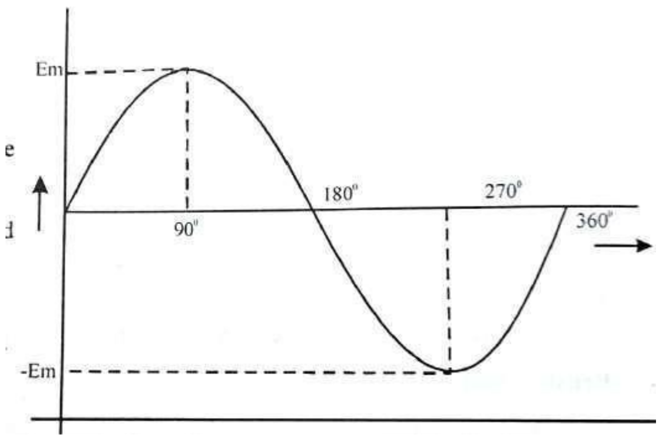


Fig: emf induced in a single turn generator in one full revolution

When the two terminals of the coil are connected to an external load (resistance in this case) through two separate rings (called slip rings) mounted on the armature current flows through the resistance and the current also would be sinusoidal.

Action of commutator: We have seen that the output from a simple single turn generator in one full revolution is a sinusoidal in nature (AC). Commutator is the most important part of a DC Generator which converts the AC to DC. The current flowing through the external load can be made unidirectional by replacing the two ***slip* rings** with two ***split* rings** as shown in the figure below which is the basis for the operation of a commutator in a practical DC Machine with more number of Poles and multiple coils .

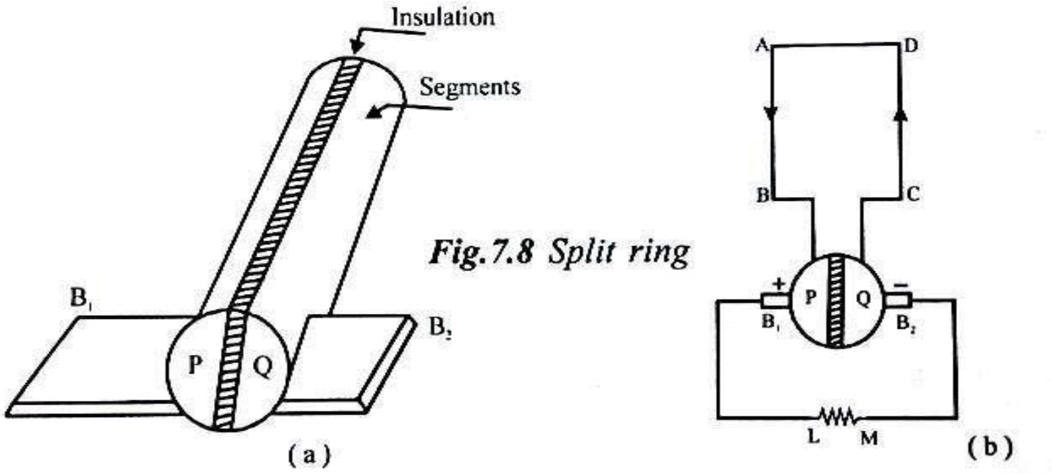


Fig.7.8 Split ring

One slip ring is split into two equal segments **P** and **Q** which are insulated from each other and the armature shaft. The two coils AB and CD are connected to the two segments **P** and **Q** .Two fixed (stationary) brushes **B_1** and **B_2** sliding along these two split rings will be collecting the current from the generator. During the first half of the revolution segment **P** is positive and current flows along **ABPLMQCD** through brush B_1 which is positive and into brush B_2 into segment Q which is negative. Next during the other half cycle, the location of the segments **AB & CD** will reverse along with the respective segments **P and Q** . Now conductor **CD** and segment **Q** are positive and current flows along **DCQLMPBA** through the Brush **B_1** which is again positive and into the brush **B_2** which is again negative as shown in the figure below.

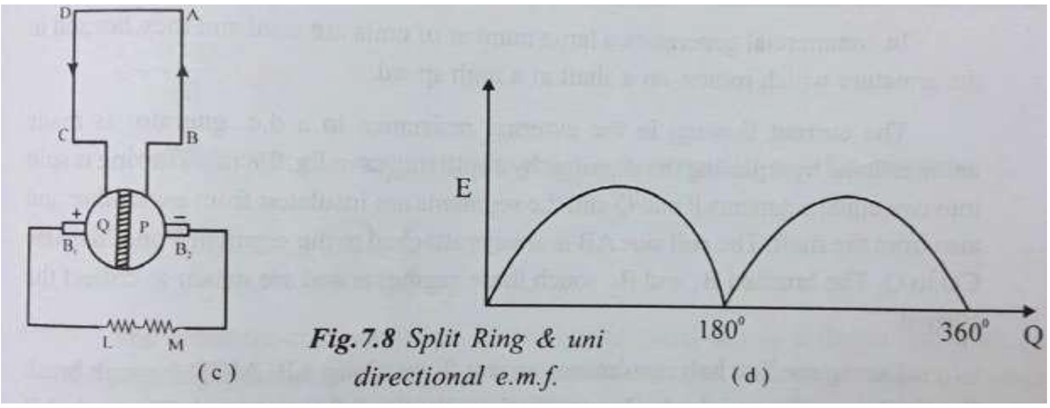


Fig.7.8 Split Ring & uni directional e.m.f.

In each half revolution the positions of the conductors **AB & CD** and the segments **P &Q** reverse but the brushes **B_1&B_2** are stationary and continue to collect current from the Positive side and deliver current to the Negative side respectively. Hence the voltage across the load will be a unipolar voltage as shown in the waveform above. The changeover of brushes **B_1&B_2** between segments **P &Q** takes place when the voltage is minimum so as to avoid or minimize the arcing between the split segments. In practical generators there will be more number of conductors and also more number of Pole pairs and hence more number of split segments are required and such a set of more number of split segments is called *commutator.*

Constructional features of a DC Generator:

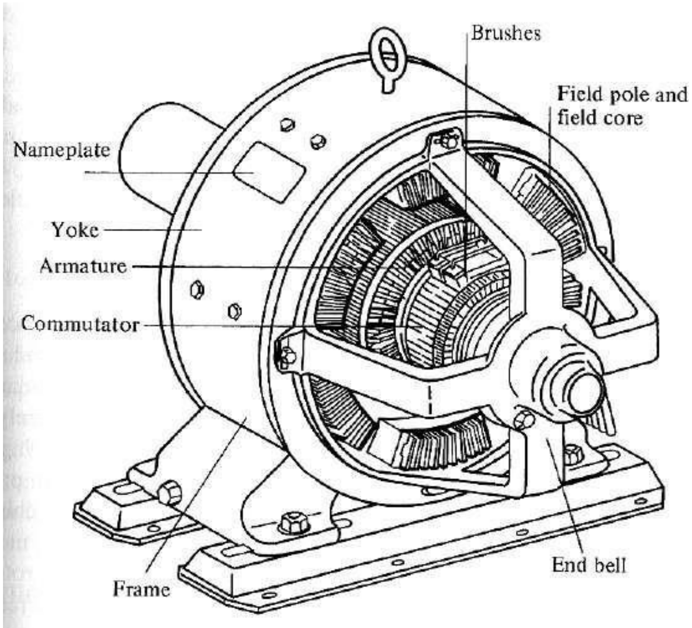


Fig: A simplified diagram of a dc machine:

Major parts of a DC generator:

- Main frame or Yoke
- Poles
- Armature
- Commutator
- Brushes ,bearings and shaft

The physical structure of the machine consists of two parts: the stator and the rotor.

The stationary part consists of the main frame (yoke), and the pole pieces, which project inward and provide a path for the magnetic flux. The ends of the pole pieces that are near the rotor spread out over the rotor surface to distribute its flux evenly over the rotor surface. These ends are called the pole shoes. The exposed surface of a pole shoe is called a pole face, and the distance between the pole face and the rotor is the air gap.

There are two principal windings on a dc machine:

- The armature windings: the windings in which a voltage is induced (rotor)
- The field windings: the windings that produce the main magnetic flux (stator)

Because the armature winding is located on the rotor, a dc machine's rotor is mostly called an armature.

The terminal characteristic of a DC Machine is a plot of the output parameters of the Machine against each other. For a DC Generator the output quantities are the Terminal Voltage and the Line (Load) current.

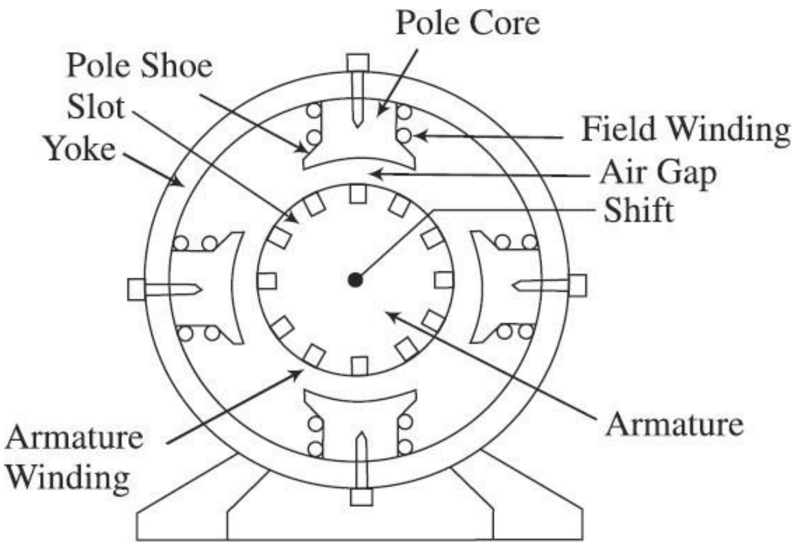


Figure: Cross sectional view of a DC Machine

The air−gap is kept very small to keep the reluctance of the magnetic circuit low. The armature is a laminated cylinder and is mounted on a shaft. The armature laminations are about 0.4−0.6 mm thick and are insulated from one another. The armature is laminated to reduce the eddy−current loss in the core. Slots are stamped on the periphery of the armature laminations. The armature slots house the armature windings. The stator core, the yoke and the poles may not be laminated as they encounter DC flux.

Due to the presence of slots on the armature surface, there is flux pulsation at the

Stator pole−face. The stator pole shoes, therefore, should be laminated to reduce the eddy−current loss. However, for mechanical reasons, in many cases the whole of the pole core is laminated. In DC Machines of high ratings slots are cut on the pole−faces to house a separate winding called the compensating winding. The compensating winding is connected in series with the armature winding and neutralises the effect of armature reaction. To neutralize the effect of armature reaction in the space in between two poles, smaller poles, called interpoles, are fixed on the yoke as shown in **Fig. 2.2.**

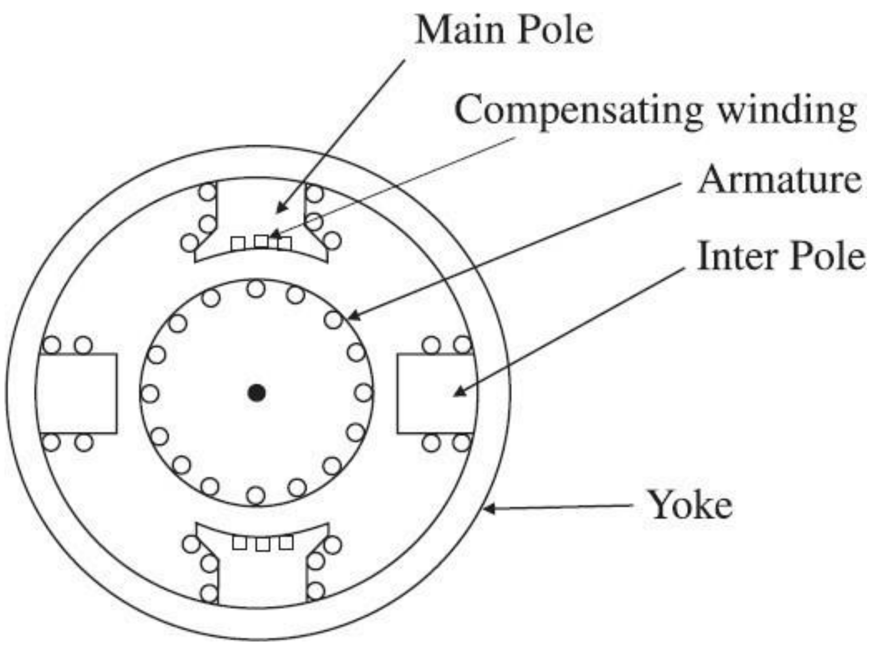


Figure: Cross sectional view of a DC Machine showing Interpoles

As mentioned earlier, the armature winding is placed inside the armature slots. The slots are lined with tough insulating material. This slot insulation is folded over the armature conductors. The conductors in the slots are secured in their places by hard wooden wedges or fiber glass wedges. The armature windings are first made on formers and then placed on slots.

Enamel insulated copper wires are used for the armature winding. Each armature coil end is connected with each segment of the commutator.

A commutator is a cylindrical body mounted on the shaft along with the armature. In fact, the armature core and the commutator form one single unit mounted on the shaft. Brushes are placed on the commutator surface to supply or collect current to the armature coils through the commutator segments. The commutator segments are insulated from each other.

The function of the commutator is to convert alternating currents induced in the armature conductors into direct currents in the external circuit in case of a DC Generator operation. In the case of a dc motor the function of the commutator is to produce a unidirectional torque. The commutator is of cylindrical structure and is built up of a wedge−shaped segment of hard−drawn copper. Mica insulation is provided between commutator segments. Brushes are made of carbon and are housed in brush−holders.

A spring in the brush−holder maintains the desirable pressure on the carbon brushes so that proper contact is maintained between the brushes and the commutator surface.

Armature windings:

The armature windings are vital part of a DC Machine. This is where emf is induced in the case of a Generator and and force is developed that results in the turning of the rotor in the case of a Motor. The design of the armature winding is more critical than the design of other parts of a DC machine. The armature winding is housed in slots made on the armature surface. Formed coils are placed on slots. The ends of the coils are joined with commutator segments.

Commutator: The commutator is made up of a number of commutator segments. Coil−ends are connected to each commutator segment. The segments of the commutator are made of hard−drawn copper and are separated by thin sheets of mica or micanite (insulator) .

The induced emf per conductor in a DC machine is small. The problem is how these conductors are to be connected together so as to form a complete winding. Figure below shows the cross−sectional view of the armature of a four−pole machine.

For ease of understanding, a developed diagram of armature of Fig. **(a)** below is drawn as shown in Fig. **(b)**. Conductors should be so connected that the total emf is maximum. Therefore, conductor 1 should be connected to conductor 6 shown by dotted line as conductor 6 is placed below conductor 5 so that they occupy identical positions under two adjacent poles. Similarly conductor 3 should be connected with conductor 8 and so on.

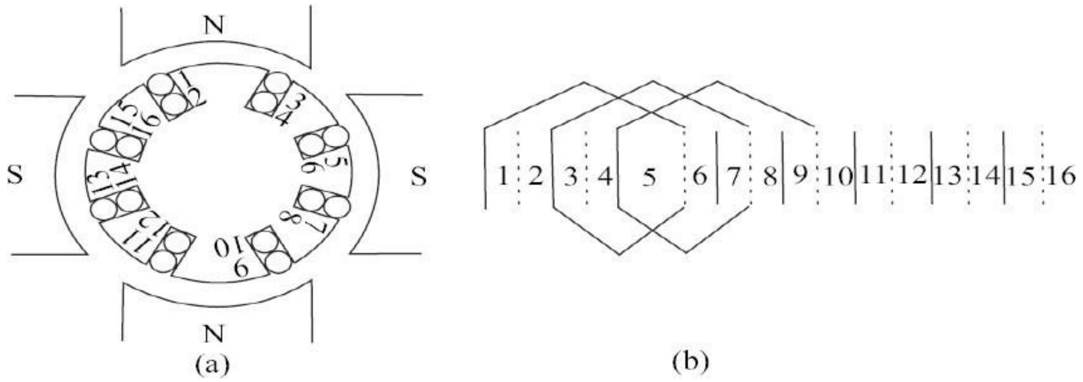


Figure : (a) Cross-sectional view of the armature of a 4-pole DC Machine (b) Incomplete developed diagram of the armature winding

Figure **below** shows the developed winding diagram of the 16 armature conductors of Fig. **(a)** Shown earlier.

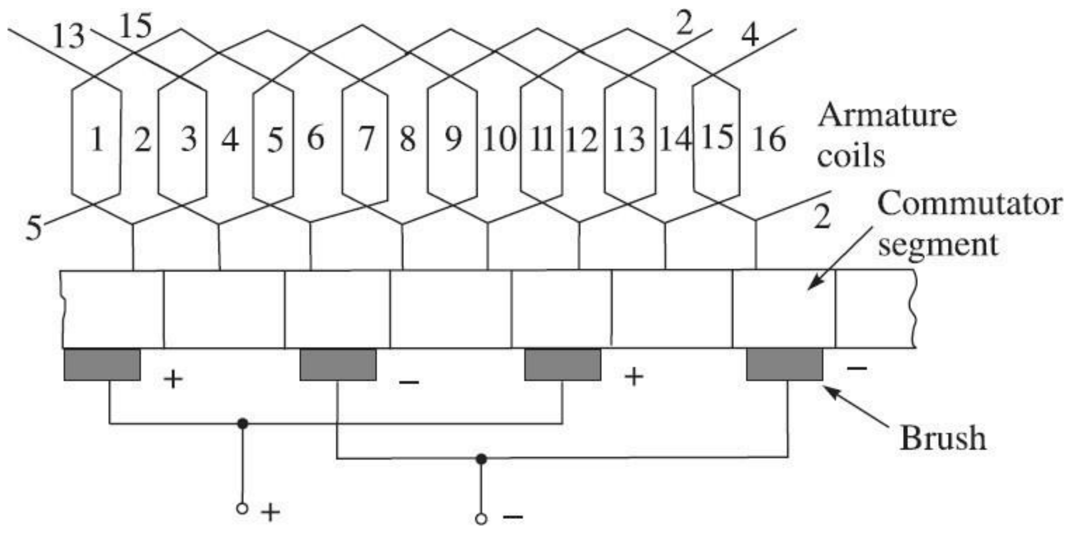


Figure: Armature winding of a DC Machine

The average pitch **Ya**, back pitch **Yb**, and the front pitch **Yf** are calculated as:
Ya = 16/4= 4
Ya = (**Yb + Yf**) /2
Yb – Yf = ±2
For progressive lap winding
Yb – Yf = 2
Yb = 5**, Yf** = 3

Figure gives the details of end connections of the conductors, connection of coils with commutator segments, and the position of brushes on the commutatorsurface with their polarities. This type of winding is called lap winding. In the winding shown in Fig. 2.11, single−turn conductors are used. As many as 16 conductors make eight coils. The coils are 1−6, 3−8, 5−10, 7−12, 9−14, 11−16, 13−2 and 15−4. The design of a lap winding of the type shown in Fig. 2.11 is described as follows.

2.3.2 Lap Winding

In a lap winding, the finishing end of one coil is connected via the commutator segment to the starting end of the adjacent coil situated under the same pole. In this way all the coils are connected. The winding is known as lap winding because the sides of successive coils overlap each other (see Fig. below). A coil may consist of any number of turns. The number of slots required on the armature is equal to the number of coil sides if two coil−sides are placed in each slot. With two coil−sides in each slot, a two layer winding is obtained. While making a winding diagram in a two−layer winding, all top coil−sides are numbered odd whereas the bottom coil−sides are numbered even (shown by dotted lines) as shown in Fig.. For an eight−coil armature, therefore, eight slots are required on the armature surface. The following terminologies are required to be understood for preparing an armature winding diagram.

Pole Pitch: It is equal to the number of coil−sides per pole. For a single turn, eight coil, four−pole armature pole pitch is calculated as:

Pole pitch (**Ya**) = (No. of coils x 2)/ No. of poles = (8 x 2) /4 = 4

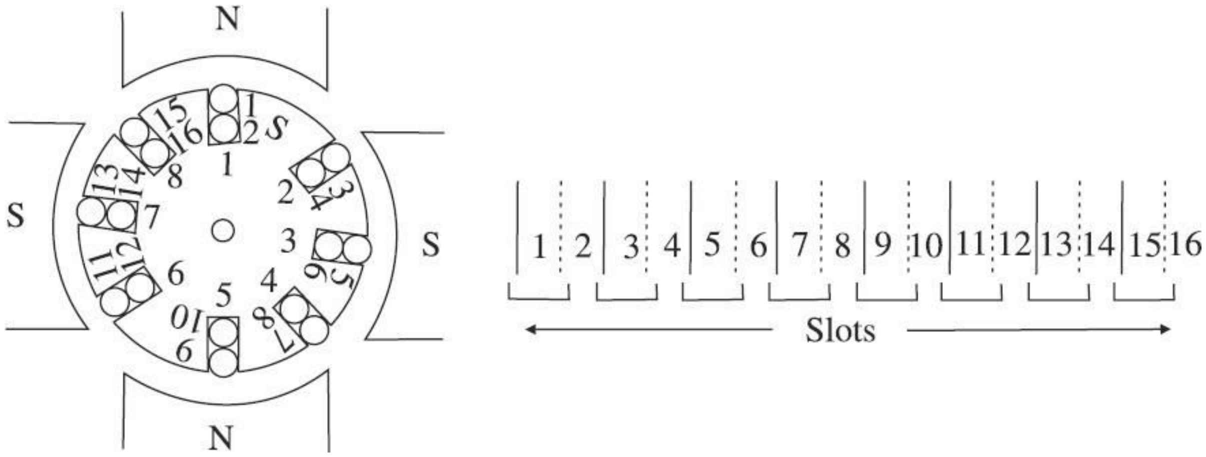
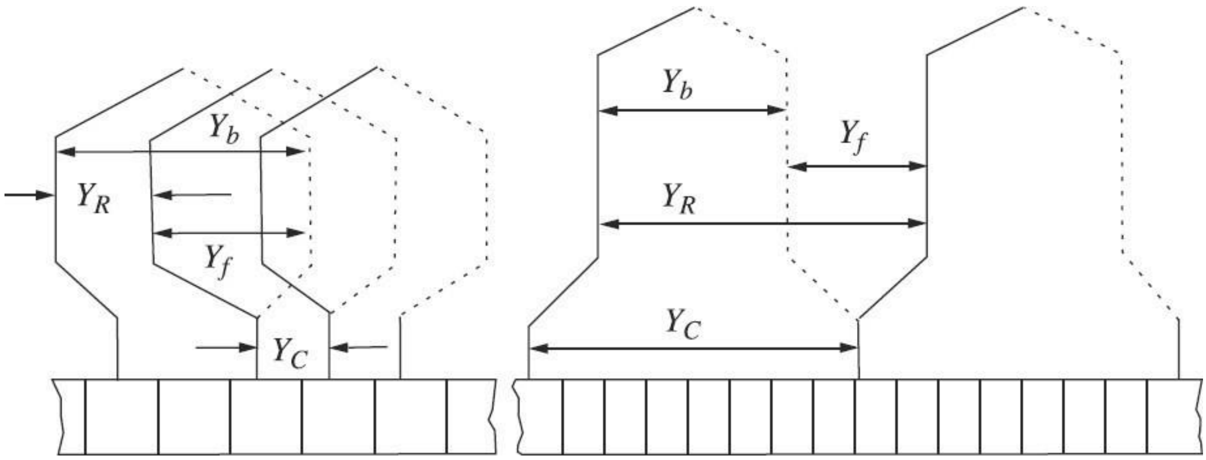


Figure: Position of coil-sides in slots of a two-layer armature winding

Coils and Coil-sides: The DC armature windings are double−layer type having at least two coil−sides per slot. Each coil consists of an upper coil−side at the top of one slot and a lower coil−side situated at the bottom of another slot. The distance between the two coil−sides of a coil is approximately equal to the pole pitch. A coil may be of single turn or of many turns. If two coil−sides are placed in one slot, then the number of slots required on the armature for housing the coils is equal to the number of coils of the winding. For low−speed high−voltage winding, however, the number of coil−sides per slot is more than two. This is because the winding will have a large number of coils and it may not be possible to have an equal number of slots on the armature.

Back Pitch: The distance measured in terms of the number of armature conductors (coil sides) between the two coil−sides of a coil measured around the back of the armature, i.e., away from the commutator end of the armature is called the back pitch, **Yb**.



(a) Lap winding (b) Wave winding

Figure: Shows back pitch **Yb** , front pitch **yf** resultant pitch **Yr**, and commutator pitch **Yc** in (a) lap winding (b) wave winding

Front Pitch: The distance between two coil−sides connected to the same commutator segment is called the front pitch, **Yf** .

Resultant Pitch: It is defined as the distance in terms of the number of coil−sides between the start of one coil and the start of the next coil to which it is connected.

Commutator Pitch: It is defined as the distance measured in terms of commutator segments between the segments to which the two ends of a coil are connected.

For calculating back pitch **Yb** and front pitch **Yf** for a lap winding, the following relations are used:

(i) Yb − Yf = ± 2m Also, Yb = (Z/P)± 1

where m = 1 for simplex winding

= 2 for duplex winding

When **Yb** is greater than **Yf** , the winding is a progressive one, i.e., it progresses from left to right. If Yb is less than Yf , the winding is called a retrogressive one, i.e., it progresses from right to left.

(ii) The back pitch and front pitch must be odd.

(iii) The average pitch, Ya = (Yb + Yf)/2 should be equal to the pole pitch, i.e., equal to Z/P, where Zis the number of coil sides.

(iv) The commutator pitch is equal to m, i.e., equal to 1, 2, etc. for simplex, duplex etc. type of winding.
(v) The number of parallel paths in the armature winding for a simplex lap winding is equal to the number of poles, P.
(vi) The resultant pitch is always even, being the difference of two odd numbers.

Example 1 : Prepare a layout winding diagram for a simplex lap−type DC armature winding. The winding is for 4 poles. The armature has 16 slots and 16 commutator segments.
Solution: Number of armature coils = Number of commutator segments = 16
Number of coil−sides (conductors) $Z = 16 \times 2 = 32$
Back pitch **Yb** =(Z/P) ±1 = (32/4) ±1 = 9 or 7
Yb – Yf = 2
Yf = **Yb** − 2 = 9 − 2 (using Yb = 9) = 7
Yb = 9
Yf = 7
Since **Yb > Yf** , the winding is a progressive one.
As there are 32 coil−sides and 16 slots, the number of coil−sides per slot is 2. The connection scheme of the coil−sides is shown in Figure . below

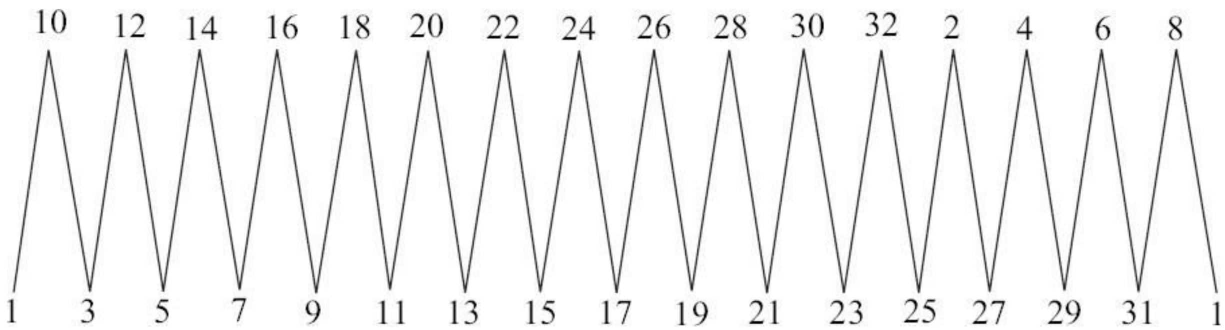


Figure: Scheme for connections of the coil-sides of a DC armature windings

Coil−side 1 is connected to coil−side 10 on the other side of the commutator (since Yb is 9, coil−side 1 is connected to coil−side 1 + 9, i.e., 10). Coil−side 10 is connected

to coil−side 3 on the commutator end (Since Yf is 7, coil−side 10 is connected to coilside10−7, i.e., 3). The winding progresses according to the above scheme. It may be noted that each coil is used once and the winding is a closed one.
The layout diagram of the winding along with commutator connections and brush positions is shown below.

Connections of the coil−sides are made as follows: for connections at the back end of the armature, add the back pitch with the coil−side which is to be connected. Thus coil−side 1 is to be connected with coil−side 1 + Yb, i.e., 1 + 9 = 10. On the commutator end side, coil−side 10 is connected to coil−side 3. This is achieved by subtracting Yf , i.e., 7 from coil−side number 10 (10 − 7 = 3). Coil−side 3 is now connected to 3 + Yb = 3 + 9 = 12. In this way the winding is completed.
The positions of the four poles are also shown in Fig. 2.15. Eight coil−sides placed in four slots are under each pole. Assuming a direction of rotation of the armature, say anticlockwise in Fig. 2.15, the direction of the induced emf in the armature conductors is determined by applying Fleming's right−hand rule. The direction of the current in the coil−sides under north poles will be downward and under south poles upward as shown in Fig. 2.15.
The position of brushes can be determined by tracing the directions of current in various coil−sides. From Fig. 2.15, it can be observed that directions of current in coil−sides 1 and 8 are downward and they are connected to commutator segment 1. A brush placed on commutator segment 1 will have positive polarity. Similarly in coil−sides 9 and 16, the current is upwards. The two coil−sides are connected to commutator segment 5. The brush placed on commutator segment 5 will have negative polarity. Similarly the positions of the other two brushes are fi xed. Two positive brushes and two negative brushes are joined together to output terminals A and B respectively.
The number of parallel paths of the armature winding across the output terminals is four (equal−to the number of poles) which can be examined as follows: Redraw the armature winding of Fig. 2.15 in a simplifi ed manner as shown in Fig. 2.16. Between terminals A and B there are four parallel paths shown as M, N, O and P. The total emf generated in the machine is equal to the emf generated in one parallel path.

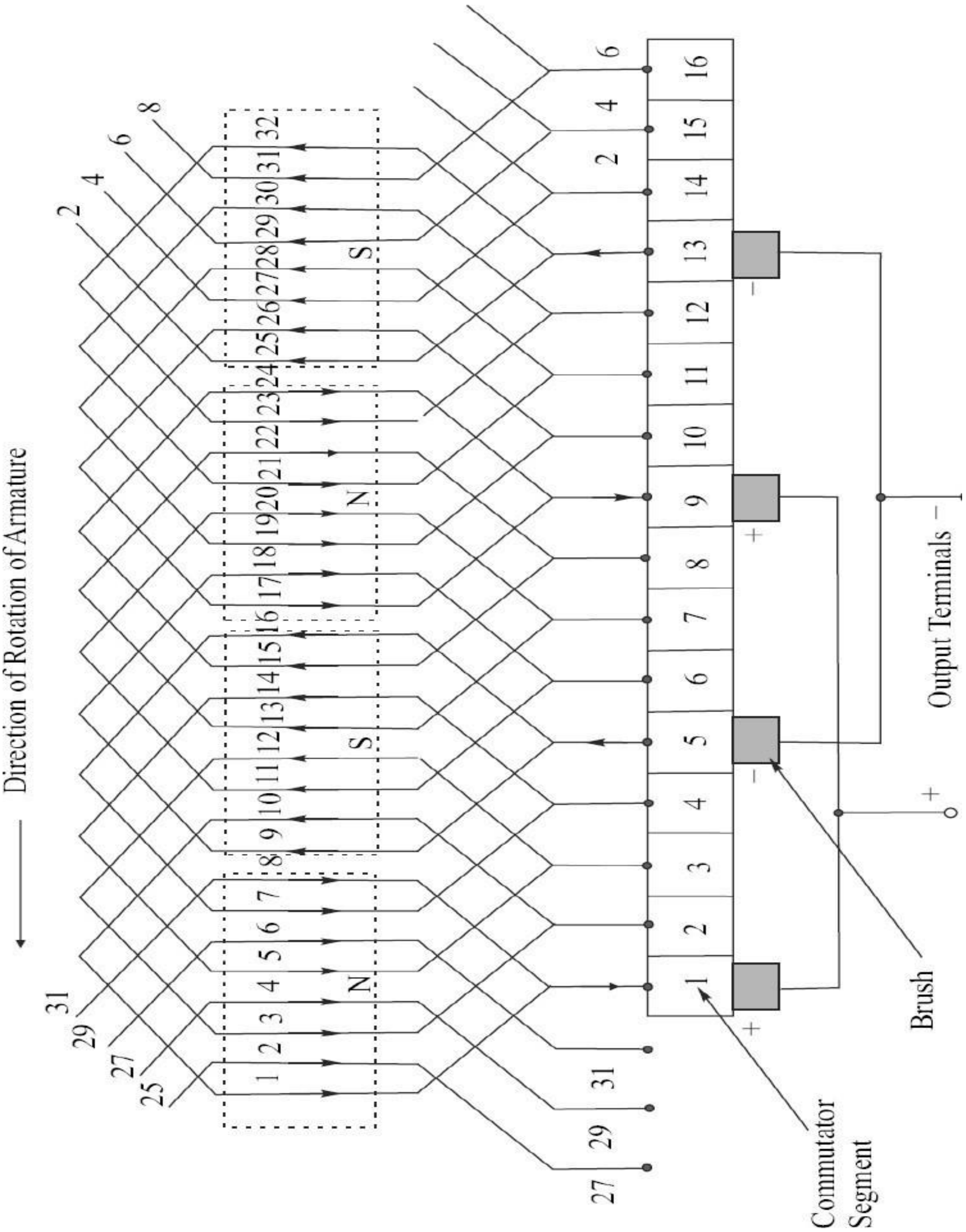


Figure: Lay out diagram for a Lap winding given in example -1

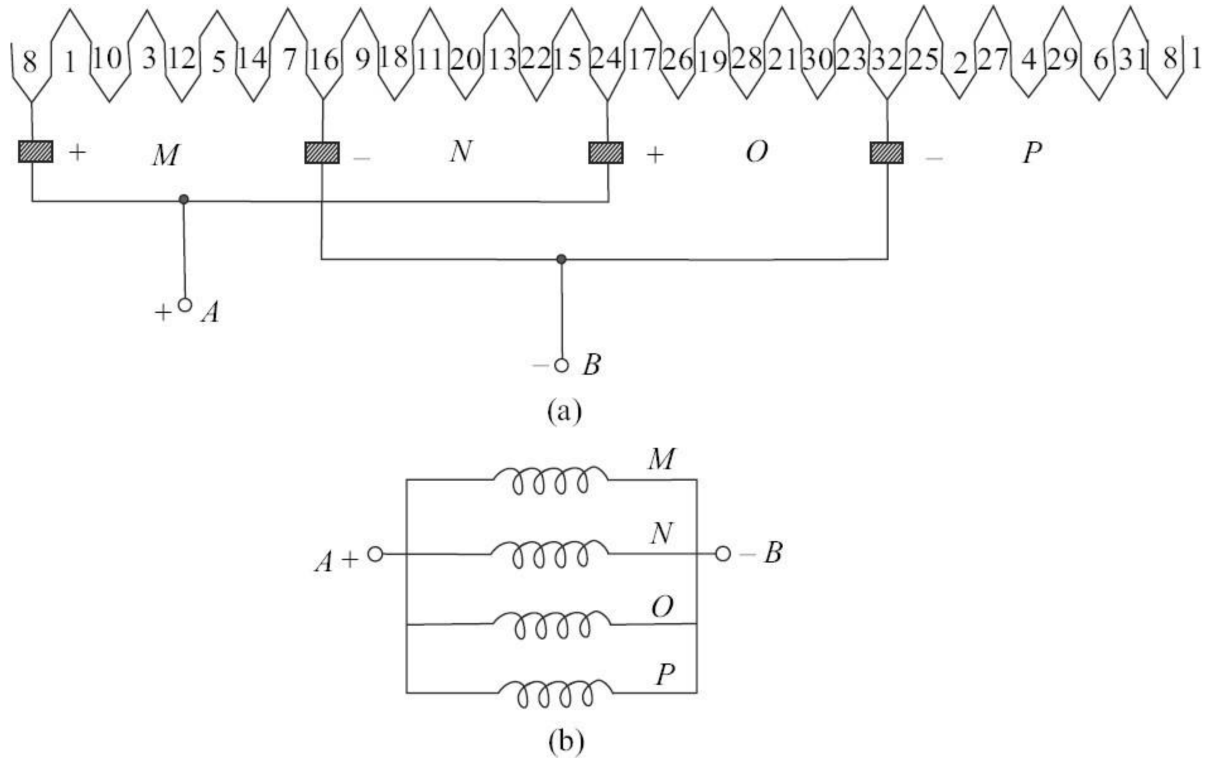


Figure: (a) Armature winding of a dc machine shown in a simplified manner (b) Shows the number of parallel paths in the armature

Equalizer Connections in Lap Winding: As mentioned earlier, a simplex lap winding has as many number of parallel paths as there are poles. The emf induced in each parallel path may not be exactly equal due to a number of reasons, such as the difference in the lengths of the air−gap under each pole, the difference in the field strength due to some error in putting field windings, etc.

Unequal values of emf generated in the parallel paths will circulate a considerable amount of current in the armature circuit without doing any useful work. This circulating current will be large as the armature circuit resistance is generally very low. This circulating current will generate heat and while circulating through the brush contacts will cause commutation diffi culties (like sparking on the commutator surface).

To overcome this problem arising from the circulating current, equalizer connections are made in lap wound armatures. These equaliser connections or equalisers are low−resistance copper conductors which connect those points in the winding which under ideal conditions should be at equal potential. The difference in potential between these points created due to reasons mentioned earlier will be equalised as a result of flow of current through these low resistance conductors which will bypass the current from flowing through the brushes.

2.3.3 Wave Winding

In a wave winding a coil−side under one pole is connected to a second coil−side which occupies approximately the same position under the next pole through back connection. The second coil−side is then connected forward to another coil−side under the next pole (in the case of lap winding the second coil is connected back through the commutator segment to a coil−side under the original pole). The difference in lap and wave winding connections has been illustrated in Fig. 2.13(a) and (b).

The characteristics of a wave winding are:

(i) Average pitch, **Ya** = **(Yb ± Yf)/2 = (Z ±2)/P**

If **Ya** is taken equal to **Z/P**, as is the case in a lap winding the winding after one round will close itself without including all the coils which is not desirable.

Hence the product of the average pitch and the number of pairs of poles must be two greater or less than the number of coil−sides.

Average pitch should be a whole number.

(ii) Both back pitch and front pitch should he odd numbers.

(iii) To make the average pitch a whole number, wave winding is not possible with any number of coil−sides. For example if Z = 32 and P = 4,

Ya = (Z±2)/P = (32±2)/4 = 8 Ъ or 7 Ъ

Thus wave winding is not possible with 32 coil−sides. In this case the number of effective coil−sides needs to be 30.

Example-2 : Prepare a winding diagram for a 4 pole wave−connected armature of a dc generator having 22 coil sides.

Ya = (Z ±2)/P = (22 ±2)/4 = 6 or 5

If **Ya** is taken to be odd, i.e., 5, then the front pitch and back pitch will be equal.
Thus, **Ya = Yb = Yf = 5**.
Connections of the coil sides will be as shown in figure. The connection diagram is achieved by adding Yb and Yf with the coil numbers progressing in the forward direction. Coil−side 1 is connected at the back with coil side 6 (1 + Yb = 6). Coil side 6 is connected at the front with coil−side 11 (6 + Yf = 11) and so on.

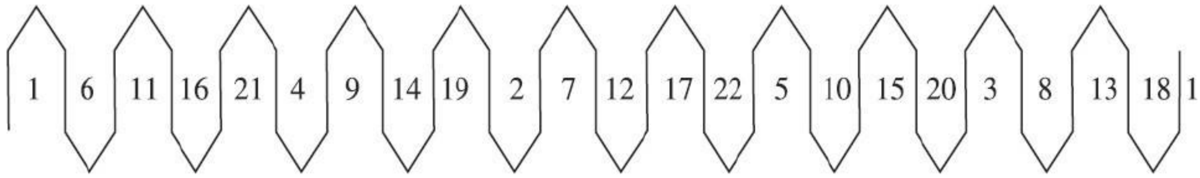


Figure: Connection diagram of the coil-sides for a DC wave winding

In Fig. 2.17 it is to be noted that coil−side 19 is connected with coil−side 2. This is obtained by adding Yb to 19 which gives 24. Coil−side 24 does not exist as there are in all 22 coil−sides. Therefore after 22 count two more numbers starting from 1. This gives coil−side 2. Similarly it can be seen that coil−side 20 is connected in the front with coil−side 3. By adding Yf (= 5) to 20, the number 25 is obtained. After 20 fi ve numbers are counted as 21, 22, 1, 2, and 3. Thus coil−side 20 should be connected to coil−side 3. In this way, the whole winding is completed by connecting all the coil−sides with one another. The actual layout diagram of the winding along with the position of the poles and the direction of induced emf in the coil−sides for a particular direction of rotation of the armature are shown in Fig. 2.18. The positions of the four brushes are also shown in the figure.

The positions of brushes are fixed as follows: for ease of understanding, the connection diagram of Fig. 2.17 is reproduced in Fig. 2.19. The directions of current in the coil−sides are also shown by observing the directions from Fig. 2.18. By carefully examining the directions of current in the coil−sides it is seen that between points P and Q current gets divided in two parallel paths. From point P the current fl ows to Q via two paths, viz. through 11−16−21− ... 6−11−18−13−

The point P in Fig. 2.19 is the separating point of the emf in the two sections of the winding and therefore corresponds to the position of one of the brushes, viz. the negative brush. For placing of the positive brush, it is seen from Fig. 2.19 that at point Q current is coming out from both the coil−sides. Therefore, point Q corresponds to the position of the positive brush.

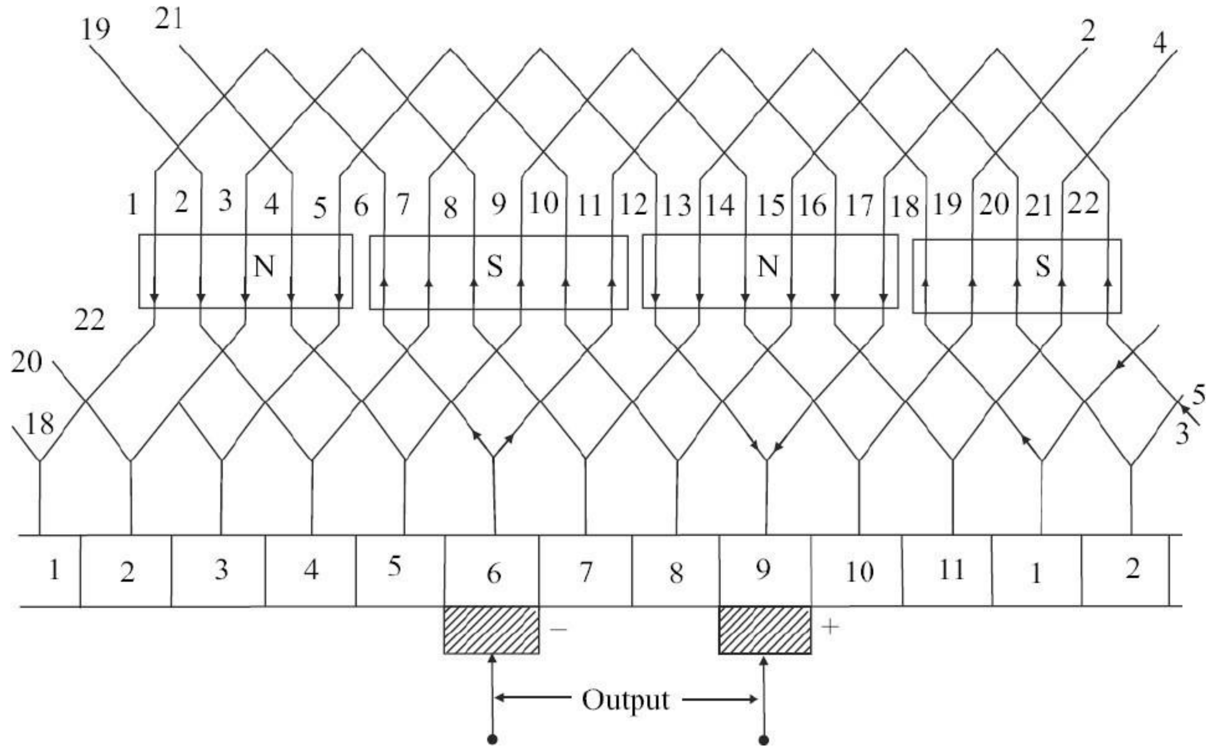


Figure: Layout diagram for the wave winding of example 2.2

It may be noted from Fig. 2.18 that coil−sides 6 and 17 lie in the interpolar region. The direction of current in these coil−sides will depend upon the direction of current in the other coil side of the respective coils, viz. coils 1−6 and 17−22.

Dummy Coils: As mentioned earlier wave winding is possible with a particular number of coil−sides. But if standard stampings with a definite number of slots are to be used, the number of coil−sides needed to be placed in all the slots may be more than the required number. In such a case, the extra coils are left unconnected. These coils are called dummy coils. Dummy coils are used so as to make the armature dynamically balanced. They, otherwise, do not contribute to the induced emf or developed torque.

Example-3 : Calculate the winding pitches and draw developed and sequence diagrams of the winding for a four−pole wave connected armature winding of a dc generator having seven coils. In the diagram, show the position of poles and the position and polarity of brushes.

Solution:

Number of coil−sides = 7 × 2 = 14

Ya = (Z ±2)/P = (14 ±2)/4 = 3 or 4

Ya should be an integer, **Yb** and **Yf** should be odd numbers.

Therefore, we choose

Ya = Yb = Yf = 3

The sequence and layout diagrams of the winding are shown in Fig**. 2.20**.

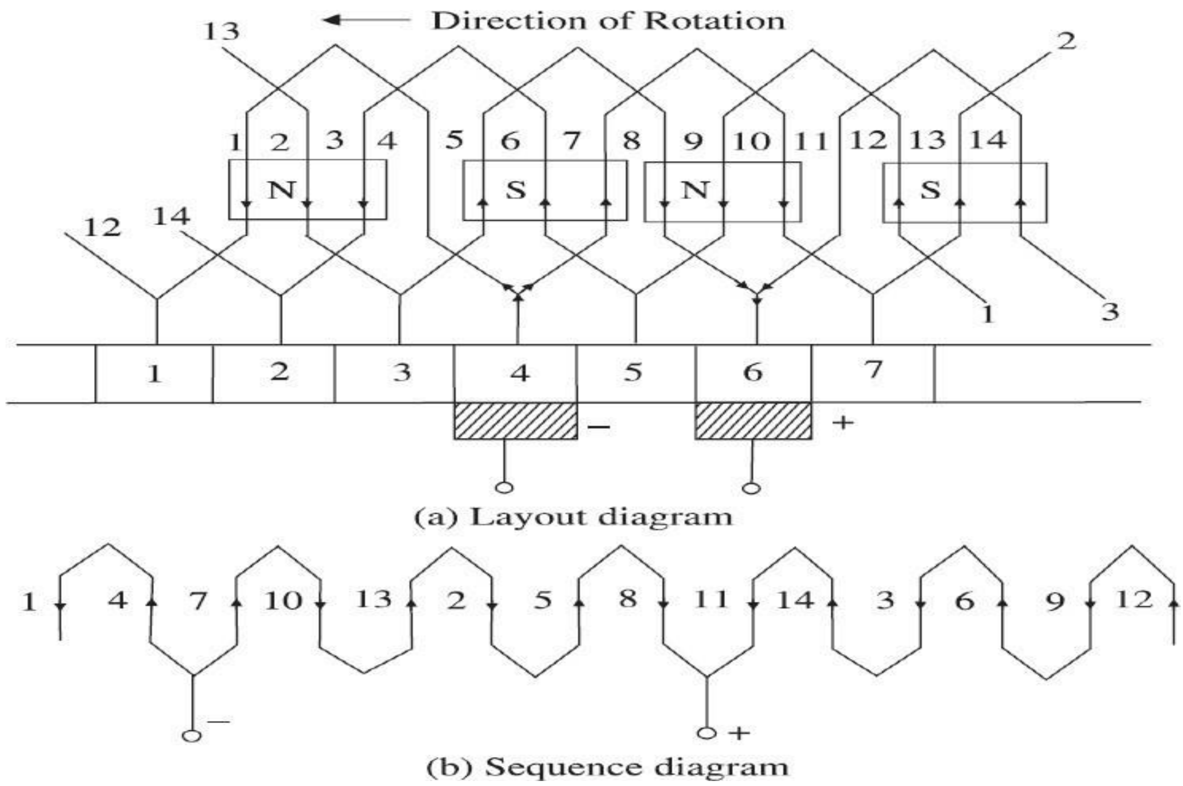


Figure: Layout and sequence diagram for a wave wound DC armature

Simplex and Multiplex windings:

Rotor (armature) windings are further classified according to the *plex* of their windings. A *simplex* rotor winding is a single, complete, closed winding wound on a rotor. A *duplex* rotor winding is a rotor with *two complete and independent sets* of rotor windings. If a rotor has a duplex winding, then each of the windings will be associated with every other commutator segment. One winding will be connected to segments l, 3, 5, etc., and the other winding will be connected to segments 2, 4, 6, etc. Similarly, a *triplex* winding will have three complete and independent sets of windings, each winding connected to every third commutator segment on the rotor. Collectively, all armatures with more than one set of windings are said to have *multiplex windings.*

Use of laminated core :

Faraday's law is the fundamental property of magnetic fields involved in transformer operation. The effect of Lenz's law in transformers is to predict the polarity of the voltages induced in transformer windings.

Faraday's law also explains the eddy current losses mentioned previously. A time–changing flux induces voltage *within* a ferromagnetic core in just the same manner as it would in a wire wrapped around that core. These voltages cause swirls of current to flow within the core, much like the eddies seen at the edges of a river. It is the shape of these currents that gives rise to the name *eddy currents.* These eddy currents are flowing in a resistive material (the iron of the core), so energy is dissipated by them. The lost energy goes into heating the iron core. The amount of energy lost to eddy currents is proportional to the size of the paths they follow within the core. For this reason, it is customary to break up any ferromagnetic core that may be subject to alternating fluxes into many small strips, or *laminntions,* and to bui ld the core up out of these strips. An insulating oxide or resin is used between the strips. so that the current paths for eddy currents are limited to very small areas. Because the insulating layers are extremely thin, this action reduces eddy current losses with very little effect on the core's magnetic properties. Actual eddy current losses are proportional to the square of the lamination thickness, so there is a strong incentive to make the laminations as thin as economically possible.

EMF Equation:

This being very important for understanding of a Generator performance we will derive a detailed expression for the exact induced emf in a generator in terms of all the following DC Machine parameters**.**

Ø The flux from a pole (webers)

Z The total number of conductors on the armature

a The number of parallel paths

- In a practical machine all the conductors are not connected in series. They are divided into groups of parallel conductors and then all the groups are connected in series to get higher voltage. In each group there are **'a'** conductors in parallel and hence there are **'a'** parallel current paths and each parallel path will have **Z/a** conductors in series.

N The Speed of rotation (RPM)

ω The speed (Radians/sec)

P The number of poles

Now consider one conductor on the armature. As this conductor makes one complete revolution it cuts **PØ** webers of flux.

Since the induced emf in a conductor is its rate of cutting of flux lines (Rate of change of Flux linkage) the emf **'e'** induced in such a single conductor is equal to

e = PØ/ Time for one revolution in seconds = PØ/(60/N) = NPØ/60 volts

There are **Z/a** conductors in series in each parallel path.

 the total induced emf **$'E' = (Z/a)\ NPØ/60\ =\ (NPØ\ Z)/(a.\ 60)$**

$$E_A = (Ø\ ZN/60).(P/a)$$

The armature conductors are generally connected in two methods. Viz. Lap winding and Wave winding**.**

In Lap wound machines the number of parallel paths **'a' = P**

 $'E' = (Ø\ ZN/60)$

In Wave wound machines the number of parallel paths **'a' = 2**

 $'E' = (Ø\ ZN/60).(P/2)$

In general the emf induced in a DC machine can be represented as **$E_A = K_a.\ Ø.N$**

Where $$K_a = ZP/60.a$$

Sometimes it is convenient to express the emf induced in terms of the angular rotation **ω (Rad/sec)** and then the expression for emf becomes:

$E_A = (Ø\ ZN/60).(P/a) = (ZP/a).\ Ø.\ N/60 = (ZP/.a).\ Ø.\ (ω/2u) = (ZP/2ua).Ø.ω = K_a.\ Ø.ω$

(since N/60 RPS = 2u. N/60 Radians /sec = ω Radians /sec and N/60 = ω/2u)

Where **K_a** is the generalized constant for the DC machine's armature and is given by :

$$K_a = (ZP/2ua)$$

Where **Ø** is the flux/per pole in the machine (Webers), **N** is the speed of rotation (RPM) **ω** is the angular speed (Radians/sec) and K_a is a constant depending on the machine parameters.

And thus finally $E_A = K_a . \text{Ø} . \omega$ *and we can say in general, the induced voltage in any DC machine will depend on the following three factors:*

1. The flux **Ø** *in the machine*

2. The angular speed of rotation **ω** *and*

3. *A constant representing the construction of the machine.* ***(ZP/2ua)***

(i.e. the number of conductors ***'Z'****, the number of poles* ***'P'*** *and the number of parallel paths* ***'a'*** *along with the other constant* ***'2u'****)*

Armature Reaction:

If the magnetic field windings of a DC machine are connected to a power supply and the rotor of the machine is turned by an external source of mechanical power,then a voltage will be induced in the conductors of the rotor. This voltage wiIl be rectified into a DC output by the action of the machine's commutator.

Now connect a load to the terminaIs of the machine, and a current will flow in its armature windings. This current flow will produce a magnetic field of its own, which will distort the original magnetic field from the machine's poles. This distortion of the flux in a machine as the load is increased is called *armature reaction.*

It causes two serious problems in real DC machines.

The first problem caused by armature reaction is *neutral−plane shift.* The *magnetic neutral plane* is defined as the plane within the machine where the velocity of the rotor wires is exactly parallel to the magnetic nux lines, so that e_{ind} in the conductors in the plane is exactly zero.

To understand the problem of neutral−plane shift, examine Figure 8− 23. Figure 8−23a shows a two−pole dc machine. Notice that the flux is distributed uniformly under the pole faces. The rotor windings shown have voltages built up out of the page for wires under the North Pole face and into the page for wires under the South Pole face. The neutral plane in this machine is exactly vertical.

Now suppose a load is connected to this machine so that it acts as a generator. Current will flow out of the positive terminal of the generator, so current will be flowing out of the page for wires under the North Pole face and into the page for wires under the South Pole face. This current flow produces a magnetic field from the rotor windings, as shown in Figure 8− 23c. This rotor magnetic field affects the original magnetic field from the poles that produced the generator's voltage in the first place. In some places under the pole surfaces, it subtracts from the pole flux, and in other places it adds to the pole flux. The overall result is that the magnetic flux in the air gap of the machine is skewed as shown in Figure 8− 23d and e. Notice that the place on the rotor where the induced voltage in a conductor would be zero (the neutral plane) has shifted.

For the generator shown in Figure 8− 23, the magnetic neutral plane shifted in the direction of rotation. If this machine had been a motor, the current in its rotor would be reversed and the flux would bunch up in the opposite corners from the bunches shown in the figure. As a result, the magnetic neutral plane would shift the other way.

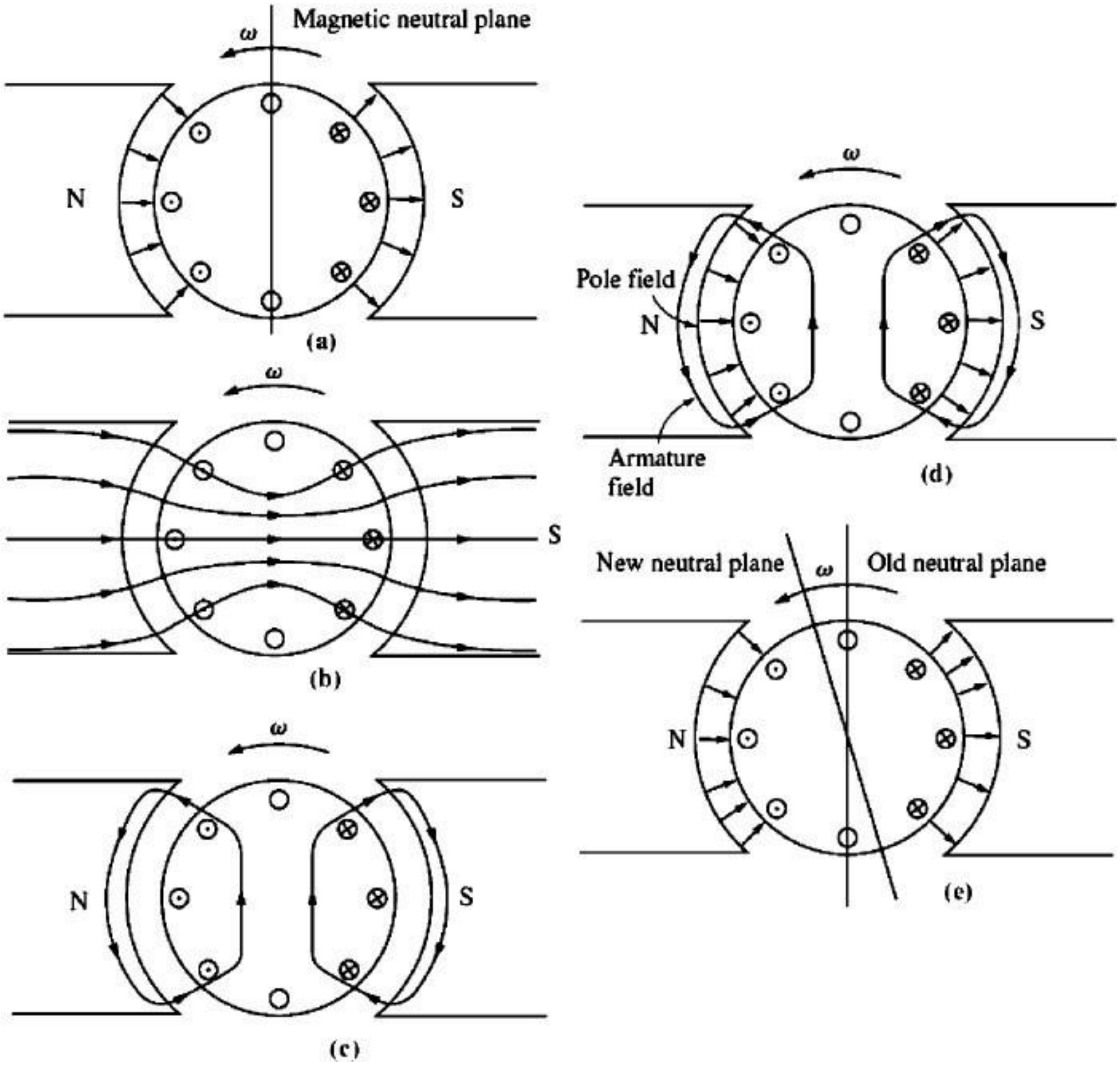


Figure 8-23: The development of armature reaction in a DC generator. (a) Initially the pole flux is uniformly distributed, and the magnetic neutral plane is vertical (b) the effect of the air gap on the pole flux distribution (c) the armature magnetic field resulting when a load is connected to the machine (d) both rotor and pole fluxes are shown indicating points where they add and subtract (e) the resulting flux under the poles. The neutral plane has shifted in the direction of motion.

In general, the neutral−plane shifts in the direction of motion for a generator and opposite to the direction of motion for a motor. Furthermore, the amount of the shift depends on the amount of rotor current and hence on the load of the machine.

So what's the problem with neutral−plane shift? It 's just this: The commutator must short out commutator segments just at the moment when the voltage across them is equal to zero. If the brushes are set to short out conductors in the vertical plane, then the voltage between segments is indeed zero *until the machine is loaded.* When the machine is loaded, the neutral plane shifts, and the brushes short out commutator segments with a finite voltage across them. The result is a current now circulating between the shorted segments and large sparks at the brushes when the current path is interrupted as the brush leaves a segment. The end result is *arcing and sparking at the brushes.* This is a very serious problem, since it leads to drastically reduced brush life, pitting of the commutator segments, and greatly increased maintenance costs. Notice that this problem cannot be fixed even by placing the brushes over the full−load neutral plane, because then they would spark at no load.

In extreme cases, the neutral−plane shift can even lead to *flashover* in the commutator segments near the brushes. The air near the brushes in a machine is normally ionized as a result of the sparking on the brushes. Flashover occurs when the voltage of adjacent commutator segments gets large enough to sustain an arc in the ionized air above them. If flashover occurs, the resulting arc can even melt the commutator's surface.

The second major problem caused by armature reaction is called *flux weakening.* To understand flux weakening, refer to the magnetization curve shown in Figure 8−24. Most machines operate at flux densities near the saturation point. Therefore, at locations on the pole surfaces where the rotor magnetomotive force adds to the pole magnetomotive force, only a small increase in flux occurs. But at locations on the pole surfaces where the rotor magnetomotive force subtracts from the pole magnetomotive force, there is a larger decrease in flux. the net result is that *the total average flux under the entire pole face is decreased* (see Figure 8− 25).

Flux weakening causes problems in both generators and motors. In generators, the effect of flux weakening is simply to reduce the voltage supplied by the generator for any given load. In motors, the effect can be more serious. As the early examples in this chapter showed, when the flux in a motor is decreased, its speed increases. But increasing the speed of a motor can increase its load, resulting in more flux weakening. It is possible for some shunt dc motors to reach a runaway condition as a result of flux weakening, where the speed of the motor just keeps increasing until the machine is disconnected from the power line or until it destroys itself.

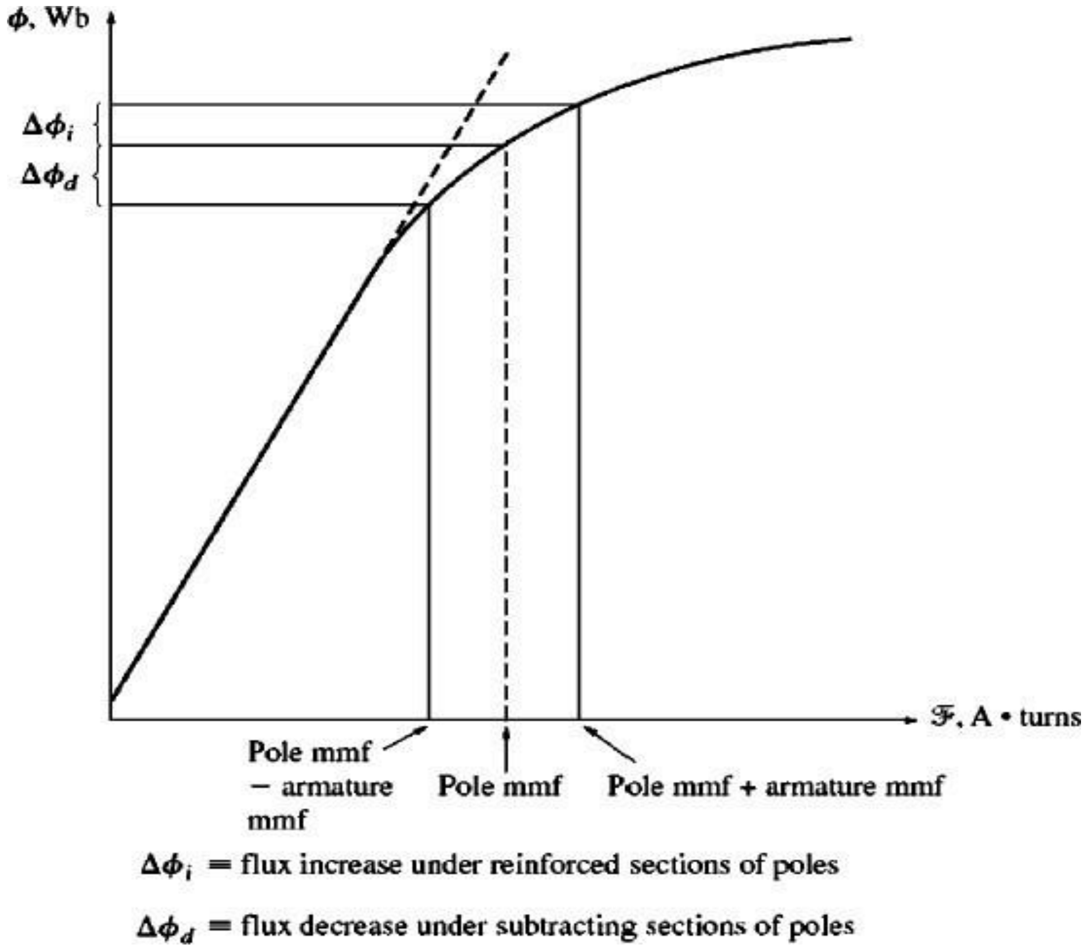


Figure 8-24: A typical magnetization curve shows the effects of pole saturation where armature and pole magnetomotive forces add.

Reactance voltage:

The voltage rise in the short circuited coil due to inductive property of the coil, which opposes the current reversal in it during the commutation period, is called the reactance voltage. It is given by :

Reactance voltage = Coefficient of self−inductance(L) x Rate of change of current (di/dt).

We know that the coil undergoes commutation when the two commutator segments get short−circuited by the brush. During this period the current say **I** changes from **+I** to **-I** . That means di= change in current = **2I**.

The time taken for this change in current is given by dt = (Wb−Wc)/v where

Wb =Width of the brush (cms)

Wm =Width of the mica insulator between the commutator segments (cms)

V = peripheral (linear) velocity of the commutator (armature) (cm/sec)

Then reactance voltage = L.di/dt = L .2I.v/(Wb−Wc)

This reactance voltage also causes sparking at the brushes resulting in the same phenomenon as that produced by neutral phase shifting due to armature reaction.

We can produce reversing e.m.f in two ways. By brush shifting. By using inter-poles or commutating poles.

Methods of improving commutation:

Shifting of Brushes: By shifting the brushes to the new MNA, sparking due to commutation can be avoided. The brushes are to be shifted by the same angle by which the MNA has shifted due to loading. They are to be shifted in the forward direction (in the direction of rotation) in a generator, and backward in a motor. The disadvantage with this method is that the angle of shift will depend upon the load on the machine and therefore is practically difficult to shift the brushes continuously with change in load.

Commutating poles or Interpoles: The basic idea behind this approach is that if the voltage in the conductors undergoing commutation can be made zero, then there will be no sparking at the brushes. To accomplish this, small poles, called *commutating poles* or *interpoles,* are placed midway between the main poles. These commutating poles are located *directly over* the conductors being commutated. By providing a suitable amount of flux with proper polarity from the commutating poles, the voltage in the coils undergoing commutation can be exactly canceled. If the cancellation is exact, then there will be no sparking at the brushes. Exact cancellation of the voltage in the commutator segments is accomplished for all values of loads by connecting the interpole windings in *series* with the windings on the rotor, as shown in the figure below.

The commutating poles do not change the operation of the machine, because they are so small that they affect only the few conductors about to undergo commutation. The *armature reaction* under the main pole faces is unaffected, since the effects of the commutating poles do not extend that far. This means that the flux weakening problem in the machine is not solved by the commutating poles.

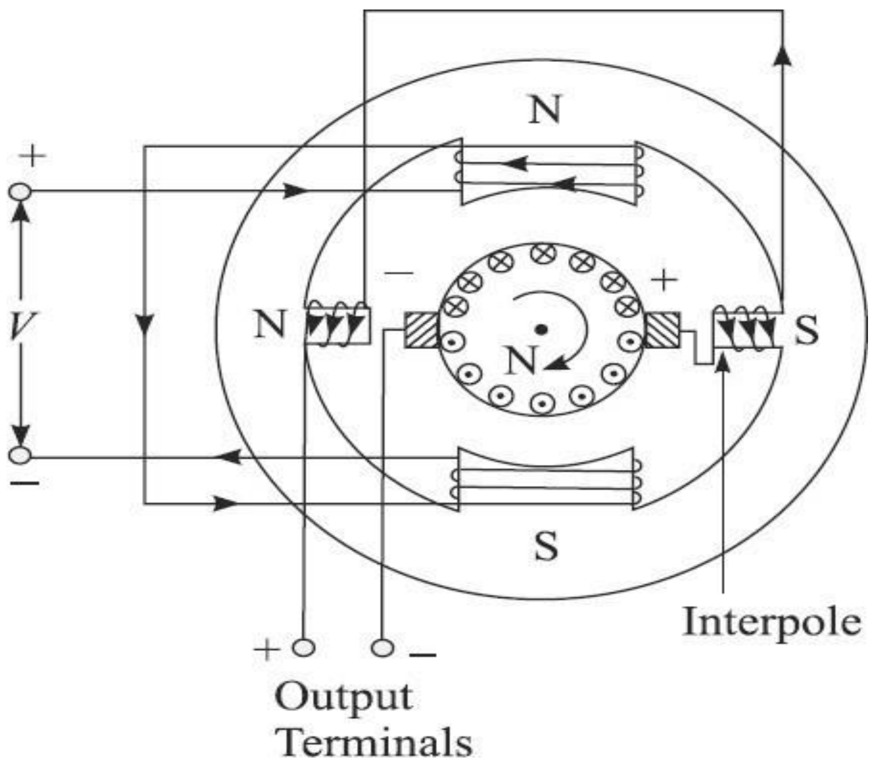


Figure: Connection of commutating poles in a DC Generator

As the load increases and the rotor current increases, the magnitude of the neutral−plane shift and the size of the *L dildt* effects increase too. Both these effects increase the voltage in the conductors undergoing commutation. However, the interpole flux increases too, producing a larger voltage in the conductors that opposes the voltage due to the neutral−plane shift. The net result is that their effects cancel over a broad range of loads. Note that interpoles work for both motor and generator operation, since when the machine changes from motor to generator, the current both in its rotor and in its interpoles reverses direction. Therefore, the voltage effects from them still cancel.

The interpoles must induce a voltage in the conductors undergoing commutation with such a polarity that is *opposite* to the voltage caused by neutral−plane shift and *L dildt* effects. In the case of a generator, the neutral plane shifts in the direction of rotation, meaning that the conductors undergoing commutation have the same polarity of voltage as the pole they just left (see Figure 8− 29).

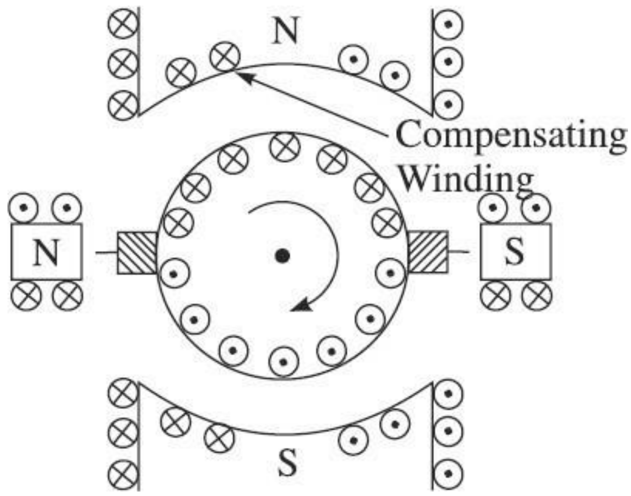


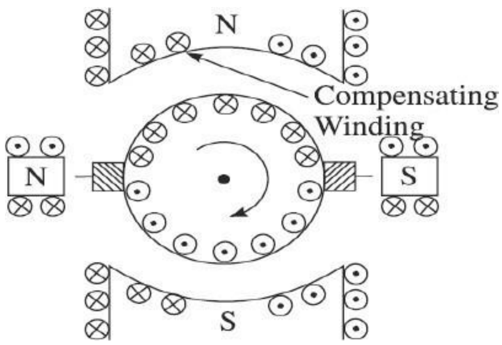
Figure : Compensating windings used to neutralize the effect of armature reaction

To oppose this voltage, the interpoles must have the opposite flux, which is the flux of the upcoming pole. In a motor, however, the neutral plane shifts opposite to the direction of rotation, and the conductors undergoing commutation have the same flux as the pole they are approaching. In order to oppose this voltage, the interpoles must have the same polarity as the previous main pole. Therefore,

1. The interpoles must be of the same polarity as the next upcoming main pole in a generator.
2. The interpoles must be of the same polarity as the previous main pole in a motor.

The use of commutating poles or interpoles is very common, because they correct the sparking problems of dc machines at a fairly low cost. They are almost always found in any dc machine of 1 hp or larger. It is important to realize, though, that they do *nothing* for the flux distribution under the pole faces, so the flux−weakening problem is still present. Most medium−size, general−purpose motors correct for sparking problems with interpoles and just live with the flux weakening effects.

Compensating windings: For very heavy, severe duty cycle motors, the flux−weakening problem can be very serious. To completely cancel armature reaction and thus eliminate both neutral−plane shift and flux weakening, a different technique was developed. This technique involves placing *compensating windings* in slots carved in the faces of the poles parallel to the rotor conductors, to cancel the distorting effect of armature reaction. These windings are connected in series with the rotor windings, so that whenever the load changes in the rotor, the current in the compensating windings also changes. Figure 8− 30 shows the basic



concept. In Figure 8−30(a), the pole flux is shown by itself. In Figure 8−30(b), the rotor flux and the compensating winding flux are shown. Figure 8−30(c) represents the sum of these three fluxes, which is just equal to the original pole flux by itself. The major disadvantage of compensating windings is that they are expensive, since they must be machined into the faces of the poles. Any motor that uses them must also have interpoles, since compensating windings do not cancel ***L di /dt*** effects. The interpoles do not have to be as strong, though, since they are canceling only ***L di /dt*** voltages in the windings, and not the voltages due to neutral−plane shifting. Because of the expense of having both compensating windings and interpoles on such a machine, these windings are used only where the extremely severe nature of a motor's duty demands them.

Important features of DC Generators:

- The terminal characteristic of a DC Machine is a plot of the output quantities of the Machine against each other. For a DC Generator the output quantities are the Terminal Voltage and the Line (Load) current.
- The various types of Generators differ in their terminal characteristics (Voltage−Current) and therefore to the application to which they are suited.
- The DC Generators are compared by their Voltages, Power ratings, their efficiencies and Voltage regulation. Voltage Regulation **(VR)** is defined by the equation: $V_R = [(V_{nl} — V_{fl}) / V_{fl}].100\ \%$ Where V_{nl} is the No load terminal voltage and V_{fl} is the Full load terminal voltage. It is a rough measure of the Generator's Voltage− Current Characteristic. A positive voltage regulation means a drooping characteristic and a negative regulation means a rising characteristic.
- Since the speed of the prime movers affects the Generator voltage and prime movers can have varying speed characteristics, The voltage regulation and speed characteristics of the Generator are always compared assuming that the ***Prime mover's speed is always constant***.

Methods of excitation: (The method by which the field current is generated)

The performance characteristics of a dc machine are greatly influenced by the way in which the field winding is excited with direct current. There are two basic ways of exciting a dc machine.

1. Separate excitation: The field is excited from a separate and independent DC source as shown in fig(a) below. It is flexible as full and independent control of both Field and Armature circuits is possible.

2. Self- excitation: The field is excited either from its own armature voltage (Shunt Excitation: fig−b) or own armature current (Series excitation : fig−c)

The dc machine excitation is also classified in three other ways:

1. Shunt excitation : Here the field winding is excited in parallel with armature circuit and hence the name shunt excitation. It is provided with a large number (hundreds or even thousands) of turns of thin wire and therefore, has a high resistance and carries a small current. Since the armature voltage of a dc machine remains substantially constant, the shunt field could be regulated by placing an external series resistance in its circuit.

2. Series excitation : Here the field winding has a few turns of thick wire and is excited with armature current by placing it in series with armature, and therefore it is known as series field winding. For a given field current, control of this field is achieved by means of a diverter, a low resistance connected in parallel to series winding. A more practical way of a series field control is changing the number of turns of the winding by suitable tappings which are brought out for control purpose.

3. Compound Excitation: In compound excitation both shunt and series fields are excited. If the two fields aid each other such that the resultant air gap flux per pole is increased (their ampere turns are additive), then the excitation is called

cumulative compound excitation as shown in Fig. (d). If the series field flux opposes the shunt field flux such that the resultant air gap flux per pole is decreased, then the excitation is called ***differential compound excitation*** as shown in Fig. (e). The series field is so designed that the increase or decrease in flux/pole is to a limited extent.

Further there are two types of compounding connections. ***Long Shunt*** and ***Short shunt*** . In long shunt compound of Fig. (f) the shunt field is connected across the output terminals. In short shunt compound, the shunt field is connected directly across the armature as shown in Fig. (g). There is no significant difference in machine performance for the two types of connections. The choice between them depends upon mechanical consideration or the reversing switches.

Figure below shows the physical arrangement of shunt and series field windings on one pole of a machine.

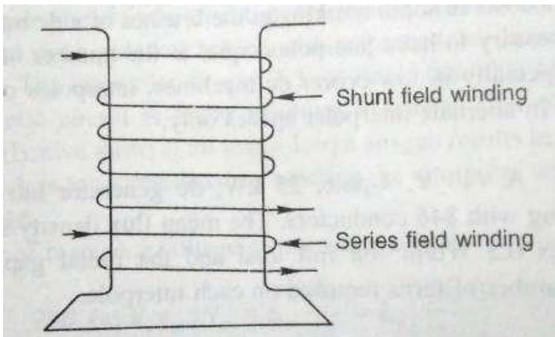
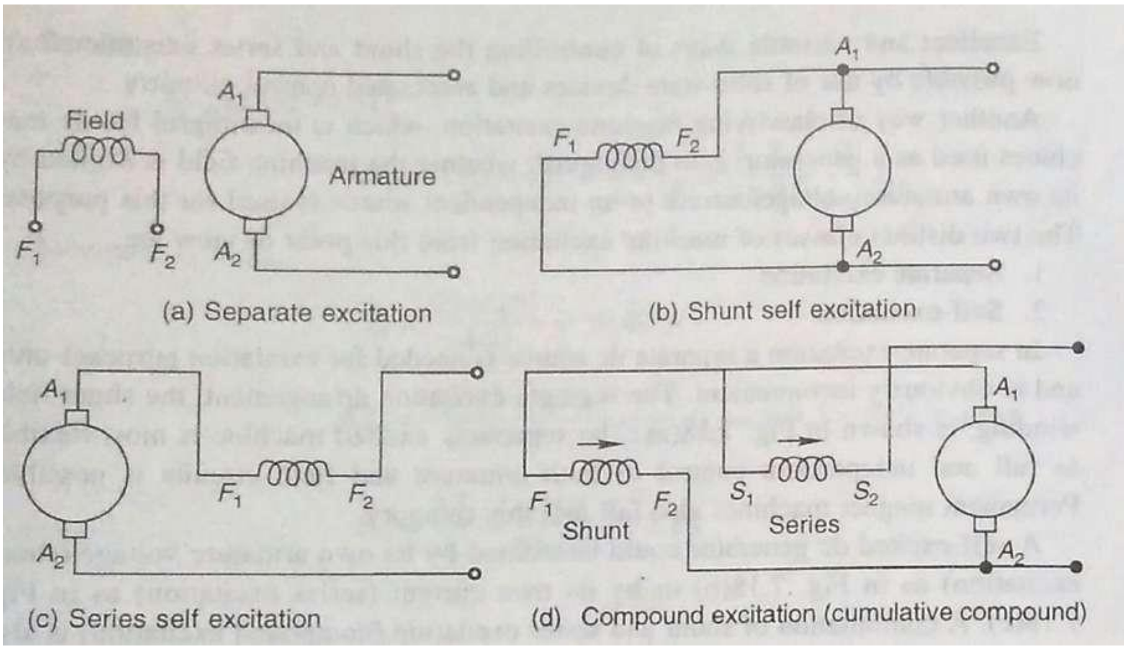
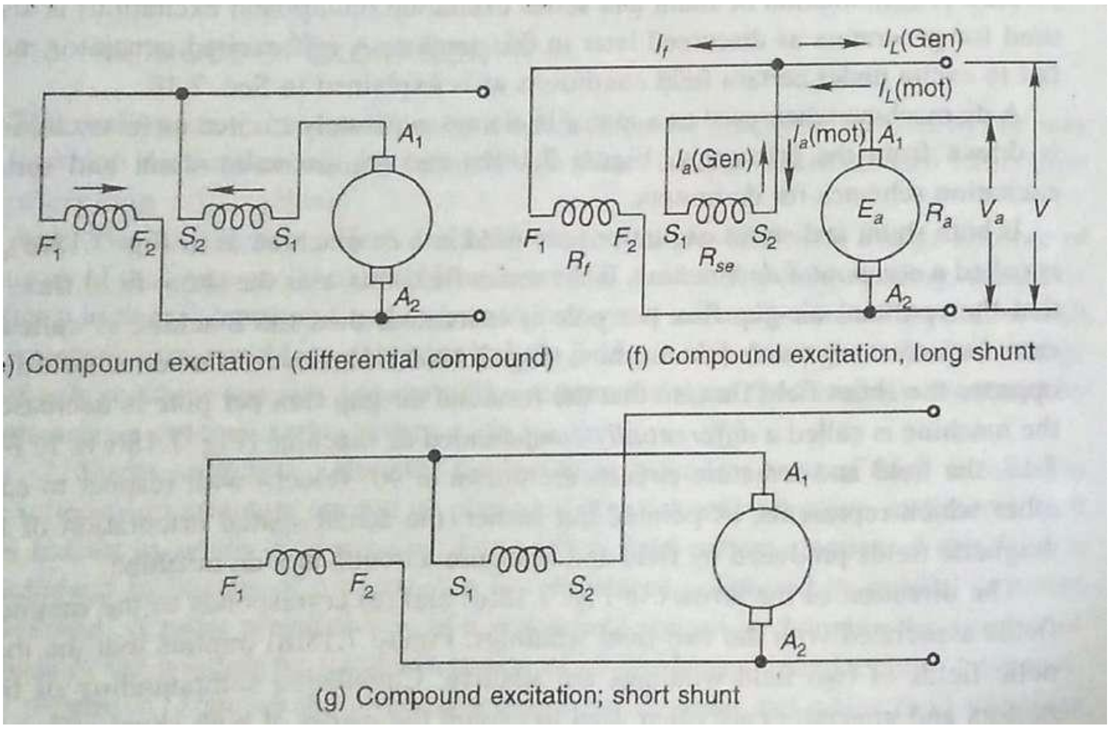


Fig: Arrangement of shunt and series field windings on one pole of a machine.

Excellent and versatile ways of controlling the shunt and series excitations are now possible by use of solid−state devices and associated control circuitry.

In showing the excitation diagrams of a dc machine, the field winding is shown to be at 90˚ (electrical) with respect to the armature circuit which is the actual spatial orientation of the magnetic fields produced by the field and armature circuits in a DC machine.





(e) Compound excitation (differential compound)

(f) Compound excitation, long shunt

(g) Compound excitation; short shunt

Magnetization characteristics of DC Generators:

No load or Open circuit magnetization characteristic of any DC Machine is a plot of the Field flux versus the magnetizing current. Since measurement of field flux is difficult we use the relation for the emf induced in a DC machine $\mathbf{E_A = K. Ø.N}$ from which we can see that the induced voltage is proportional to the Flux in the machine when the speed is maintained constant. Hence we conduct a test on the given DC machine to obtain data on the induced voltage as a function of the field current.

The diagram of the test setup required to obtain the above data is shown in the figure below.

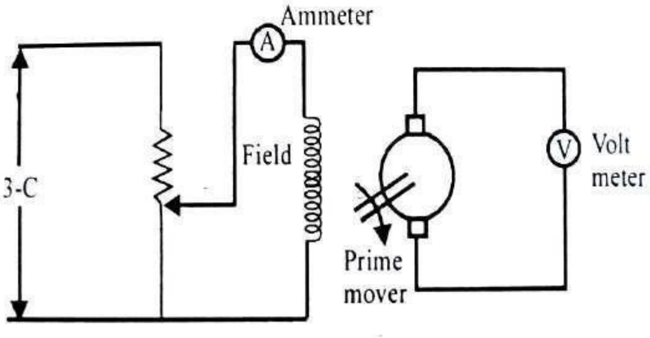


Fig: Test setup with a DC machine to obtain the No load magnetization Characteristic

The prime mover gives the required mechanical energy to the DC Machine and it can be a small Diesel engine. The rheostat connected between the DC Input and the field winding is used to adjust and get the required field current. The field current is initially set to Zero and the Armature volatage is measured. Then the field current is gradually increased and the corresponding values of Armature voltage are measured until the output voltage saturates. Next the field current is brought back to zero gradually and the corresponding Armature voltages are measured at a few points. The corresponding data on Armature voltage is plotted against field current and is shown in the figure below.

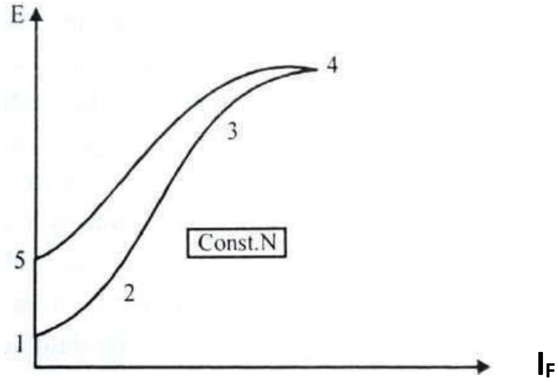


Fig: No load magnetization curve (or OCC) of a DC Machine (Plot of Armature Voltage E_a Vs.Field current I_F)

Though the field current is zero we get a small value of Armature voltage as seen at point 1 due to the residual magnetism present in the field coil. Subsequently armature voltage increases with field current upto some point 3 and then the rate of rise decreses. Finally at poin 4 field flux gets saturated and hence the emf also gets saturated. The plot of armature voltage vs.field current is not same during the field current reduction as that during the field current increase and this is due to the property of magnetic hysteresis in the Ferro magnetic materials. In the return path the induced volatage at zero field current is higher than that during the field current increase. This is due to the combined effect of Hysterisis and the residual magnetism.

Different Types of DC Generators and their Terminal (or Load) Characteristics:

The DC generators are classified according to the manner in which the field flux is produced. Let us consider the following important types of DC Generators and their characteristics along with their equivalent circuits.

The following notation is used uniformly in all the following circuits/characteristics:

- V_T = Generator's Terminal Voltage
- I_L = Load or line current
- I_A = Armature current
- E_A = Armature voltage
- R_A = Armature Resistance
- I_F = Field current
- V_F = Field voltage
- R_F = Field Resistance

Separately Excited Generator: In this type the field flux is derived from a separate power source which is independent of the Generator. The equivalent circuit of such a machine along with the governing equations is shown in the figure below.

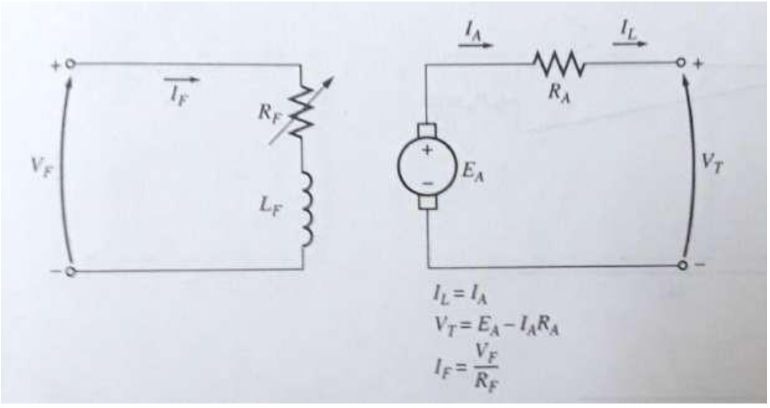


Fig: Equivalent circuit of a separately excited DC Generator

The terminal characteristic of this type of Generators is a plot of $\mathbf{V_T}$ **vs.** $\mathbf{I_L}$ for a constant speed **ω** and the governing equations are :

- The Load or line current $\mathbf{I_L}$ = The armature current $\mathbf{I_A}$
- Generator's Terminal Voltage = $\mathbf{V_T}$ = $\mathbf{(E_A - I_A R_A)}$
- $\mathbf{I_F = V_F / R_F}$

Since the internally generated voltage is independent of $\mathbf{I_A}$,the terminal characteristic of a Separately Excited Generator is a straight line as shown in the figure below.

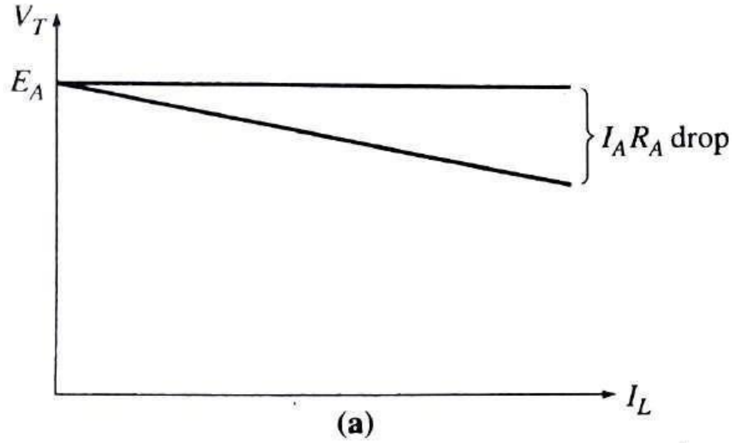
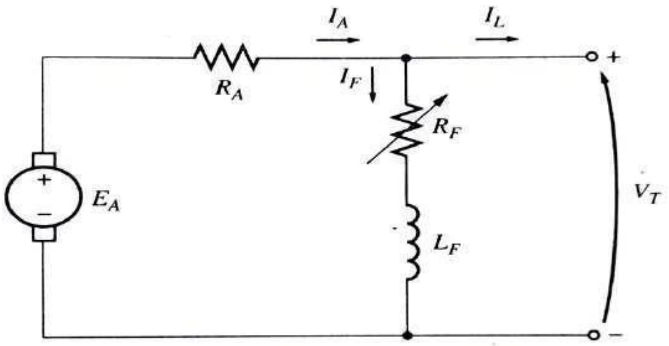


Fig: The terminal Characteristics of a Separately Excited DC Generator

When the load supplied by the generator increases, the load current $\mathbf{I_L}$ increases and hence the armature current $\mathbf{I_A}$ also increases. When the armature current increases, the $\mathbf{I_A R_A}$ drop increases, so the terminal voltage of the generator droops (falls). It is called a drooping characteristic.

Shunt Generator: In this the field flux is derived by connecting the Field directly across the Armature terminals. The equivalent circuit of such a generator is shown in the figure below along with the governing equations.



$$I_A = I_F + I_L$$
$$V_T = E_A - I_A R_A$$
$$I_F = \frac{V_T}{R_F}$$

Fig: The equivalent circuit of a DC Shunt generator along with the relevant governing equations

As could be seen, in this machine the armature current supplies both the load current and the field current. Using the Kirchhoff's voltage law the terminal voltage is seen to be same as that of a separately excited voltage i.e. $\mathbf{V_T = (E_A — I_A R_A)}$**.** In this the advantage is that no external supply is required for the field circuit. *But this leaves an important question. If the generator supplies its own field current how does it get the initial field flux that is required to start the machine and generate voltage when it is first turned on? This is explained below.*

Build-up of E.M.F, Critical Field Resistance and Critical Speed :

Voltage build up in a Shunt Generator :

The voltage build up in a shunt generator depends upon the presence of a ***residual flux*** in the poles of the generator. When a Shunt generator first starts to turn on an internal voltage is generated which is given by $\mathbf{E_A = k.\ Ø_{res}.\ ω.}$ This voltage(which may be just one or two volts) appears at the generator terminals. This causes a current to flow in the generator's field coil $\mathbf{I_F = V_T / R_F}$. This produces a m.m.f. in the poles which in turn increases the flux in them. The increase in the flux causes an increase in $\mathbf{E_A = k.Ø . \omega}$ which in turn increases the terminal voltage $\mathbf{V_T}$. When $\mathbf{V_T}$ rises, $\mathbf{I_F}$ increases further, increasing the flux more which increases $\mathbf{E_A}$ and so on. This voltage build up phenomenon is shown in the figure below.

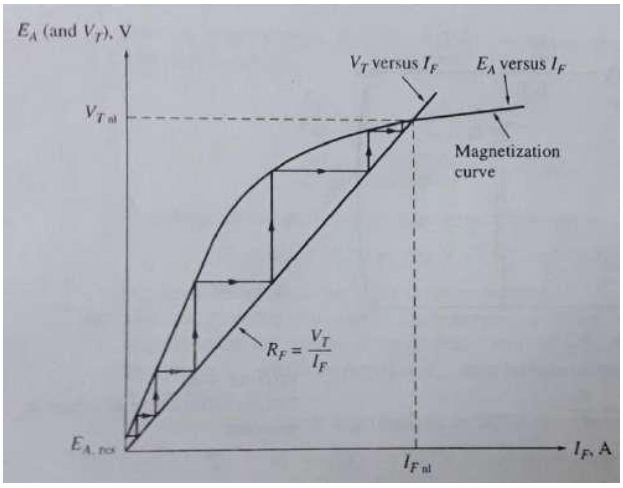


Fig: Voltage build up on starting in a DC Shunt generator

It is to be noted here that it is effect of ***magnetic saturation*** in the Pole faces which eventually limits the build of the terminal voltage.

The voltage build up in the figure above shows up as though it is building up in discrete steps. It is not so. These steps are shown just to make it clear the phenomenon of positive feedback between the Generator's internal voltage and the field current. In the DC Shunt generator both $\mathbf{E_A}$ and $\mathbf{I_F}$ increase simultaneously until the steady state conditions are reached.

Critical Resistance: For understanding the terms *critical Resistance* and *critical speed*, the open circuit characteristic (OCC) or the magnetization characteristic of a DC machine is shown again in the figure below along with *air gap* line and R_f line. The extension of the liner portion of the magnetization curve, shown dotted in the figure below is known as the *air−gap line* as it represents mainly the magnetic behavior of the machine’s air−gap.

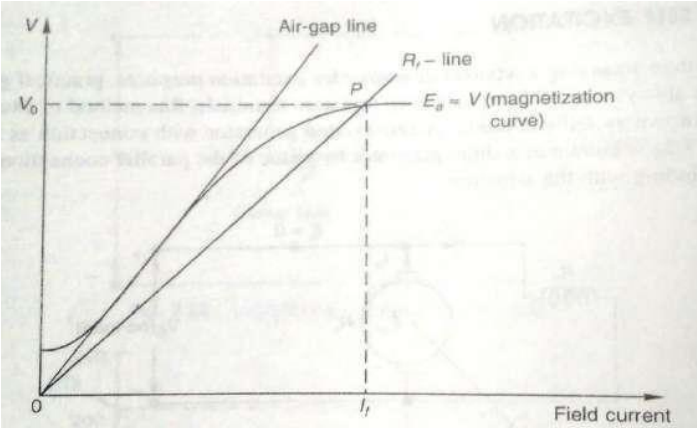


Fig: Open Circuit Characteristic of DC machine along with Air gap and R_f lines

As already explained in the topic *Build up of EMF in a DC shunt generator:* At the instant of switching on the field after the armature has been brought to rated speed, the armature voltage corresponds to a small residual value which causes a small field current to flow. If the field is connected such that this current increases the field mmf and therefore the induced emf, the machine voltage cumulatively buildups and settles at a final steady value because of the saturation characteristic of the machine’s magnetic circuit.

Since the generator is assumed to be on no−load during the build−up process, the following circuit relationships apply with reference to the machines’ equivalent circuit shown in the figure below.

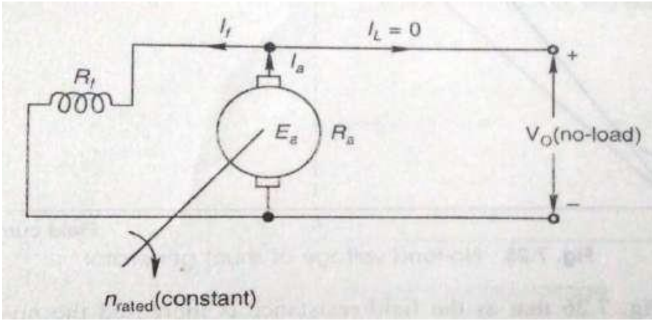


Fig: The equivalent circuit of DC shunt generator

$$I_a = I_f$$
$$V = E_a - I_f R_a$$

The field current in a shunt generator being very small, the voltage drop $I_f R_a$ can be neglected so that : $V_o = E_a(I_f)$ (magnetization characteristic)

And for the field circuit:

$$V_o = I_f R_f$$

which is a straight line relationship, called the R_f−line as shown in the OCC plot earlier. The no−load terminal voltage is the solution of the above two eqations for V_o.Thus the intersection point P of the R_f −line with the magnetization characteristic as shown in the OCC gives the no−load terminal voltage (V_O) and the corresponding field current. Further, it is easy to visualize from this figure that the no−load voltage can be adjusted to a desired value by changing the field resistance.

It can be seen in the figure below that as the field resistance is increased the no−load voltage decreases.

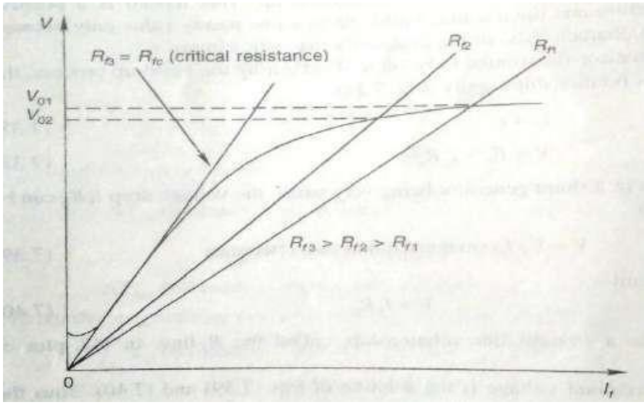


Fig: Variation of No load voltage with field resistance

The no−load voltage is undefined for a field resistance (R_{f3} = R_{fc}) whose line coincides with the linear portion of the magnetization curve. With field resistance even slightly more than this value, the machine does not excite to any appreciable value and would give no−load voltage close to the residual value. The machine with this much resistance in the field fails to excite and the corresponding resistance is known as the *critical resistance* ($\boldsymbol{R_{fc}}$).

Critical speed: Consider now the operation with fixed $\boldsymbol{R_f}$ and variable armature speed as illustrated in the figure below. It can be observed that as the speed is reduced, the OCC proportionally slides downwards so that the no−load voltage *reduces.* At a particular speed, called the *critical speed,* the OCC becomes tangential to the R_f line and as a result the generator would fail to excite.

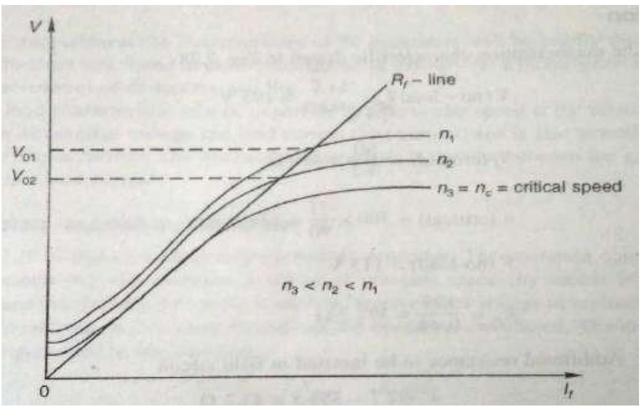


Fig: Effect of speed on No load voltage

Causes of failure to excite in a Shunt Generator:

A shunt generator may not get excited in certain conditions. The causes of such failure to excite, the method of detection and the corresponding remedial measures are given in the table below.

S.No	Cause	Method of detection	Remedy
1	Absence of residual magnetism due to ageing	Zero reading on Voltmeter after rotating the machine	Operate the Generator as separately excited machine first and then as separately excited
2	Wrong field winding connections. Due to this the flux gets produced in opposite direction to that of the residual flux and they cancel each other.	Voltmeter reading decreases rather than increasing as the field current is increased	Interchange the field connections
3	Field resistance is more than the Critical field resistance.	Voltmeter shows zero reading	Field resistance to be reduced using suitable field diverter
4	Generator is driven in opposite direction	This wipes out the residual flux and the machine fails to excite	Generator to be driven in the proper direction

Terminal characteristics of a shunt generator :

The terminal characteristics of the shunt generator differ from that of the separately excited generator because the amount of field current depends on its terminal voltage. As the generator load is increased, the load current I_L increases and so $I_A = I_F + I_L$ also increases . An increase in I_A increases the I_AR_A drop causing $V_T = (E_A - I_A R_A)$ to decrease. This is precisely the same behavior we have seen in the case of separately excited generator. However, in the shunt generator when V_T decreases the field current decreases ,hence the field flux deceases thus decreasing the generated Voltage E_A **.** Decreasing the E_A causes a further decrease in the terminal voltage $V_T = (E_A M - I_A R_A)$**.** The resulting characteristic is shown in the figure below.

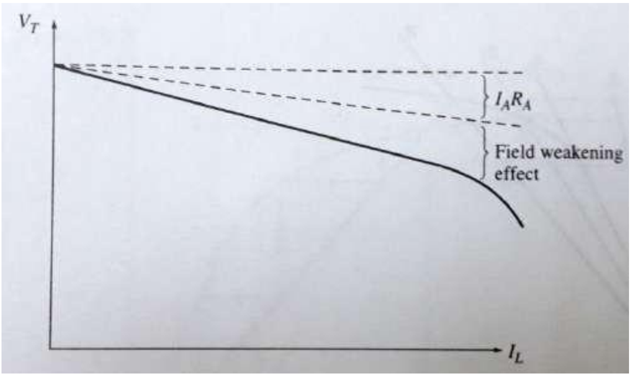


Fig: Terminal Characteristic of DC Shunt Generator

It can be noticed that the drop with load is steeper than that of a separately excited motor due to the field weakening affect. This means that the regulation of a Shunt Generator is worse than that of a Separately Excited Generator.

DC Series Generator: In this the field flux is derived by connecting the Field coil in series with the Armature of the Generator as shown in the figure below.

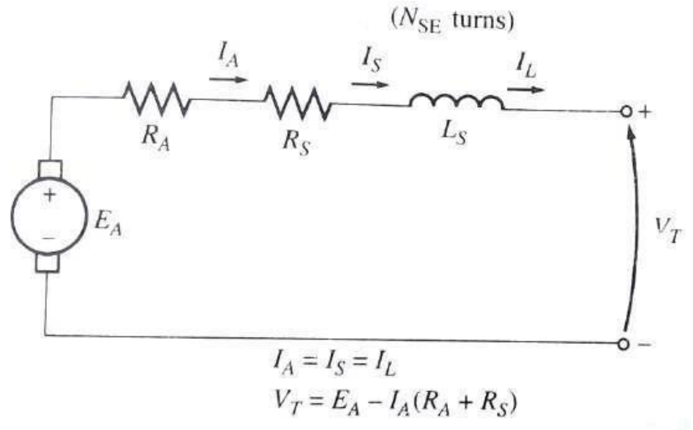


Fig: Equivalent circuit of DC Series Generator along with the governing equations

As shown, the armature current, load current and field current are same in a DC series generator. i.e **$I_A = I_F = I_L$** . Since the mmf produced by the fields is given by $\mathcal{F}$, **= NI** and the field current is more in the DC series generator , the fileld winding is wound with lesser number of turns and also with a thicker gauge so as to offer less field resistance since full load current flows through the field winding .

The terminal characteristic of a DC Series Generator looks very much like the magnetization curve of any other type of generator and is shown in the figure below.

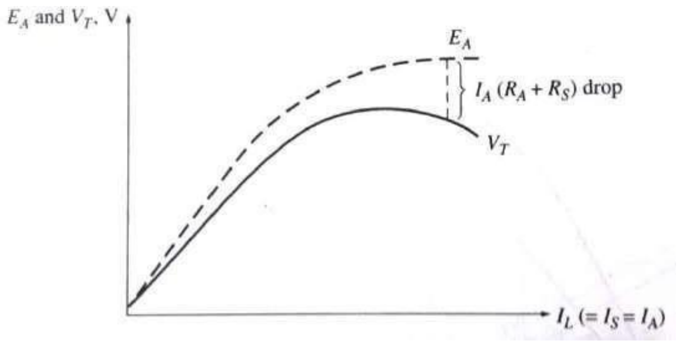


Fig: Terminal Characteristic of DC Series Generator

At no load however since there is no field current,armature voltage $\mathbf{E_A}$ and also the terminal voltage $\mathbf{V_T}$ are very small (generated by the small amount of residual flux.) As the load increases ,field current rises hence $\mathbf{E_A}$ also increases rapidly. The $\mathbf{I_A\ (R_A+R_F\ )}$ drop also goes up but this rise is less predominant compared to the rise in $\mathbf{E_A}$ initaially and hence $\mathbf{V_T}$ also rises initially. After some time field flux gets saturated and hence the induced voltage $\mathbf{E_A}$ will be constant without any further rise. At this stage the resistive drop predominates and hence the ***terminal voltage V_T starts drooping*.**

DC Compound generator:

As we know in DC shunt Generator the terminal Voltage falls and in a DC series generator the terminal voltage increases on loading. A compound DC Generator is the one in which there will be both Series and shunt field coils. If they are wound such that they aid each other then it is called a Cumulative Compound DC Generator and if they are wound such that the two fields oppose each other, then it is called a differential Compound DC Generator. The equivalent circuit diagram of such Cumulative DC Generator along with relevant governing equations is shown in the figure below.

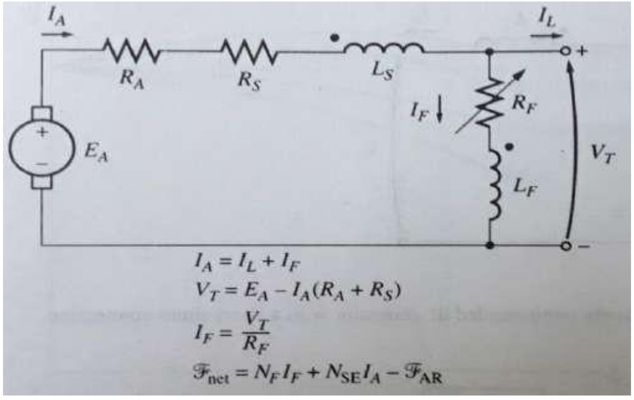


Fig: Equivalent circuit of a Cumulative compound DC Generator

The circuit diagram is shown with standard ***dot convention*** on the field windings.

i.e. The current flowing into the dot side of the winding produces a positive mmf .

And as can be seen that both $\mathbf{I_F}$ in the shunt winding and $\mathbf{I_A}$ in the series winding flow into the dot side and hence both produce magnetic fields which are positive and hence aid each other.

When the two fields are aiding each other we get a characteristic which will have the combined effect of ***drooping*** (due to the shunt coil) and ***rising*** (due to the field coil). Whichever coil current is more its effect will be more predominant. The terminal characteristics of a cumulative compound DC Generator are shown in the figure below for all the three cases.

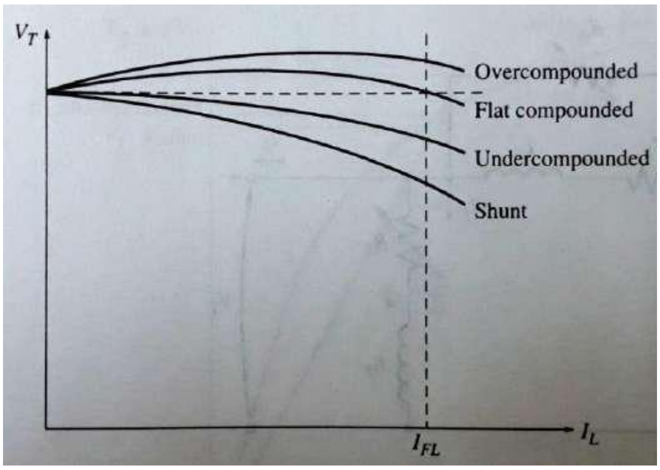


Fig: Terminal Characteristics of a DC Compound Generator

1. If the Series field effect is more dominating than that of the Shunt field coil then we get the ***Over compounded*** characteristic where the full load terminal voltage is higher than the no load terminal voltage.
2. If the Series field effect is equal to that of the Shunt field coil then we get the ***Flat compounded*** characteristic where the full load terminal voltage is equal to the no load terminal voltage.
3. If the Shunt field effect is more dominating than that of the Series field coil then we get the ***Under compounded*** characteristic where the full load terminal voltage is lower than the no load terminal voltage.
   The normal shunt characteristic is also shown in the figure for comparison.

Important concepts and Formulae:

- **Voltage generated in a DC machine:** $E_A = (\text{Ø}\, ZN/60).(P/a)$ **and in terms of angular speed ω :**
  $E_A = K_a\, \text{Ø}\, \omega$ **where** $K_a = ZP/2ua$

Illustrative Examples:

Ex.1: Calculate the e.m.f. generated by a 6 pole DC Generator having 480 conductors and driven at a speed of 1200 RPM. The flux per pole is 0.012 webers. (a) When the machine is lap wound (b) When the machine is wave wound

Solution: We know that the e.m.f. generated by a DC Generator is given by

E_A **= (Ø ZN/60)(P/a)** where

Ø Flux per pole (webers) = 0.012Wb

Z Total number of conductors on the armature = 480

a The number of parallel paths **=** No of Poles P (= 6) when Lap wound and = 2 when

wave wound

N Speed of rotation of the machine (RPM) = 1200 RPM

P The number of poles = 6

(a) For Lap wound machine **a** = P = 6

E_a = [(0.012 x 480 x 1200) / 60] [6/6] **= 115.2 Volts**

(b) For wave wound machine **a** = 2

E_a = [(0.012 x 480 x 1200) / 60] [6/2] **= 345.6 Volts**

Ex.2 : A 50 Kw ,250 V shunt generator operates at 1500 RPM .The armature has 6 poles and is lap wound with 200 turns. Find the induced e.m.f and the flux per pole at full load given that the armature and the field resistances are 0.01 Ω and 125 Ω respectively.

Solution:

Output line current = Output power / Line voltage = 50 x 1000 / 250 = 200 A

Field current = Line Voltage / Field resistance = 250 / 125 = 2 A

Armature current in a shunt generator: $= I_l + I_f$ = 200 + 2 = 202 A

Induced e.m.f E_a : = Line Voltage + Armature drop ($I_a R_a$ drop)

= 250 + 202 x 0.01 = **252.02 V**

But we know that armature voltage in terms of the basic machine parameters is also given by

E_A = (Ø ZN/60)(P/a) where

Ø c: Flux per pole (webers) = To be determined

Z : Total number of conductors on the armature = Number of turns x 2 (since each turn has two conductors) = 200 x 2 = 400

a : The number of parallel paths **=** No of Poles P (= 6) (since Lap wound)

N : Speed of rotation of the machine (RPM) = 1500 RPM

P : The number of poles = 6

 Ø = (E_A x 60 x a/ ZNP) = 252.02 x 60 x 6 / 400 x 1500 x 6 = **0.025202 Wb**

Ex.3: A shunt generator connected in parallel to supply mains is delivering a power of 50 Kw at 250 V while running at 750 RPM. Suddenly its prime mover fails and the machine continues to run as a motor taking the same 50 Kw power from 250 V mains supply. Calculate the speed of the machine when running as a motor given that R_a = 0.01 Ω, R_f = 100 Ω and brush drop is 1 V per brush.

Solution:

First let us calculate the Voltage generated by the machine while running as a generator under the given conditions:

Output line current = Output power / Line voltage = 50 x 1000 / 250 = 200 A

Field current = Line Voltage / Field resistance = 250 / 100 = 2.5 A

Armature current : $I_l + I_f$ = 200 + 2.5 = 202.5 A

Induced e.m.f E_a : = Line Voltage + Armature drop (I_aR_a drop)+ Brush drop(two brushes)

= 250 + 202.5 x 0.01 + 2 x1 = 254.025 V

Next let us calculate the Voltage generated by the machine while running as a motor under the given conditions :

Input line current = Input power / Line voltage = 50 x 1000 / 250 = 200 A

Field current = Line Voltage / Field resistance = 250 / 100 = 2.5 A

Armature current : $I_l - I_f$ = 200 − 2.5 = 197.5 A

Induced e.m.f or back e.m.f E_b : = Line Voltage − Armature drop(I_aR_a drop) − Brush drop(two brushes)

= 250 − 197.5 x 0.01 − 2 x1 = 246.025 V

We know that the voltage induced in the machine is proportional to the speed i. e

Generator armature voltage is proportional to Generator speed : E_a a N_G and similarly

Motor back e.m.f is proportional to Motor speed : E_b a N_M

Hence $E_a / N_G = E_b / N_M$ or $\mathbf{N_M} = (E_b / E_a) N_G = (246.025 / 254.025) \times 750 =$ **726 RPM**

Example 4 : The following figures give the open−circuit characteristics of a dc shunt generator at 300 rpm:

I_f (A)	0	0.2	0.3	0.4	0.5	0.6	0.7
V_{oc} (V)	7.5	93	135	165	186	202	215

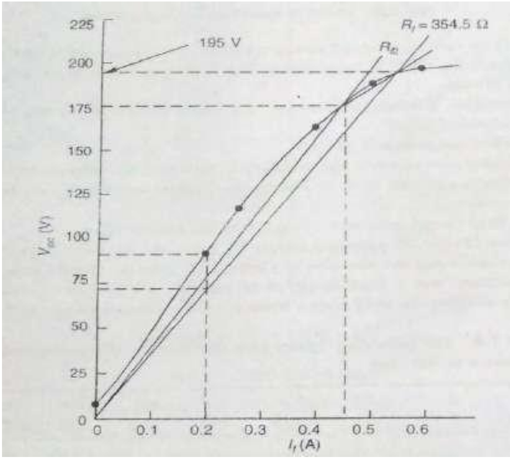
The field resistance of the machine is adjusted to 354.5 W and the speed is 300 rpm.

(i) Determine graphically the no−load voltage.
(ii) Determine the critical field resistance.
(iii) Determine the critical speed for the given field resistance.
(iv) What additional resistance must be inserted in the field circuit to reduce the no−load voltage to 175 V.

Solution:

Step−1 : Let us draw the Field resistance line corresponding to 354.5 Ω on the OCC (magnetization characteristic). This can be done by identifying a point corresponding to a Voltage and current below the OCC corresponding to 354.5 Ω and extending the line joining this point with the origin.

1. $V_{\text{No load}}$: This is the voltage corresponding to the point of intersection of the OCC and the R_f line corresponding to 354.5 Ω and is seen to be = 195 V
2. R_f critical : To obtain this draw a line tangential to the OCC starting from the Voltage at I_f = 0 A. Note down the voltage at which the tangential deviates from the OCC and the corresponding I_f. Dividing this voltage by the corresponding I_f we get the critical resistance viz. 90/0.2 = 450 Ω



3. Critical Speed: We know that as speed reduces the armature voltage reduces. i.e. the OCC leans down wards with decrease in speed and becomes tangential to the existing R_f line itself. So to find out the critical speed we have to find out the new E_a from the OCC corresponding to the lesser speed which deviates from the existing R_f line. This is done by dropping a vertical perpendicular line from the point of deviation of the critical resistance line from the original OCC and identifying its intercept on the existing R_f line. Then by drawing a line parallel to the I_f axis from this point and locating its intercept with the Voltage axis, the new E_a is found out.

   Then Critical speed = Original RPM x new E_a / Original E_a = 300 x71/90 = 236.7 RPM

4. To find out the additional resistance to be introduced into the field to get a new no load voltage of 175 V first we have to find out the value of I_f corresponding to the new no load voltage. This can be directly read from the OCC and then from these voltage and current values we can directly get the new value of R_f and thus the additional value of R_f to be introduced into the field circuit.

   Thus new R_f = 175/0.44 = 397.7 Ω and

   The additional resistance to be introduced into the field = 397.7 − 354.5 = 43.2 Ω

UNIT – II
D.C. MOTORS

CONTENTS:

- **Principle of operation**
- **Back E.M.F - Torque equation**
- **Characteristics and application of shunt, series and compound motors**
- **Armature reaction and commutation.**
- **Speed control of D.C. Motors - Armature voltage and field flux control methods.**
- **Motor starters (3 point and 4-point starters)**
    - **Important concepts and Formulae**
    - **Illustrative examples**

Principle of operation: DC Motors are DC machines used as motors. A DC Motor converts the input DC power into output rotational mechanical power from the following principle. A current carrying conductor placed in a magnetic field experiences a mechanical force given by **F= i (l X B).**

When a group of such conductors is placed on a rotor and are connected properly the force experienced by the all the conductors together gets translated into a torque on the rotor (armature) and it starts rotating. We will derive an expression for such a Torque developed by a DC Motor from the first principles and its equivalent circuit by equating the Electrical power given to the motor (excluding the losses) to the mechanical power developed by the motor.

Torque developed by a DC Motor:

Consider the equivalent circuit of a DC motor as shown in the figure below.

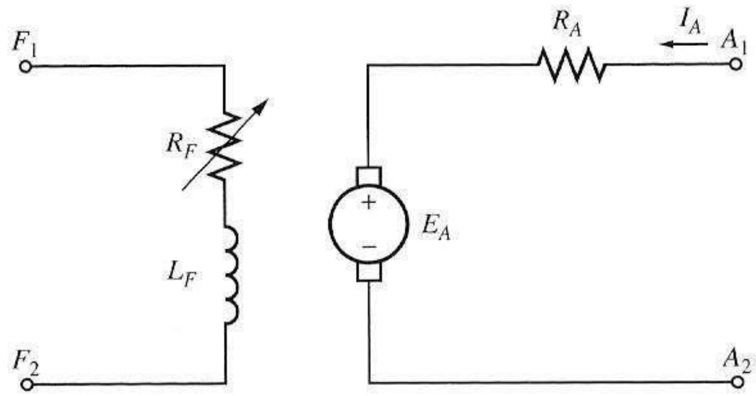


Fig: Equivalent circuit of a DC motor

In this figure, the armature circuit is represented by an ideal voltage source $\mathbf{E_A}$ and the armature resistance $\mathbf{R_A}$. The field coils, which produce the magnetic flux in the motor, are represented by inductor $\mathbf{L_F}$ and the field resistance $\mathbf{R_F}$. The separate external variable resistor used to control the amount of current in the field circuit is also combined with the field resistance and is together shown as $\mathbf{R_F}$.

We know from the earlier study of generators that the voltage generated in a DC Machine when It's armature is rotating in a magnetic flux of **Ø** webers/pole is given by $E_A = K_A. Ø.\omega$ where K_A is given by:

$$K_A = (ZP/2ua)$$

Now in the DC Motor also, when it is rotating, from the same fundamental principle of Generator a Voltage is generated across the armature and it is now called back EMF and is normally shown as E_b to distinguish it from the voltage generated in the armature of a generator which was shown as E_A **.**

The governing equation of the DC Motor armature circuit now becomes:

$$V_T = E_b + I_aR_A \text{ or } E_b = V_T - I_aR_A$$

(as against $V_T = E_A - I_aR_A$ in the case of a generator where I_A flows from armature towards the external terminals i.e external load)

Since now an external voltage V_T is applied to the motor terminals , direction of armature current changes i.e. now it flows from external terminals towards the armature.

The power delivered to the motor is given by : $P_{in} = V_T . I_a$. From this, the loss of power in the armature is equal to $I_a^2R_A$ and hence the net power given to the motor armature is given by :

$$P_m = V_T . I_a - I_a^2R_A = I_a (V_T - I_aR_A) = I_a . E_b$$

$$P_m = I_a . E_b$$

This net electrical power is converted into mechanical power. We know that in mechanical rotational systems the power is equal to Torque times the speed. In the SI system of units which is the present Industry standard it is given by :

$$P_{mech} \text{ (watts)} = \gamma \text{ (Nw.mtrs)}.\omega \text{ (Radians/second)}$$

For simplification if we ignore the mechanical losses in the motor,then :

$$P_m = I_a . E_b = P_{mech} = \gamma .\omega$$

$$\text{i.e. } \gamma .\omega = I_a . E_b = E_b . I_a$$

Substituting the value of $E_A = K_A \cdot Ø \cdot \omega$ we got in generators here for E_b since they are the same induced emfs we get $\Upsilon \cdot \omega = I_a \cdot K_A \cdot Ø \cdot \omega$ or

$$\Upsilon = K_A \cdot Ø \cdot I_a$$

It is to be noted that this expression for the torque induced in a motor is similar to the voltage induced in a DC Generator except that the speed **ω** in the DC Generator is replaced by the Armature current I_a. The constant K_A is same and is given by K_A = **(ZP/2ua)**

In general, the torque c in the DC motor will depend on the following 3 factors:

1. ***The flux Ø in the machine***
2. ***The armature current I_a in the machine***
3. ***The same constant K_A representing the construction of the machine***

Types of DC Motors and their output (or terminal) Characteristics:

There are three important types DC Motors: DC separately excited, Shunt and Series motors. We will explain their important features and characteristics briefly.

The terminal characteristic of a machine is a plot of the machine's output quantities versus each other. For a motor, the output quantities are shaft ***torque*** and ***speed,*** so the terminal characteristic of a motor is a plot of its output ***torque versus speed.*** (Torque/Speed characteristics)

They can be obtained from the Motor's Induced voltage and torque equations we have derived earlier plus the Kirchhoff's voltage law around the armature circuit and are again given below for quick reference.

- The internal voltage generated in a DC motor is given by: $E_b = K_a \cdot \Phi \cdot \omega$
- The internal Torque generated in a DC motor is given by: $\Upsilon = K_a \cdot \Phi \cdot I_a$
- KVL around the armature circuit is given by : $V_T = E_b + I_a \cdot R_a$

Where	Φ	=	Flux per pole	....	Webers
	I_a	=	Armature current	....	Amperes

V_T = Applied terminal Voltage .... Volts
R_a = Armature resistance .... Ohms
ω = Motor speed .... Radians/sec
E_b = Armature Back EMF .... Volts
K_a = **(ZP/2ua)** : Motor Back EMF/Torque constant

From the above three equations we get the relation between Torque and speed as:

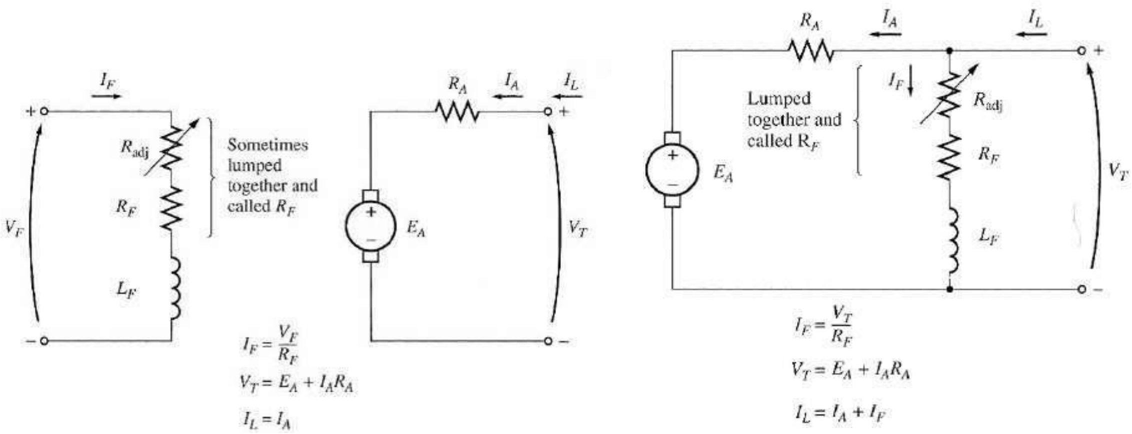
$$\omega = (V_T / K_a.\,\Phi\ ) -- (R_a/ K_a.\,\Phi).\ I_a$$

$$\omega = (V_T / K_a.\,\Phi\ ) -- [R_a/ (K_a.\,\Phi)^2+.\text{Y}$$

We will use this generalized equation in different types of motors and obtain their ***Torque vs. Speed*** characteristics.

DC separately excited and Shunt Motors:

The Equivalent circuits of DC separately excited and Shunt Motors along with their governing equations are shown in the figure below.



(a) Separately Excited (b) Shunt

Fig: Equivalent circuit of DC separately excited and Shunt Motors

In a separately excited DC motor the field and armature are connected to separate voltage sources and can be controlled independently. In a shunt motor the field and the armature are connected to the same source and cannot be controlled independently. When the supply voltage to a motor is assumed constant and is same to the field and armature circuits, there is no practical difference in behavior between these two machines. Unless otherwise specified, whenever the behavior of a shunt motor is described, it would be same as that of a separately excited motor.

In both their cases, with a constant field current the field flux can be assumed to be constant and then **($K_a.\Phi$)** Would be another constant **K**. Then the above Generalized Torque speed relations would become:

$$\omega = V_T / K -- (R_a / K).\ I_a$$

Substituting the value of I_a in terms of **Y (I_a = Y / $K_a.\Phi$ = Y / K)** we get

$$\omega = V_T / K -- [R_a / (K)^2].\text{Y}$$

This equation is just a straight line with negative slope. The resulting Speed/ Torque Characteristics of a DC Separately Excited /Shunt Motor for a rated terminal voltage and full field current are shown in the figure below. It is a drooping straight line.

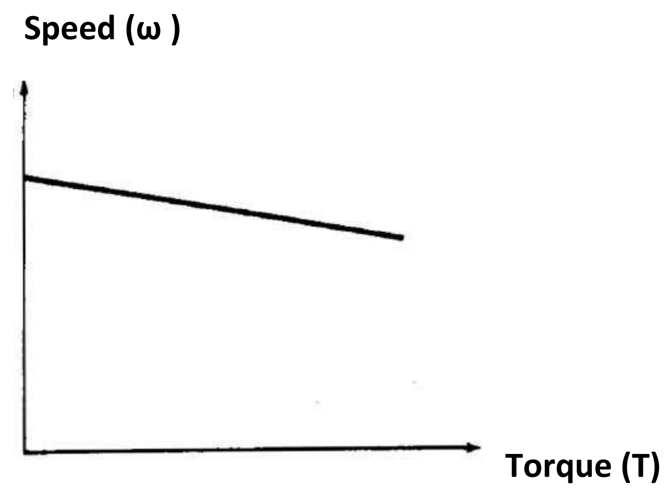


Fig: Speed/ Torque Characteristics of a DC Separately Excited/Shunt Motor

The no load speed is given by the Applied armature terminal voltage and the field current. Speed falls with increasing load torque. The speed regulation depends on the Armature circuit resistance. The usual drop from no load to full load in the case of a medium sized motor will be around 5%. Separately excited motors are mostly used in applications where good speed regulation and adjustable speed are required.

If motor armature reaction is taken into account, then as its load increases, the flux−weakening effects reduce its flux. From the motor speed equation above, the effect of reduction in flux is to increase the motor's speed at any given load over the speed it would run at without armature reaction. Though at a first glance of the Speed torque equation it may appear that the effect of reduction in flux is to decrease the motor's speed at any given load (since **Φ^2** is in the denominator**)** actually since the first positive term contains V_T which is much larger quantity compared to the second negative term, IaRa drop the net effect would only be to increase the motor's speed at any given load. The torque−speed characteristic of a shunt motor with armature reaction is shown below:

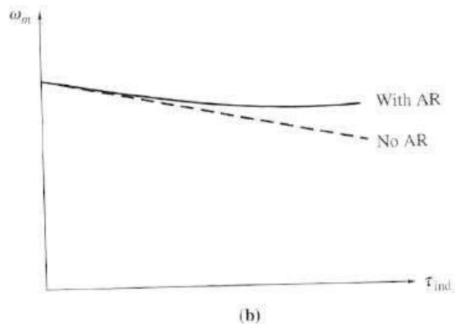


Fig: Torque-speed characteristic of the motor with armature reaction considered

Motor's Other Characteristics: Though the terminal characteristics **(Speed vs. Torque)** are only important for analysis of a DC motor performance, study and understanding of the other characteristics like speed vs. I_a and Torque vs. I_a would also give additional insight into the performance of the motor and hence they are obtained from the basic equations and presented below:

- **Speed vs.** I_a **:** $$E_b = K_a . \Phi . \omega = V_T - I_a . R_a$$
$$\omega = (V_T - I_a . R_a) / K_a . \Phi$$

- **Torque vs.** I_a: $$\gamma = K_a . \Phi . I_a$$

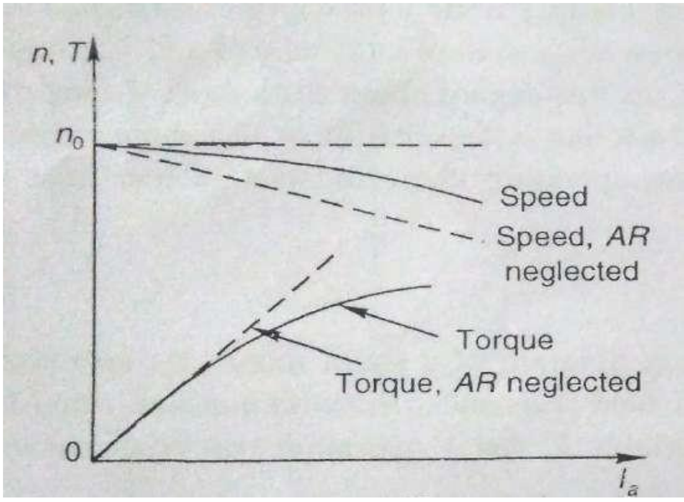


Fig: Speed and torque vs. Armature current for a DC shunt motor

DC Series Motor:

The equivalent circuit of a DC Series motor is shown in the figure below.

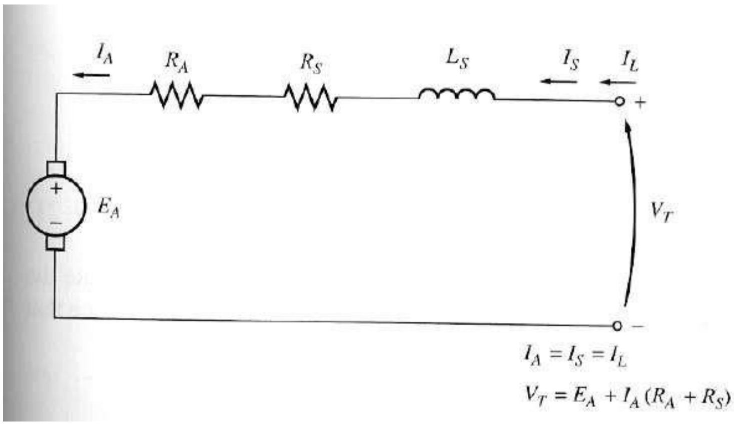


Fig: Equivalent Circuit of a DC Series Motor

In a series motor the field current and armature current are same and hence the field flux is directly dependent on the armature current. Hence during the initial i.e unsaturated region of the magnetization characteristic the flux **Φ** can be assumed to be proportional to the armature current.

$$\text{Then } \boldsymbol{\Phi = K_f . I_a}$$

And using this value in the first basic motor relation given earlier we get:

$$\boldsymbol{\Upsilon = K_a.\ \Phi . I_a = K_a.\ K_f I_a^2}$$

$$\boldsymbol{\Upsilon = K_{af} . I_a^2 \quad (\text{ where } K_{af} = K_a . K_f\ )}$$

Substituting the above two values of **Φ** and **ϒ** in the second basic motor equation

$$\boldsymbol{\omega = (V_T / K_a.\ \Phi\ ) \text{ -- } [R_a / (K_a.\ \Phi)^2] . \Upsilon}$$

We get

$$\boldsymbol{\omega = V_T / K_a.\ K_f . I_a \text{ -- } [R_a / (K_a.\ K_f\ . I_a)^2] . K_{af} I_a^2}$$

$$\boldsymbol{\omega = V_T / K_{af} . I_a \text{ -- } [R_a / (K_{af}\ . I_a)^2] . K_{af} I_a^2}$$

$$\boldsymbol{\omega = V_T / K_{af} . I_a \text{ -- } [R_a / (K_{af})]}$$

From the relation $\Upsilon = K_a.\ \Phi.I_a = K_a.\ K_f.I_a^2$ **we get** $I_a = \sqrt{\Upsilon/K_{af}}$ and substituting this in the above equation $\omega = V_T / K_{af}.I_a -- [R_a/ (K_{af})]$

We get $\omega = *V_T /\sqrt{(K_{af}.\Upsilon)}] -- [R_a/(K_{af})]$

Where R_a is now the sum of armature and field winding resistances and $K_{af} = K_a.K_f$ is the total motor constant. The Speed−Torque characteristics of a DC series motor as obtained from the above relation are shown in the figure below.

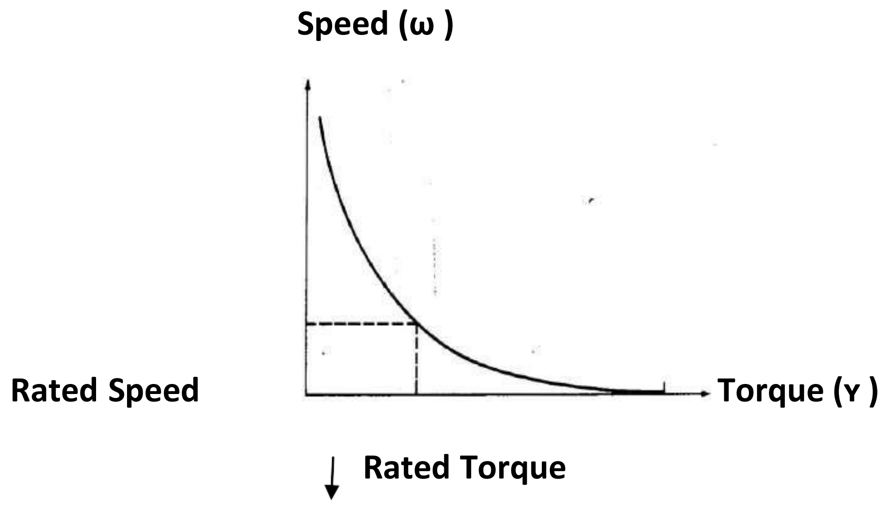


Fig: Speed-Torque characteristics of a DC series motor

Motor's Other Characteristics: Though the terminal characteristics **(Speed vs. Torque)** are only important for analysis of a DC motor performance, study and understanding of the other characteristics like speed vs. I_a and Torque vs. I_a would also give additional insight into the performance of the motor and hence they are obtained from the basic equations and presented below:

- **Speed vs.** I_a **:** $E_b = K_a.\ K_f.\ I_a\ \omega = V_T - I_a.R_a$
  i.e. $= K_{af}.\ I_a\ .\omega = V_T - I_a.R_a$ and
  $\omega = (V_T - I_a.R_a) / K_{af}.\ I_a = (V_T / K_{af}.\ I_a) - (R_a / K_{af})$

This is an inverse relationship and is shown plotted in the figure below.

- **Torque vs. I_a**: $\Upsilon = K_a . \Phi . I_a = K_a I_a^2$

This is a direct relationship and is shown plotted in the figure below.

Saturation and armature reaction demagnetization both cause the flux per pole to increase (with respect to **I_a**) at a rate slower than the assumed linear relationship. Actual characteristics are shown in dotted lines.

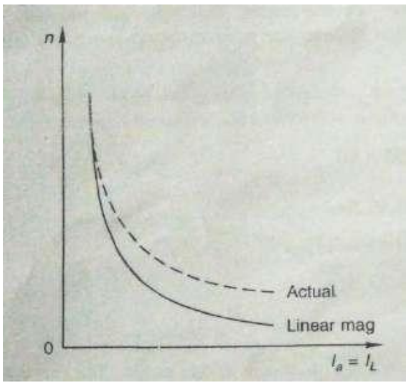


Fig: Speed Vs. Armature current in a Series Motor

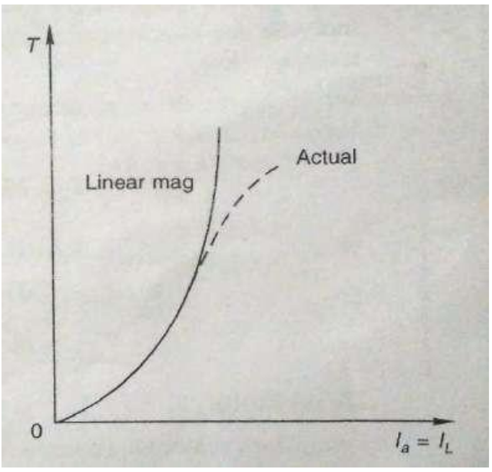


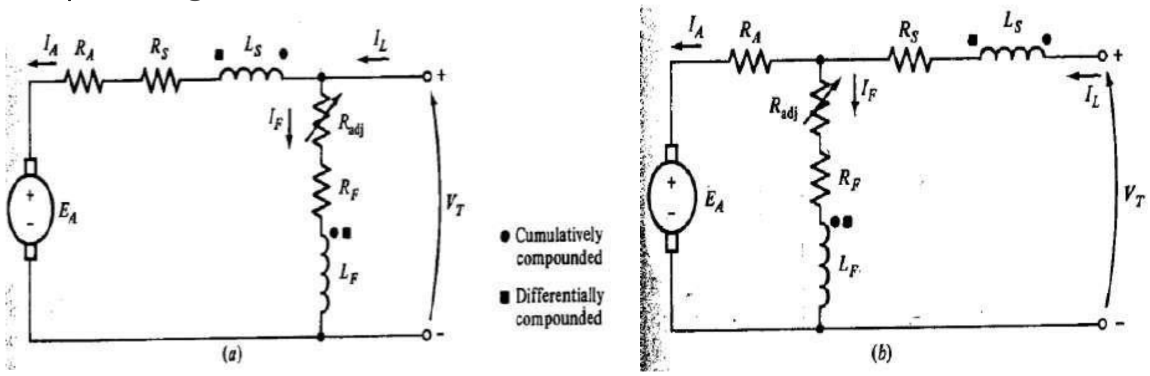
Fig: Torque vs. Armature current in a DC Series Motor

Series motors are suitable for applications requiring high starting torque and heavy overloads. Since Torque is proportional to square of the armature current, for a given increase in load torque the increase in armature current is less in case of series motor as compared to a separately excited motor where torque is proportional to only armature current. Thus during heavy overloads power overload on the source power and thermal overload on the motor are kept limited to reasonable small values. According to the above Speed torque equation, as speed varies inversely to the square root of the Load torque, the motor runs at a large speed at light load. Generally the electrical machine's mechanical strength permits their operation up to about twice their rated speed. Hence the series motors should not be used in such drives where there is a possibility for the torque to drop down to such an extent that the speed exceeds twice the rated speed.

DC Compound Motor:

A compound motor is a motor with *both a shunt and a series field.* Such a motor is shown in the Figure below. The dots that appear on the two field coils have the same meaning as the dots on a transformer: *Current flowing into a dot produces a positive magneto motive force.* If current flows into the dots on both field coils, the resulting magneto motive forces add to produce a larger total magneto motive force.

This situation is known as *cumulative compounding.* If current flows into the dot on one field coil and out of the dot on the other field coil , the resulting magneto motive forces subtract. In the Figure below the round dots correspond to cumulative compounding of the motor, and the squares correspond to differential compounding.



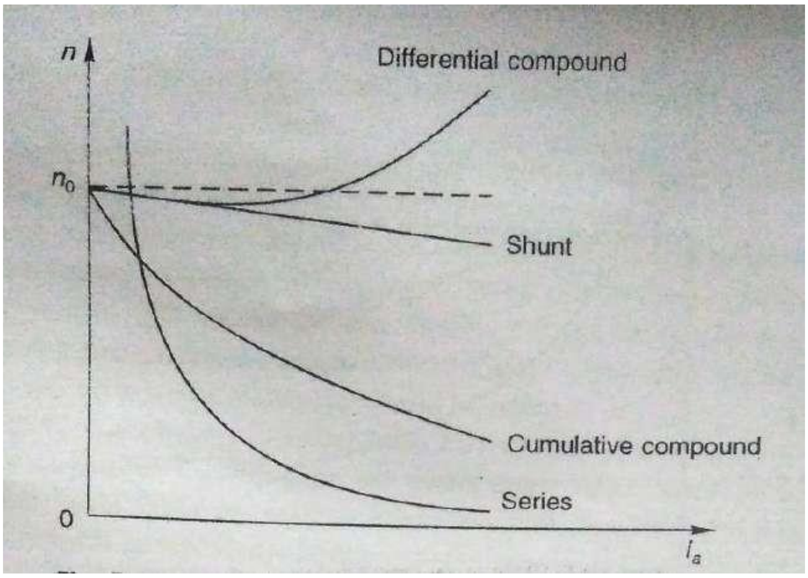


Fig: Speed vs. Armature current in a DC Compound Motor Compared with other Motors

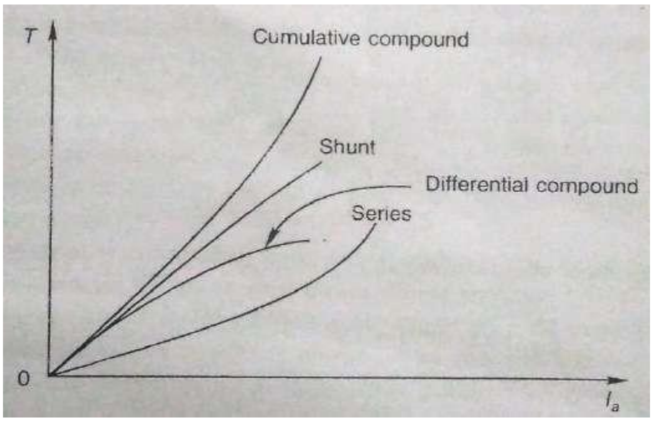


Fig: Torque vs. Armature current in a DC Compound Motor Compared with other Motors

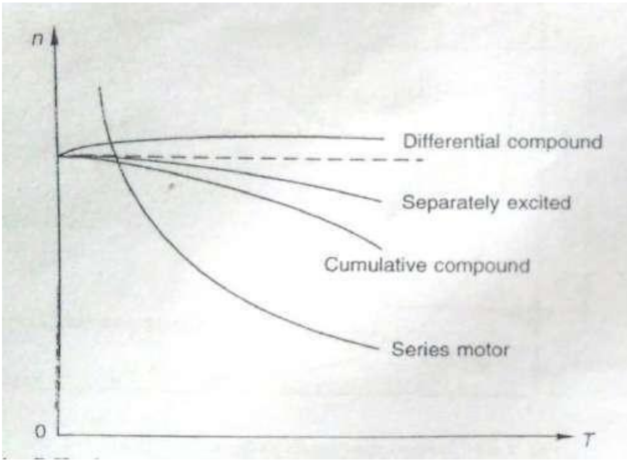


Fig: Speed vs. Torque in a DC Compound Motor Compared with other Motors

The Torque-Speed Characteristic of a Cumulatively Compounded DC Motor :

- In the cumulatively compounded DC motor, there is a component of flux which is constant and another component which is proportional to its armature current (and thus to its load) which aid each other. Hence the flux per pole increases with armature current and as consequence (n−Ia) curve lies between that of a shunt motor(Ø − constant) and series motor (Ø a Ia).
- Therefore, the cumulatively compounded motor has a higher starting torque than a shunt motor (whose flux is constant) but a lower starting torque than a series motor (whose entire flux is proportional to armature current).
- In a sense, the cumulatively compounded DC motor combines the best features of both the shunt and the series motors. Like a series motor, it has extra torque for starting; like a shunt motor, it does not over speed at no load.
- At light loads, the series field has a very small effect, so the motor behaves approximately as a shunt DC motor. As the load gets very large, the series flux becomes quite important and the torque−speed curve begins to look like a series motor's characteristic. A comparison of the torque−speed characteristics of each of these types of machines is shown in Figures.

The Torque-Speed Characteristic of a Differentially Compounded DC Motor:

- In a differentially compounded dc motor, *the shunt magneto motive force and series magneto motive force subtract from each other.* This means that as the load on the motor increases, $\mathbf{I_a}$ increases and *the flux in the motor decreases.* But as the flux decreases, the speed of the motor increases. This speed increase causes another increase in load, which further increases $\mathbf{I_a}$ further decreasing the flux, and Increasing the speed again. The result is that a differentially compounded motor is unstable and tends to run away. This instability is *much* worse than that of a shunt motor with armature reaction. It is so bad that a differentially compounded motor is unsuitable for any application.
- Because of the stability problems of the differentially compounded DCmotor, it is almost never *intentionally* used.
- However, a differentially compounded motor can result if the direction of power flow reverses in a cumulatively compounded generator. For that reason, if cumulatively compounded DC generators are used to supply power to a system, they will have a reverse−power trip circuit to disconnect them from the line if the power flow reverses. No motor− generator set in which power is expected to flow in both directions can use a differentially compounded motor, and therefore it cannot use a cumulatively compounded generator.

Typical terminal characteristics of differentially compounded dc motor are also included in the Figures.

Speed control of DC Motors:

Speed control of DC Motors is easier as compared to the speed control of AC motors and much wider range of speeds is possible. That is one reason why even today they are preferred in modern industrial drives. From the two basic equations of DC machines

- $E_b = K_a. \Phi.\omega$
- $V_T = E_b + I_a.R_a$

We have the expression for the speed ω **= (V_T - $I_a.R_a$)/ K_a. Φ.** From this equation we can (Since K_a is a constant and I_a is load dependent) easily see that the speed can be controlled by two methods:

1. By varying the terminal voltage known as : ***Armature Voltage Control (AVC)*** and
2. By varying the field current and thus the flux per pole **Φ** known as **: *Flux control***

Let us study them one by one for all the three types of Motors

Speed control of DC Shunt Motor:

Armature Voltage Control (AVC):

This method involves changing the voltage applied to the armature of the motor without changing the Voltage applied to the field. This is possible with a separately excited DC Motor only and not with DC Shunt Motor. So first we shall explain for a DC separately excited motor and extend the same logic to a shunt Motor. If the armature terminal Voltage V_T is increased, then the I_A will rise since **[$I_A = (V_T \uparrow - E_b)/R_A$].** As I_A increases, the induced torque **$Y = K_a. \Phi.I_a \uparrow$ increases**, making $Y_{ind} > Y_{load}$, and the speed of the motor increases.
But, as the speed **ω** increases, **$E_b = K_a. \Phi.\omega \uparrow$** increases, causing the armature current I_A to decrease since **[$I_A = (V_T - E_b \uparrow)/R_A$].** This decrease in I_A decreases the induced torque, causing Y_{ind} to become equal to Y_{load} at a final higher steady state rotational speed **ω**. Thus we can see that an increase in Armature voltage results in a higher speed and the resulting Speed Torque characteristics with **AVC** is shown in the figure below.

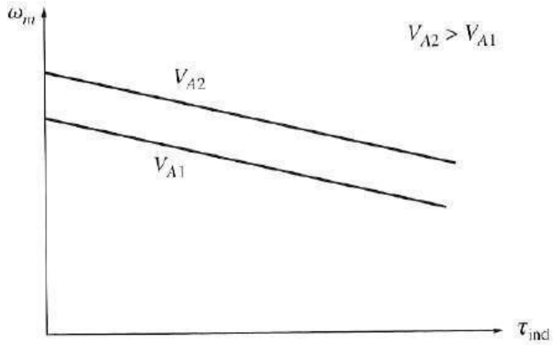


Fig: The effect of armature voltage speed control

Notice that the no−load speed of the motor is shifted by this method of speed control , but the slope of the curve remains constant
The cause−and−effect behavior in this method of speed Control can be summarized as below:

1. An increase in $\mathbf{V_T}$ increases **[** $\mathbf{I_A = (V_T\ j - E_b)/R_A}$
2. Increasing $\mathbf{I_A}$ increases $\mathbf{\gamma_{ind} = K_a.\ \Phi.I_a}$
3. Increasing γ_{ind} makes $\gamma_{ind} > \gamma_{load}$ increasing $\boldsymbol{\omega}$.
4. Increasing $\boldsymbol{\omega}$ increases $\mathbf{E_b = K_a.\ \Phi.\omega}$
5. Increasing $\mathbf{E_b}$ decreases $\mathbf{I_A = (V_T - E_b)/R_A}$
6. Decreasing $\mathbf{I_A}$ decreases γ_{ind} until $\gamma_{ind} = \gamma_{load}$ corresponding to a higher $\boldsymbol{\omega}$.

In the case of a DC Shunt motor since changing the voltage applied to the armature of the motor without changing the Voltage applied to the field is not possible, a Variable resistance is introduced in series with the Armature which results in a reduction in the Armature current I_A. Effectively reduction of Armature current is equivalent to reduction in Armature voltage as seen in the above logic. Hence we get the same type of Speed control as shown in the figure above except that the characteristic with $\boldsymbol{V_{A2}}$ *represents the nominal rated speed and that with* $\boldsymbol{V_{A1}}$ *represents with additional resistance introduced in series with the Armature. With this method, speed control is possible but speed can only be reduced from the rated or nominal speed. Even for a separately excited DC Motor it can provide* *speed control below Base speed only because armature voltage cannot exceed the rated value.*

Inserting a resistor in series with the armature circuit : If a resistor is inserted in series with the armature circuit, the effect is to drastically increase the slope of the motor's torque−speed characteristic, making it operate more slowly if loaded as shown in the figure below. This fact can easily be seen from the basic Equation:

$$\boldsymbol{\omega = (V_T / K_a . \Phi) -- [R_a / (K_a . \Phi)^2 + .\gamma}$$

The insertion of a resistor is a very wasteful method of speed control, since the losses in the inserted resistor are very large. For this reason, it is rarely used.

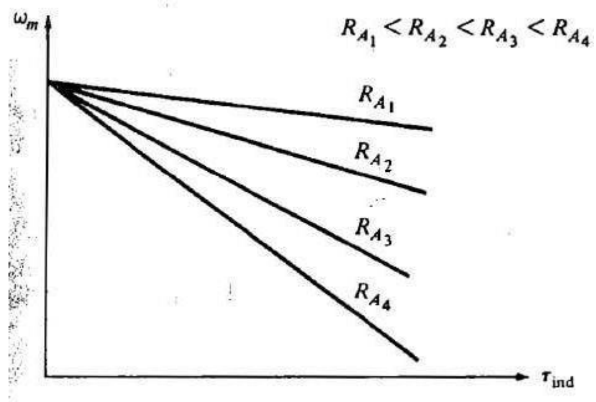
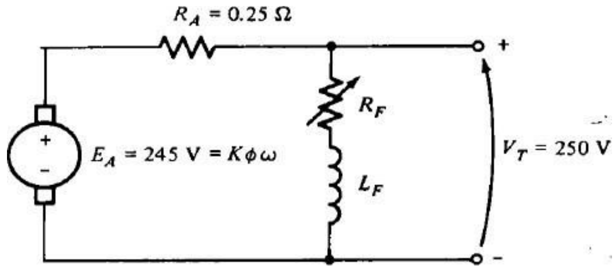


Figure: Effect of Armature Resistance on the Speed Torque characteristic of a DC shunt motor

Flux control:

Another method of Shunt motor speed control is to change the flux in the field. In a shunt motor Field current and hence field flux cannot be changed without changing the armature voltage. Hence flux control in Shunt motor is achieved by changing the Field resistance. Field coil resistance being fixed we cannot reduce it but increase the field circuit resistance by adding a variable resistance in series with the field coil as shown in the figure below.



Accordingly, when the resistance increases, the field current decreases **(I_FM = V_T/R_F),** and as the field current decreases, the flux decreases. A decrease in flux causes an instantaneous decrease in the back emf (**E_bM = K_a. ΦM.ω**) which causes an increase in the machine's armature current since,

I_A j = (V_T -- E_BM)/R_A

The induced torque in a motor is given by **γ_{ind} = K_a. ΦM.I_a** .

Here since the flux in this machine decreases while the current ***I_A*** *increases, which way does the induced torque change?*

From practical data it is seen that for an increase in field resistance the decrease in flux is much lesser than the increase in armature current i.e. the increase in current predominates over the decrease in flux.

Hence, γ_{ind} increases i.e. $\gamma_{ind} > \gamma_{load}$, and the motor speeds up.
However, as the motor speeds up, E_b rises, causing I_A to fall. Thus, induced torque γ_{ind} too drops, and
Finally γ_{ind} equals γ_{load} at a higher steady−state speed than the original speed .

The cause−and−effect behavior involved in this method of speed control is summarized below :

1. Increasing R_F causes **I_FM = V_T/R_Fj̿** to decrease
2. Decreasing **I_FM** decreases **ΦM**
3. Decreasing **ΦM** lowers **E_bM = K_a. ΦM.ω**
4. Decreasing **E_bM** increases **I_A** since **I_A j = (V_T -- E_BM)/R_A**
5. Increasing **I_A** increases **γ_{ind} = K_a. ΦM.I_aj̿**, with the change in **I_A** being dominant over the change in flux).
6. Increasing **γ_{ind}** makes **$\gamma_{ind} > \gamma_{load}$**, and the speed **ω** increases.
7. Increasing **ω** increases **E_bj̿ = K_a. Φ.ωj̿** again.
8. Increasing **E_b** decreases **I_a**
9. Decreasing **I_a** decreases " **γ_{ind}** until " **γ_{ind}** = " **γ_{load}** at a higher speed **ω**.

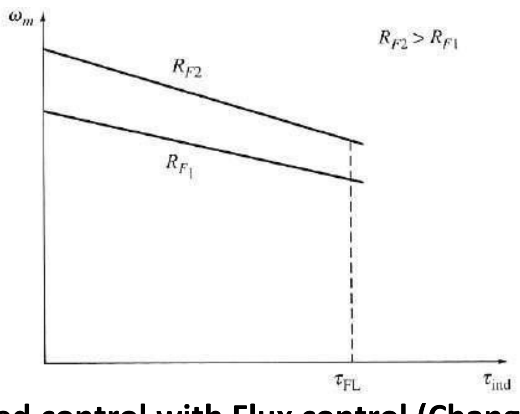


Fig: Shunt Motor Speed control with Flux control (Change in field resistance)(over the normal operating Range)

The Speed Torque characteristics with change in Field Resistance are shown in the figure below. Notice that with flux control i.e. with insertion of additional resistance in the field circuit, the flux in the machine decreases and hence:

- The no− load speed of the motor increases, while the slope of the torque−speed curve becomes steeper and also
- Speeds above base speed only can be achieved. (as against with Armature resistance insertion control , speeds below base speed only can be achieved) since to achieve speed below base speeds field current has to be increased beyond its rated value which is not permitted. In a normally designed motor the maximum speed can be twice the rated speed and in specially designed motors it can be up to six times the rated speed.

Other important Limitation of field resistance speed control:

The effect of increasing the field resistance on the output characteristic of a DC shunt motor as seen and explained above is a consequence of the Equation

$$\omega = (V_T / K_a. \Phi) -- [R_a/ (K_a. \Phi)^2 +.\gamma$$

which describes the technical characteristic of the motor. In this equation, the no−load speed is proportional to the reciprocal of the flux in the motor, while the slope of the curve is proportional to the reciprocal of the flux squared. Therefore, a decrease in flux causes the slope of the torque− speed curve to become steeper. The earlier figure shows the technical characteristic of the motor over the range from no−load to full−load conditions. Over this range, an increase in field resistance increases the motor's speed, as described above. Hence for motors operating between no− load and full−load conditions, an increase in $\mathbf{R_F}$ may reliably be expected to increase the operating speed.

Now let us examine the figure shown below. This figure shows the technical characteristic of the motor over the full range i.e. from no− load to stall conditions. It is apparent from the figure that at *very slow* speeds an increase in field resistance will actually *decrease* the speed of the motor. This effect occurs because , at very low speeds, the increase in armature current caused by the decrease in $\mathbf{E_b}$. is no longer large enough to compensate for the decrease in flux in the induced torque equation. With the flux decrease being actually larger than the armature current increase, the induced torque decreases, and the motor slows down.

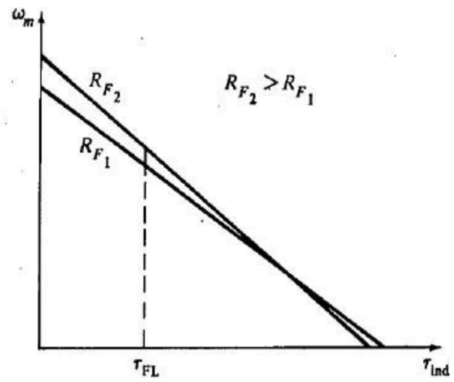


Fig: Shunt Motor Speed control with Flux control (Change in field resistance)(over the complete operating Range i.e. from no load to stall condition)

Some small DC motors used for control purposes actually operate at speeds close to stall conditions. For these motors, an increase in field resistance might have no effect, or it might even decrease the speed of the motor. Since the results are not predictable, field resistance speed control should not be used in these types of dc motors. Instead, the armature voltage method of speed control should be employed.

Speed Control of Series DC Motors:

Unlike with the shunt dc motor, there is only one efficient way to change the speed of a series dc motor. That method is to change the terminal voltage of the motor. If the terminal voltage is increased, the first term in Equation

$$\omega = *V_T / \sqrt{(K_{af}.Y)}] -- [R_a/(K_{af})]$$

is increased, resulting in a *higher speed for any given torque.*

The speed of DC series motors can also be controlled by the insertion of a series or parallel (Diverter) resistor into the motor circuit as shown in the figures below along with the resulting effect on Speed torque characteristics. But in this technique large amount of power is dissipated as heat and thus wasted. Hence this method is used only for intermittent periods during the start−up of some motors.

Until the last 40 years or so, there was no convenient way to change $\mathbf{V_T}$, so the only method of speed control available was the wasteful series resistance method. That has all changed today with the introduction of solid−state control circuits. We will study the techniques of obtaining variable terminal voltages subsequently in another subject '***Power Electronics*** '

Speed Control of Cumulatively Compounded DC Motor:

The techniques available for the control of speed in a cumulatively compounded DC motor are the same as those available for a shunt motor:

1. Change the field resistance $\mathbf{R_F}$
2. Change the armature voltage $\mathbf{V_T}$
3. Change the armature resistance $\mathbf{R_A}$.

The analysis describing the methods and effects of changing $\mathbf{R_F}$ or $\mathbf{V_T}$ or R_A are similar to the analysis given earlier for the shunt motor.

The ward Leonard system:

The speed of a separately excited, shunt, or compounded dc motor can be varied in one of three ways: by changing the field resistance, changing the armature voltage, or changing the armature resistance. Of these methods, perhaps the most useful is armature voltage control, since it permits wide speed variations without affecting the motor's maximum torque.
A number of motor−control systems have been developed over the years to take advantage of the high torques and variable speeds available from the armature voltage control of DC motors. In the days before solid−state electronic components became available, it was difficult to produce a varying DC volt age. In fact, the normal way to vary the armature voltage of a dc motor was to provide it with its own separate dc generator.

An armature voltage control system of this type known as ward Leonard Speed control system is shown in the Figure below .

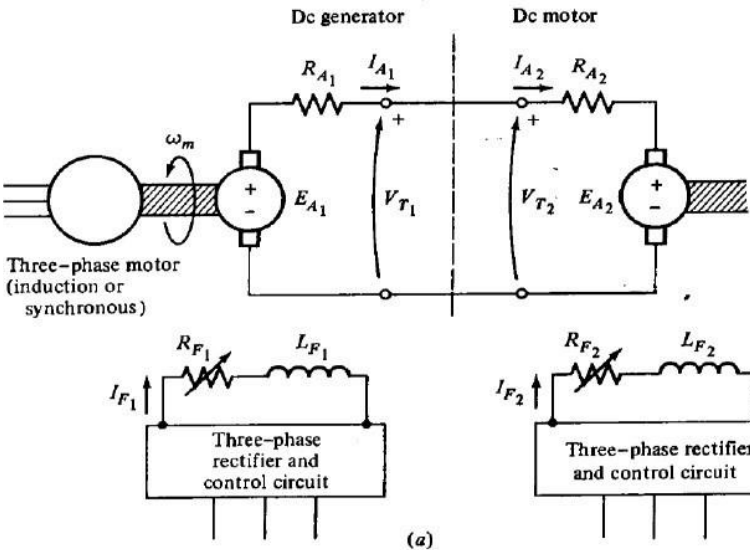


Figure: Ward Leonard DC Motor Speed control system

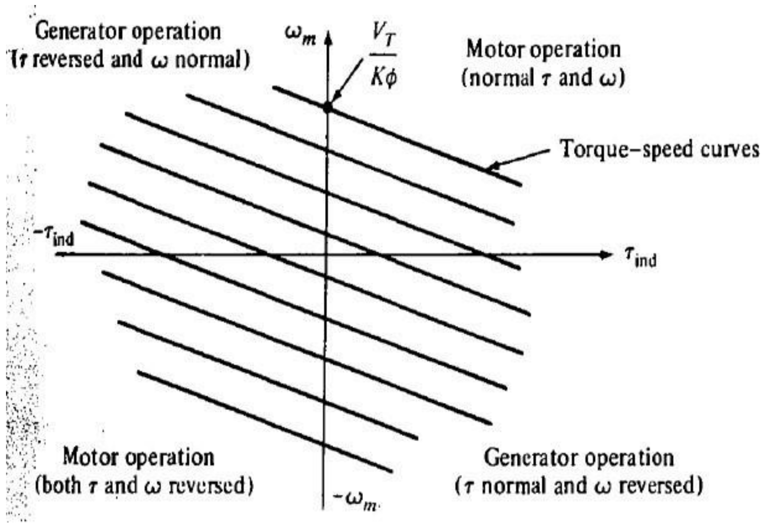


Figure: The operating range of a Ward-Leonard motor-control system. The motor can operate as a motor in either the forward (quadrant - 1) or reverse (quadrant -3) direction and it can also regenerate in quadrants 2 and 4.

In this an AC motor is serving as a prime mover for a DC generator, which allows the motor's speed to be smoothly varied between a very small value and the base speed. The speed of the motor can be adjusted above the base speed by reducing the motor's field current. With such a flexible arrangement, total motor speed control is possible.

Furthermore, if the field current of the generator is reversed, then the polarity of the generator's armature voltage will be reversed, too. This will reverse the motor's direction of rotation. Therefore, it is possible to get a very wide range of speed variations in *either direction of rotation* using a Ward−Leonard DC motor control system.

Another advantage of the Ward−Leonard system is that it can "regenerate," or return the machine's energy of motion to the supply lines. If a heavy load is first raised and then lowered by the DC motor of a Ward−Leonard system, when the load is being lowered, the DC motor acts as a generator and supplying power back

to the power system. In this fashion, much of the energy required to lift the load in the
first place can be recovered, reducing the machine's overall operating costs.
The possible modes of operation of the DC machine are shown in the torque−speed diagram shown in the above Figure. When this motor is rotating in its normal direction and supplying a torque in the direction of rotation, it is operating in the first quadrant of this figure. If the generator's field current is reversed, that will reverse the terminal voltage of the generator, in turn reversing the motor's armature voltage. When the armature voltage reverses with the motor field current remaining unchanged, both the torque and the speed of the motor are reversed, and the machine is operating as a motor in the third quadrant of the diagram. If the torque or the speed alone of the motor reverses while the other quantity does not, then the machine serves as a generator, returning power to the dc power system. Because a Ward−Leonard system permits rotation and regeneration in either direction, it is called *a four−quadrant control system.*
The disadvantages of a Ward−Leonard system should be obvious. One is that the user is forced to buy *three* full machines of essentially equal ratings, which is quite expensive. Another is that three machines will be much less efficient than one. Because of its expense and relatively low efficiency, the Ward−Leonard system has been replaced in new applications by SCR−based controller circuits.

Principle of 3 point and 4 point starters:

Before studying the principle of operation of these starters let us understand the basic principles underlying the starters.

- DC motors are by themselves self starting type. Once the appropriate field and armature supply are given the motors start automatically. They do not need any additional device for the purpose of starting.
- But DC motor starters are required for safe starting of the motors. Initially just at the starting of the motor, the speed is zero and hence the back emf $\mathbf{E_b}$ is also zero. In this condition if the Rated terminal voltage $\mathbf{V_t}$ is applied to the motor we can see from the basic governing equation

$$I_a = (V_t - E_b)/R_a$$

That the motor draws excessive current which would be easily 10 to 15 times that of the nominal rated current of the motor. This excessive current would flow till the motor develops the rated speed.

- During this transient period when the excessive current flows the torque developed also would be excessive. With the result the motor would get damaged both electrically and mechanically.
- To protect the motor from such damage, a resistance is introduced in series with the motor as shown in the figure below which would be withdrawn gradually in steps as the motor picks up speed.
- This is a basic arrangement of a DC motor starter and its operation is totally manual. But practical starters have been developed with additional protective and automatic starting features. They are called 3 ***point starters and 4 point starters***, the subject of our study.

3 Point starter:

The circuit diagram and the arrangement of a three point starter are shown in the figure below.

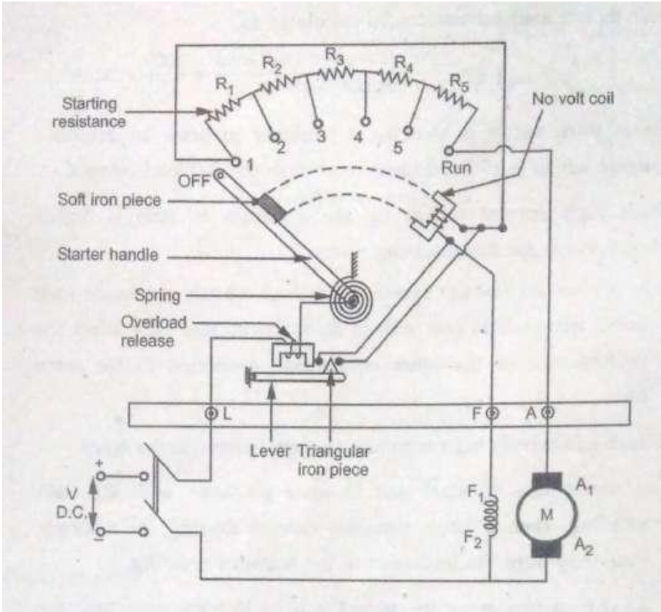


Figure: 3 Point starter

Basic features and working principles:

- The basic component viz starter resistance comes in steps with contact points brought out as studs 1,2,3.. Run.
- The three points are:
  - L − The line terminal to be connected to the DC positive terminal through a two pole switch
  - A − The terminal to be connected to the terminal A_1 of the armature.
  - F − The terminal to be connected to the terminal F_1 of the Field winding
- The other ends A_2 of the armature and F_2 of the field are connected to the other contact of the two pole switch which gets connected to the negative terminal of the DC power supply when switched on.
- Point L in turn is connected to the pivot point of the handle through a protective device called OLR (Over Load Relay)

- The handle which is spring loaded comes back to the OFF position under its own force until locked in the RUN position due to the electromagnetic pull of the other protective device known as NVC (No Volt Coil)
- The field terminal F is connected to starting point 1 of the resistance in a parallel path through the NVC.

Operation of the starter: The starter is gradually moved from the initial position to the final RUN position manually against the spring force. When the handle comes in contact with stud −1 , the field supply gets extended to the field coil through the parallel path connected directly from stud −1 through NVC . In the starter initially entire resistance comes in series with the armature and as the handle is moved towards RUN, the portion of the series resistance that comes out of the armature circuit gets added to the field circuit. Finally when the handle is brought to the Run position, the entire resistance gets removed from the Armature circuit and the motor runs at the rated speed. The handle is held in RUN position due to the action of the NVC.

Action of the NVC: When the field current flows through the NVC it attracts the handle with the soft iron piece and keeps it in contact the NVC electromagnet. Hence NVC is also called as ***Hold On Coil***. In addition to holding the handle in the final RUN position, the NVC works as a safety/protection device by releasing the handle back to the start position from the RUN position whenever there is a power failure or when the field circuit breaks. Thus the entire starting resistance comes into the armature circuit every time the motor is started from zero speed and prevents high inrush currents during every fresh starting attempt after a power failure.

Action of the OLR: As can be seen from the figure there is another protective relay called OLR (Over Load Relay) which is also an electromagnet which works in conjunction with an arm fixed on a fulcrum at one end and with a triangular iron piece fixed on the other end. Whenever there is an overload current beyond a set safety value, the electromagnet activates and pulls the arm upwards and the triangular iron piece short circuits the two terminals which are connected to the two ends of the NVC coil. Thus with any overload due to a fault in the motor or associated circuit , the NVC gets deactivated and releases the handle back to the initial safe start condition. After the fault is rectified the motor can be started afresh with full resistance brought back into the armature circuit.

3 Point starter with a brass/copper arc:

In the earlier version of the 3 Point starter as we have seen, as the handle is moved towards RUN position, the portion of the series resistance that comes out of the armature circuit gets added to the field circuit. Thus finally when the motor is running, the entire starter resistance gets added to the field circuit. But since the starting resistance value is very small compared to field winding resistance, this hardly reduces the field current and hence there is no any practical impact. However this addition of resistance in the field circuit can be avoided by providing a brass or copper arc with one end connected to the stud −1 and the other end connected to the NVC as shown in the figure below.

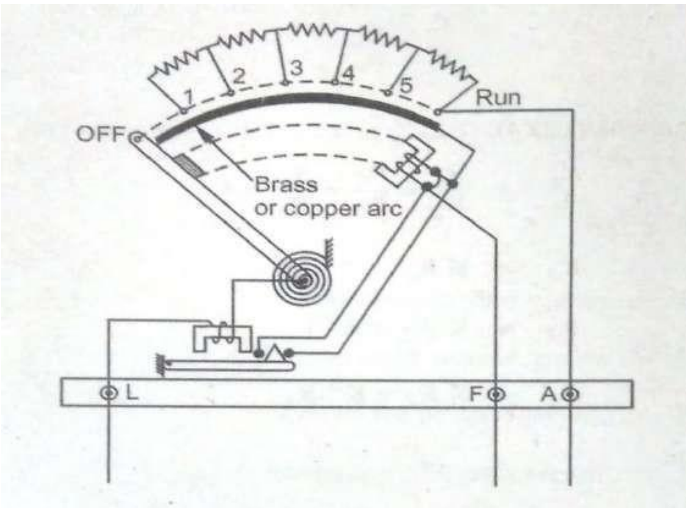


Figure: 3 Point starter with Brass arc

With such an arrangement when the handle moves on the arc the field current directly flows through the arc to the NVC thus avoiding the starting resistance. With such an arc in place, the earlier parallel connection from stud −1 to the NVC start terminal is no more required and hence is removed.

4 Point starter: The operation of the 4 point starter is explained along with the schematic diagram shown below.

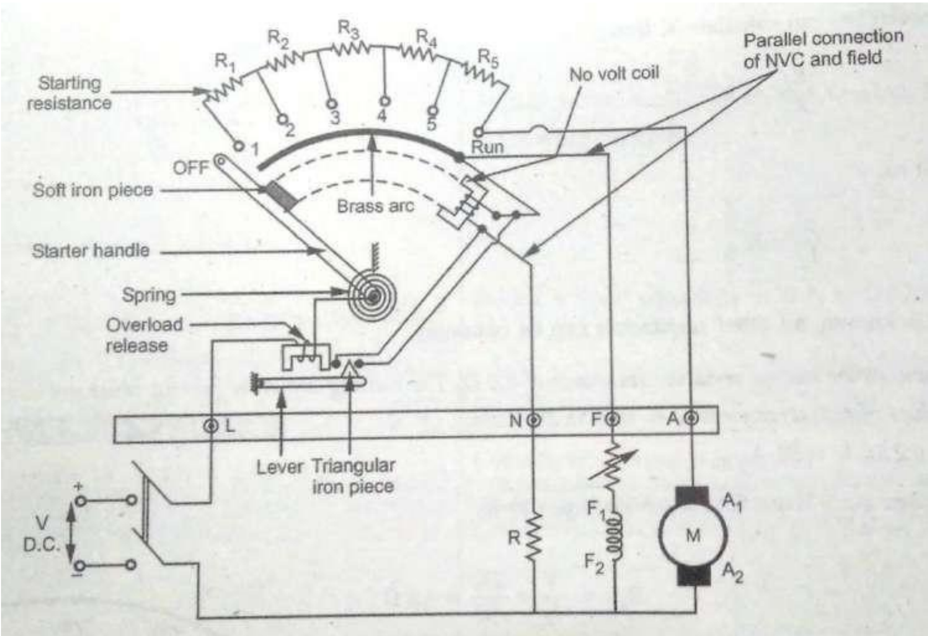


Figure: 4 Point starter

- The basic difference between a 3 Point starter and a four point starter is : In a 3 point starter NVC was connected in series with the field coil while in a 4 point starter the NVC is connected independently to the supply through a fourth terminal termed as N in addition to L, F and A.
- With this arrangement any change in the field current due to the change field control resistance will not affect the performance of the NVC. This ensures

that NVC always produces a force enough to hold on the handle irrespective of the amount of field current. Adequate current required for the confirmed operation of the NVC is obtained by adjusting the resistor R connected in series with the NVC coil.

- However the 4 point starter has a separate disadvantage: Since now the NVC is connected separately excluding the field current, it cannot detect the field failure and hence the resulting ***over speed*** cannot be prevented.

Important concepts and Formulae:

- **Torque generated in a DC machine** : $\Upsilon = K_a . \Phi . I_a$
- **KVL around the armature circuit:** $V_T = E_b + I_a . R_a$
- **Generalized *Torque vs. Speed* equation in different types of motors:**

$$\omega = (V_T / K_a . \Phi) -- [R_a / (K_a . \Phi)^2 + . \Upsilon$$

- **Shunt Motor's Other Characteristics:**

  Speed vs. I_a : $\omega = (V_T - I_a . R_a) / K_a . \Phi$

  Torque vs. I_a: $\Upsilon = K_a . \Phi . I_a$

- **Series Motor:**

$$\omega = *V_T / \surd(K_{af} . \Upsilon)] -- [R_a / (K_{af})]$$

- **Speed vs. I_a :** $\omega = (V_T - I_a . R_a) / K_{af} . I_a = (V_T / K_{af} . I_a) - (R_a / K_{af})$
- **Torque vs. I_a:** $\Upsilon = K_a . \Phi . I_a = K_{af} I_a^2$
- **Speed control with armature voltage control is possible only below the rated or nominal speed (also known as base speed).**
- **Speed control with flux control is possible only above the base speed**

Illustrative examples:

Ex.1: A 500 V shunt motor with R_f = 250 Ω and R_a = 0.2 Ω runs at 2500 RPM taking a current of 25 A from the mains supply . Calculate the resistance to be added to the armature circuit to reduce the speed to 1500 RPM keeping the armature current constant.

Solution:

First let us calculate the back e.m.f developed by the motor in the given first set of conditions:

Field current I_f = Rated terminal voltage / R_f = 500 / 250 = 2 A

Armature current $I_a = I_l - I_f = (25 - 2) = 23$ A

Back e.m.f $E_b = V_T - I_a R_a = 500 - 23 \times 0.2 = 495.4$ V

We know that the back e.m.f is proportional to the speed

 $E_{b1} / E_{b2} = N_1 / N_2$ i.e $495.4 / E_{b2} = 2500/1500$ $E_{b2} = 495.4 \times 1500 / 2500$

$= 297.24$ V

But we also know that $E_{b2} = V_T - I_a R_{a2}$ (Since the terminal voltage and the armature current remain the same)

 $297.24 = 500 - 23 \times R_{a2}$ from which we get $R_{a2} = (500 - 297.24)/23 = 8.82$ Ω

This is the total new resistance of the armature circuit (including the original armature resistance of 0.2 Ω to get a speed of 1500 RPM)

Hence the new resistance to be added into the armature circuit = 8.82− 0.2 = **8.62 Ω**

Ex.2: A DC shunt motor takes 22 A from 250 V supply. R_a = 0.5 Ω , R_f = 125 Ω. Calculate the resistance required to be connected in series with the armature to halve the speed (a) when the load torque is constant (b) When the load torque is proportional to the square of the speed

Solution :

First let us calculate the speed of the motor when the load current I_l is 22 A :

Field current I_f = Rated Terminal voltage / Field resistance = 250/125 = 2 A

Armature current $I_a = I_l - I_f = 22 - 2 = 20$ A

Back e.m.f $E_b = V_T - I_a R_a = 250 - 20 \times 0.5 = 240$ V

(a) *we have to find out the New R_a when the speed is halved with torque maintained constant* :

We know that Torque T = $K_a . Ø . I_a$. In this case since change is only in the armature resistance field current and hence flux Ø remains the same. Further since the torque is maintained constant the armature currents are also equal and hence $I_{a1} = I_{a2} = 20$ A

We also know that $E_b = K_a . Ø . \omega$. As already explained, K_a. Ø remains same and hence when the speed is halved the back e.m.f also gets halved.

Hence $E_{b2} = 120 \text{ V} = V_T - I_a R_{a2}$ i.e $250 - 20 \times R_{a2} = 120$ V i.e $R_{a2} = (250 - 120)/20 = 6.5\Omega$

Hence the **Resistance to be added to halve the speed** = $R_{a2} - R_a = 6.5 - 0.5 =$ **6.0 Ω**

(b) Next we have to find out the New R_a when the speed is halved when torque is proportional to square of speed.

When the torque is proportional to the square of the speed $\Upsilon_1 = K \omega_1^2$ and $\Upsilon_2 = K \omega_2^2$

$\Upsilon_1 / \Upsilon_2 = K \omega_1^2 / K \omega_2^2 = \omega_1^2 / \omega_2^2 = (1/0.5)^2 = 4$

But Torque is also proportional to the product of flux (and hence field current)and Armature current. Here field circuit is not disturbed and hence the field current is same. Using this relation we can find out new armature current I_{a2}

□ Υ_1 / Υ_2 = K x I_f x I_{a1} / K x I_f x I_{a2} = I_{a1} / I_{a2} = 4 i.e $I_{a2} = I_{a1} / 4$ = 20/4 =5 A

Next using the relation between the speeds and the back emfs we can find out the armature resistance to be added.

ω_1 / ω_2 = 2 and also

ω_1 / ω_2 = E_{b1}/E_{b2} = 240 / (250 − 5 x R_{a2}) i.e 250− 5R_{a2} = 240/2 = 120 From which we get

R_{a2} = (250−120)/5 = 26 Ω □ Finally **Resistance to b added** is = 26−0.5 = **25.5 Ω**

Ex.3: A 250 V DC series motor takes 40 A and runs at 1000 RPM. Find the speed at which it runs if its torque is halved. Assume that the motor is operating in the unsaturated region of its magnetization. R_f = 0.25 Ω R_a = 0.25 Ω

First we will use the relation between torque and armature current and get the back e.mf when the torque is halved :

In a DC motor we know that the torque is proportional to Ø.I_a. In the case of a series DC motor flux is proportional to the armature current itself since $I_f = I_a$. Hence in a series motor **ϒ** a **I_a^2**

Hence **Υ_1/Υ_2 = I_{a1}^2 / I_{a2}^2 =** 2 (Since torque is halved) **I_{a1} / I_{a2} =** √2

I_{a1} = 40 A and I_{a2} = 40/√2 = 28.28 A

E_{b1} = 250 − 40 (0.25 + 0.25) = 230 V and E_{b2} = 250 − 28.28 (0.25 + 0.25) = 235.86 V

Next we will use the relation between back emf and speed and get the speed when the torque is halved:

We know that $E_{b1} = K_a Ø_1 N_1$ and $E_{b2} = K_a Ø_2 N_2$. But since the flux is proportional to I_a the relations become $E_{b1} = K I_{a1} N_1$ and $E_{b2} = K I_{a2} N_2$ where K is a new constant.

Hence $E_{b1}/ E_{b2} = K I_{a1}N_1 / K I_{a2}N_2 = I_{a1}N_1 / I_{a2}N_2$ and $N_2 = (I_{a1} / I_{a2}) (E_{b2}/ E_{b1}) N_1$

Substituting the above values we get $\mathbf{N_2} = \sqrt{2}\ (235.86/230)1000$ = **1450 RPM**

Ex.4: A 500 V DC shunt motor runs at 1900 RPM taking an armature current of 150 A. The armature resistance is 0.16 Ω. Find the speed of the motor when a resistance is inserted in the field circuit which reduces the field current to 80 % and the armature current is 75 A.

Solution:

We know that the back e.m.f of a DC motor is proportional to the Flux and speed. And in the unsaturated region of the magnetization region the flux in turn is proportional to the field current. So Back e.m.f is proportional to field current and speed. We will find out the new speed by calculating the back e.m.fs [from the relation ($E_b = V_T - I_a R_a$)]and using the above proportionality relation in both the conditions as below.

$E_{b1} = V_T - I_{a1} R_a = 500 - 150 \times 0.16 = 476$ V and is equal to $K_a.Ø_1. N_1$

$E_{b2} = V_T - I_{a2} R_a = 500 - 75 \times 0.16 = 488$ V and is equal to $K_a . 0.8Ø_1. N_2$

 $476 /488 = K_a.Ø_1. N_1 / K_a . 0.8Ø_1. N_2$

And $\mathbf{N_2} = (488 /476)(N_1 / 0.8) = (488 /476)(1900 / 0.8)$ = **2435 RPM**

TESTING OF D.C. MACHINES

CONTENTS:

- Losses – Constant & Variable losses
- Calculation of efficiency
- Condition for maximum efficiency.
- Methods of Testing
    - Direct, indirect and regenerative testing
    - Brake test
    - Swinburne's test
    - Hopkinson's test
    - Field's test
    - Retardation test
- Separation of stray losses in a DC motor test
    - **Important concepts and Formulae**
    - **Illustrative examples**

Losses:

DC Generators convert Mechanical power into Electrical power and DC Motors convert Electric power to Mechanical power. In the process of conversion some power is lost. The difference between the input power and the output power of a machine is the **Power loss** that occurs inside the machine.

Constant & Variable losses:

The losses are broadly classified as *constant losses* and *variable losses*. Constant losses are constant and are independent of the load where as the variable losses are dependent on the load. They are further classified in detail as below.

Detailed Classification of Losses:

1. **Electrical or Copper Losses (I^2R Loss):** Current flow through the resistance of Armature and Field coils gives rise to I^2R losses and since the coils are normally made up of copper these losses are called Copper losses.

   Armature copper loss: $P_A = I_A^2 R_A$

   Field copper loss: $P_F = I_F^2 R_F$

2. **Brush losses**: The brush drop loss is the power lost across the contact potential at the brushes of the machine. It is given by the equation:

$$\mathbf{P_{BD} = V_{BD} \times I_A}$$

where **P_{BD}** = brush drop loss

V_{BD} = brush voltage drop

I_A = armature current

The brush losses are calculated in this manner because the voltage drops across a set of brushes are approximately constant over a large range of armature currents. Unless otherwise specified. The brush voltage drop is usually assumed to be about 2 V.

3. **Core Losses:** *Hysteresis* and *eddy current* losses occurring in the Armature and Field cores together are called core losses.
   - **Hysteresis loss**: in an iron core is the loss of power due to the hysteresis loop in the magnetization characteristic of the core in each cycle of the alternating current applied to the core. In the case of DC machines though there is no alternating current applied to the core, the change in the magnetic flux within the machine due to its constructional features result in a small *hysteresis loss*
   - **Eddy current losses**: A time−changing flux induces voltage within a ferromagnetic core in just the same manner as it induces voltage in the conductors around the core of the armature. These voltages cause swirls of current to flow within the core, much like the eddies seen at the edges of a river. It is the shape of these currents that gives rise to the name ***eddy currents****.* These eddy currents flowing in a resistive material (the iron of the core) cause power loss thus heating the iron core and the resulting loss is called ***eddy current loss***. This loss is proportional to the thickness of the core material and hence to minimize this loss the core is made up of thin sheets called laminations instead of a single thick block. An insulating oxide or resin is used between the strips so that the current paths for eddy currents are limited to very small areas. Thus the eddy current losses have a very little effect on the core's magnetic properties.
4. **Mechanical Losses:** They are associated with the mechanical effects and they are mainly *Friction* and *windage* losses.
   - ***Friction losses*** are losses caused by the friction in the bearings of the machine and
   - ***Windage losses*** are due to the friction between the moving parts of the machine and the air flow in the machine housing.
5. **Stray Losses:** They are other miscellaneous losses that cannot be grouped into any of the above categories.

Out of the above, Core Losses and Mechanical Losses are grouped under Constant losses. Electrical or Copper Losses and Stray Losses are grouped under variable losses.

The losses and their classification explained above is summarized in the form of a tree and is shown below.

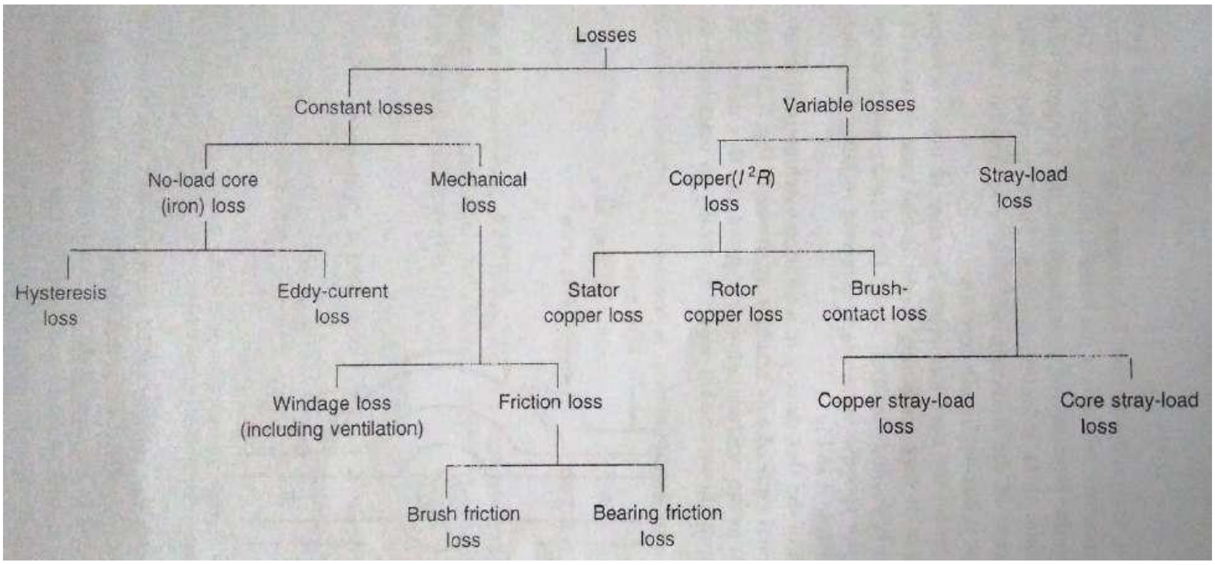


Figure: Classification of losses in DC Machine

The power flow in DC machines showing the stages where the different losses occur is shown clearly in the figure below.

Power flow diagram:

One of the most convenient techniques for accounting for power losses and showing them clearly in the order in which they occur in a machine is the *power−flow diagram.* A power−flow diagram for a DC generator is shown in the figure (a) below. In this figure, mechanical power is input into the machine, and then the stray losses, mechanical losses, and core losses are subtracted. After they have been subtracted, the remaining power is ideally converted from mechanical to electrical form at the point labeled $\mathbf{P_{conv}}$. The mechanical power that is converted is given by:

$$\mathbf{P_{CONV} = \gamma_{ind} \cdot \omega_m}$$

and the resulting electric power produced is given by: $\mathbf{P_{CONV} = E_A . I_A}$

However, this is not the power that appears at the machine's terminals. Before the terminals are reached, the electrical power losses like the copper losses and the brush losses must be subtracted.

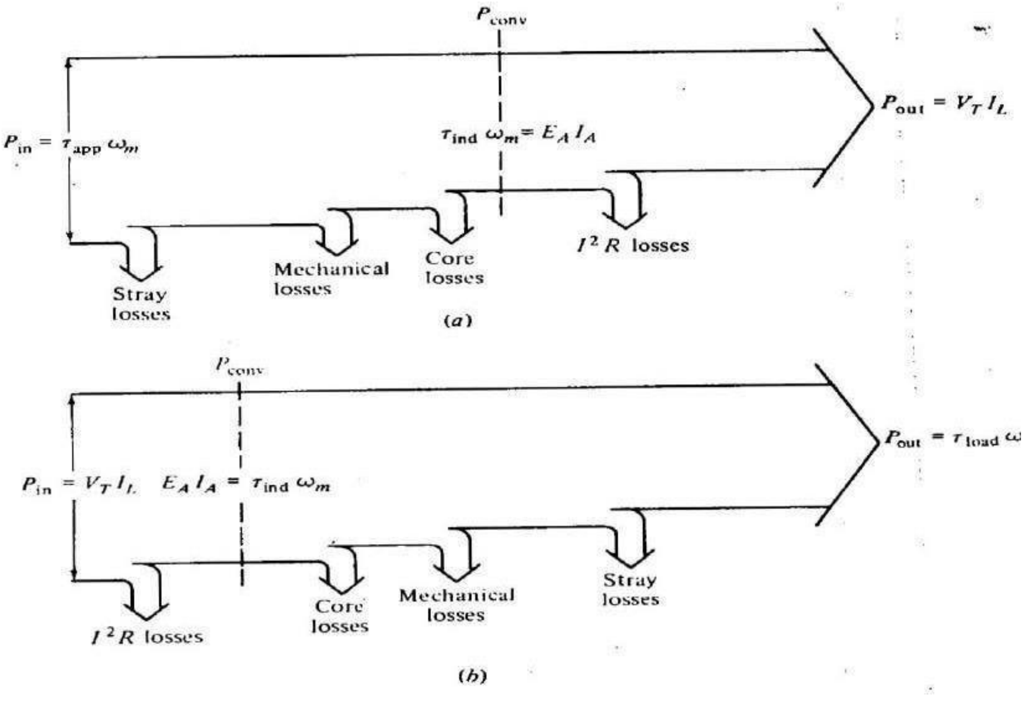


Figure: Power flow diagram of a DC Machine (a) Generator (b) Motor

In the case of dc motors, this power−flow diagram is simply reversed. The Power flow diagram for a motor is shown in the above figure (b) above

Efficiency:

The efficiency of a DC Machine is defined as $\eta = (P_{out}/P_{in}).\ 100\ \% = [(P_{in} - P_{loss}) / P_{in}] \times 100\ \%$

Using this basic relation and from a clear understanding of the above Power flow the ***y*** *calculations when the machine is working as a Generator and as a Motor are given below.*

Efficiency calculations of Generator:

- If I_L is the load current supplied by the Generator at a terminal voltage of V_T then the output power is given by $P_{out} = V_T.I_L$
- The armature current $I_A = I_L + I_F$
- Armature copper loss $P_A = I_A^2 R_A$
- Field copper loss $P_F = I_F^2 R_F$
- Total losses $= I_A^2 R_A + I_F^2 R_F + W_C$ where W_C is the sum of the core losses and stray losses. (also known as constant losses)
- Therefore Input $= P_{out} + \text{Total losses} = P_{out} + I_A^2 R_A + I_F^2 R_F + W_C$

Hence $\eta = (P_{out}/P_{in}).\ 100\ \% = (V_T.I_L)/(V_T.I_L + I_A^2 R_A + I_F^2 R_F + W_C).100\%$

Efficiency calculations of Motor:

- If I_L is the line current taken by the Motor at a terminal voltage of V_T then the input power is given by $P_{in} = V_T.I_L$

- The losses are same as in the Generator
- Therefore output **P_{out} = P_{in}-- Total losses = P_{in}-($I_A^2 R_A + I_F^2 R_F + W_C$)**

Hence **$\eta = (P_{out}/P_{in}). 100\ \% = [\{P_{in} - (I_A^2 R_A + I_F^2 R_F + W_C)\}/(V_T.I_L)]\ .\ 100\%$**

Of these losses **($I_F^2 R_F + W_C$) are** called constant losses **P_C** since they are almost independent of load. The armature copper losses i.e. **($I_A^2.R_A$)** is called the variable loss and is dependent on the load. The variable loss varies approximately as the square of load current. We say approximately since loss varies as the square of the armature current and not as the square of the load current. Hence if we know the loss at full load, the loss at half load, one fourth load etc can be calculated.

Condition for Maximum efficiency:

The condition for maximum efficiency is developed by differentiating the expression for efficiency as a function of load current and equating it to zero since the variable losses are dependent on the load current.

Generator:

The efficiency is obtained as: **$\eta = (P_{out}/P_{in}). 100\ \% = (V_T.I_L)/(V_T.I_L + I_A^2 R_A + I_F^2 R_F + W_C).100\%$**

Neglecting the field current which is small compared to armature current we get

$$\eta = (V_T.I_L)/(V_T.I_L + I_L^2 R_A + W_C).100\% = 1/[1 + I_L^2 R_A/(V_T.I_L) + W_C/(V_T.I_L)].100$$
$$= 1/[1 + I_L R_A / V_T + (W_C/(V_T I_L))].100$$

The efficiency is maximum when the denominator is maximum. Hence the condition for maximum efficiency becomes: **$d/dI_L [1 + I_L R_A / V_T + (W_C/(V_T I_L))] = 0$**

i.e. $R_A / V_T - (W_C/(V_T I_L^2)) = 0$

And finally the condition for maximum efficiency becomes: $I_L^2 R_A = W_C$
Which means **Variable losses = Constant Losses**

And the current at maximum efficiency becomes:

$$I_L = \sqrt{W_C / R_A} = \sqrt{\text{Constant Losses/Armature resistance}}$$

Motor:

The efficiency is obtained as: $\eta = (P_{out}/P_{in}) . 100\ \% = [\{P_{in}-(I^2_A R_A + I^2_F R_F + W_C)\}/(V_T.I_L)] . 100\%$

$$= [\{V_T.I_L - (I^2_A R_A + I^2_F R_F + W_C)\}/(V_T.I_L)] . 100\%$$

Neglecting the field current which is small compared to armature current we get

$$\eta = [\{V_T.I_L - (I_L^2 R_A + W_C)\}/(V_T.I_L)] . 100\%$$
$$= 1 - [(I_L^2 R_A + W_C)/(V_T.I_L)] . 100$$

η becomes maximum when the term in the square brackets becomes minimum and thus the condition for maximum efficiency becomes $\mathbf{d/d\,I_L\,[(I_L^2 R_A + W_C)/(V_T.I_L)] = 0}$ which again finally becomes **:**

$$I_L^2 R_A = W_C$$

Or Variable losses = Constant Losses

And the current at maximum efficiency also becomes:

$$I_L = \sqrt{W_C / R_A} = \sqrt{\text{Constant Losses/Armature resistance}}$$

Both same as that for the generator.

Testing of DC machines:

Involves the measurement of the various losses and then finding out the efficiency of the machine by various methods. The methods are broadly classified as:

1. Direct 2. Indirect and 3. Regenerative methods of testing

1. Direct method of testing : In this method the DC machine is actually loaded to the required extent, the Input and output are measured and then the η is calculated as η = **Output/Input**

   1. This method is generally employed only for small motors. The motor is loaded by a friction pulley arrangement.(braking)
   2. The main drawback of this method is accuracy of output power measurement is limited.
   3. Difficult to provide braking load for a large capacity motor.

2. Indirect method of testing: In this method the machine is not subjected to full load. First on no load the constant losses are measured and then efficiency is estimated at various loads. Swinburne's test and Hopkinson's test come under this category. Only shunt motors can be tested using these methods. Series motors cannot be tested with this method since they cannot be run on no load.

3. Regenerative method of testing: In this method a motor generator is pair is used which are powered by each other. Thus only losses are drawn from the mains power supply. Hopkinson's test comes in this category.

Brake Test: This is a direct method of testing. In this method the motor is put on a direct friction load arrangement with a belt and a pulley as shown in the figure below.

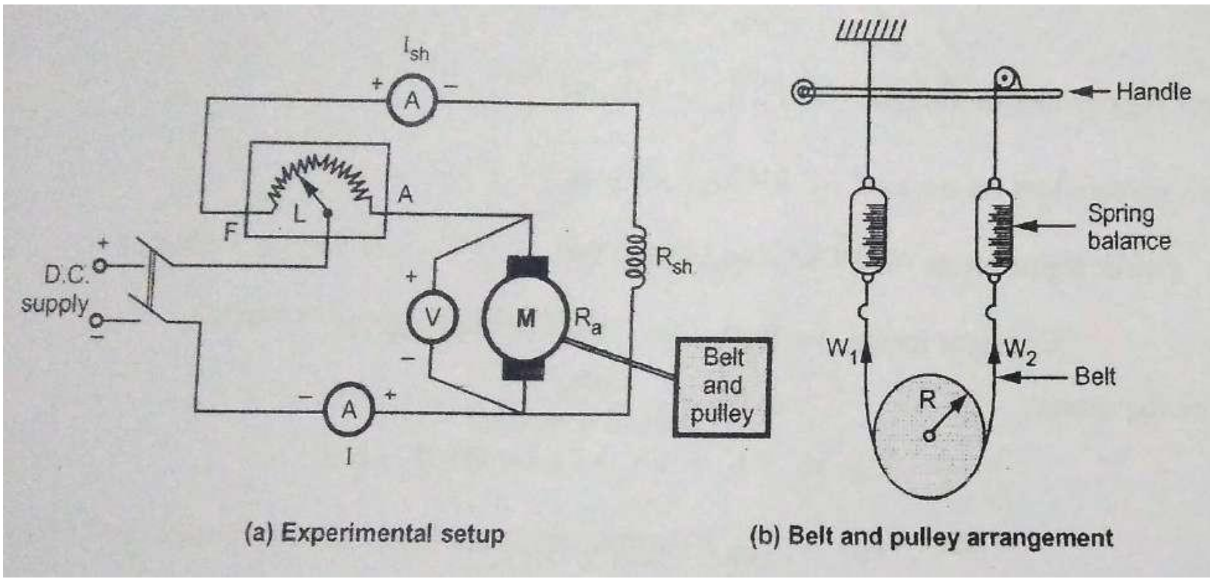


Figure: Brake Test setup

By adjusting the tension in the pulley the motor can be subjected from no load to its full load capability. Since the load is applied by the physical braking action, the test is called the ***Brake test***.

The tension in the belt is adjusted by using the handle. The tension (kgf) is obtained from the spring balance readings. The net force applied on the pulley by this braking arrangement is given by:

Net force $= (W_1 - W_2)$ **Kgf** $= 9.81(W_1 - W_2)$ **Nw**

Where R = Radius of the pulley in meters

N = Speed in RPM

W_1 and W_2 = Spring balance readings on the tight side and on the slack side of the

Pulley respectively.

With this force exerted on the pulley, the load torque applied on the motor shaft is given by:

$$\Upsilon_{load} = \textbf{Net force x Radius of the pulley} = 9.81(W_1 - W_2)R \quad \text{Nw.m}$$

With this applied load torque Υ_{load}, the output power(mechanical) of the motor is given by:

$$P_{out} = \Upsilon_{load} \text{ x } \omega = \Upsilon_{load} \text{ x} 2uN/60 \text{ W}$$

The input power (electrical) to the motor is given by : $P_{in} = VI$

Thus we have $ɻ = P_{out}/ P_{in} = [\Upsilon_{load} \text{ x}2uN/60+/VI$

Apart from the efficiency, we can also find out all the characteristics like ***Torque vs Speed*** , ***Speed vs Armature current*** and ***Torque vs armature current*** of the motor by noting down the currents and voltage along with the speed N at various load settings. The speed is measured by using physical contact type Tachometer.

Advantages:

1. Efficiency can be found out in the actual working conditions.
2. The method is simple and easy to perform.
3. The test can be performed on any type of DC machine.

Disadvantages:

1. Due to the friction lot of energy is wasted in the form of heat. Hence the test is quite expensive and is suitable for only small machines.
2. Since heat energy is not accounted for, the efficiency observed would be inaccurate to that extent.

Swinburne's test:

This is a test to determine the efficiency of any DC Machine (Motor or Generator) without conducting the actual test at the required load. The test is conducted just at no load and the constant losses are found out when the machine is running as a motor. Then the efficiency is found out by calculating the variable losses at the required load. This method is formulated by Sir James Swinburne and hence it is called Swinburne's test. This comes under the category *indirect method of testing*. The test setup required to conduct this test is shown in the figure below.

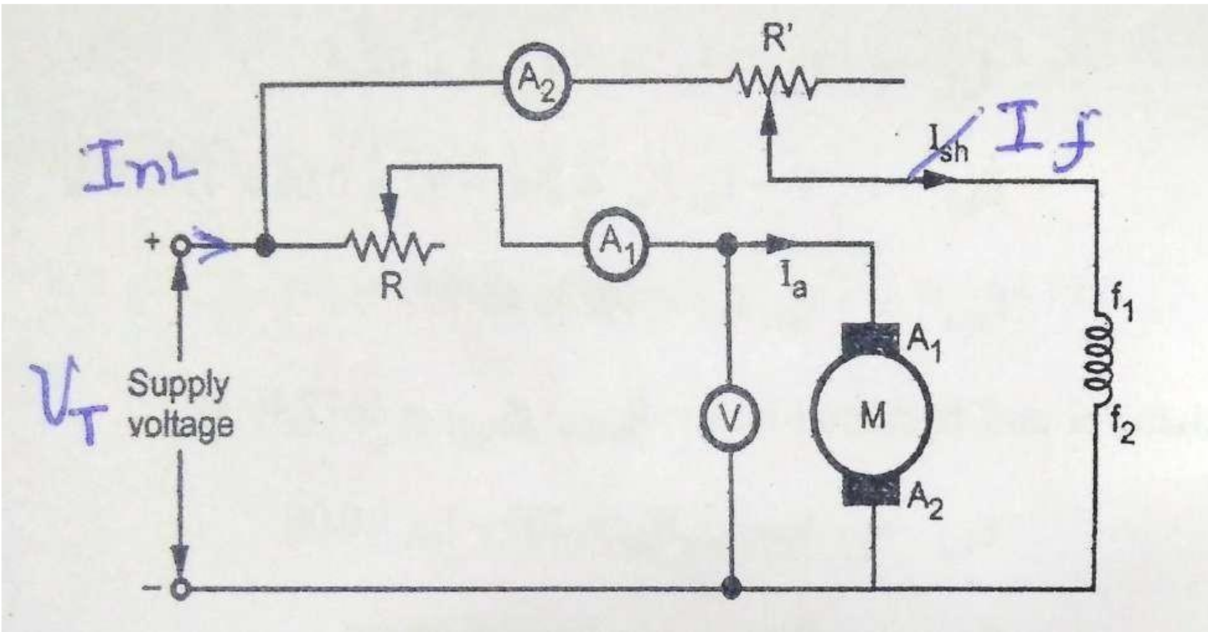


Figure: Swinburne's Test Setup

The machine is run as a motor on no load at normal terminal voltage $\mathbf{V_T}$, at normal speed and the armature current $\mathbf{I_A}$ & field current $\mathbf{I_F}$**(**$\mathbf{I_{sh}}$ in figure) are measured.

- Then the no load armature current $\mathbf{I_{NL} = I_A + I_F}$
- Variable losses on no load = $\mathbf{I_A^2 . R_A}$ (Machine's armature resistance can be measured directly and these losses can be calculated)

- Input to the motor = $V_T. I_{NL}$ = **Total losses** (Since the machine is on no load there is no output. i.e. the entire input power on no load goes as losses.)
- Therefore constant losses **P_C = (Total losses - Variable losses) = $(V_T. I_{NL})-(I_A^2 .R_A)$**

Using these constant losses P_C, the efficiency of the machine can be estimated at any other load when working either as a Motor or as a Generator.

Working as a Generator delivering a load current of I_L amperes at a terminal voltage of V_T volts:

Power output = $V_T. I_L$

Armature current $I_A = I_L + I_F$ (I_F is same as obtained in the No load test)

Variable loss = $I_A^2 R_A$ (R_A is obtained from the no load test or from Machine data)

Efficiency = (output/Input) = [output/(output + Total losses)] = $(V_T. I_L)/(V_T. I_L + I_A^2 R_A + P_C)$

(P_C is obtained from the No load test and $I_A^2 R_A$ is calculated using I_A corresponding to the required I_L at which the efficiency is to be calculated)

Working as a Motor drawing a load current of I_L amperes from a supply terminal voltage of V_T volts:

- Power in put = $V_T. I_L$

Armature current $I_A = I_L - I_F$ (I_F is same as obtained in the No load test)

Variable loss = $I_A^2 R_A$ (R_A is obtained from the no load test or from Machine data)

Efficiency = (output/Input) = [(Input-Total losses)/ Input] = [$V_T. I_L$-($I_A^2R_A$+P_C)]/($V_T. I_L$)

(**P_C** is calculated and obtained from the No load test and **$I_A^2R_A$** is calculated using **I_A** corresponding to the required **I_L** at which the efficiency is to be calculated)

Advantages of Swinburne's test:

- This is a very simple to determine the efficiency of the machine at any load just by conducting the no load test.
- The power required is very less compared to the direct full load test.

Disadvantages of Swinburne's test:

- This test can be done on Shunt machines only.
- The speed and flux are assumed constant. But the speed will fall with loading. Fall in speed results in lesser friction and windage losses. Change in flux will change the core losses.
- The temperature of the machine changes while running on load. Hence the assumption that **R_A** is same as that of the No load test is not correct.
- These reasons contribute to the difference in the efficiency obtained from the Swinburne's test and actual load test.

Hopkinson's test: In this set up two identical DC machines are coupled mechanically and tested together. One of the machines works as a motor and drives the other machine which works as generator and it's electrical output in turn is connected back to the motor. Hence this comes under the regenerative category and is also called a Back to Back test. The motor is connected to the mains supply and it draws power from the mains only to compensate for the losses in the two machines since the major power required by each machine is derived from the other machine. Since the power consumption is only to the extent of the losses they can be tested up to full load.

Figure below shows the Hopkinson's test up with all the measured voltage and current parameters marked clearly.

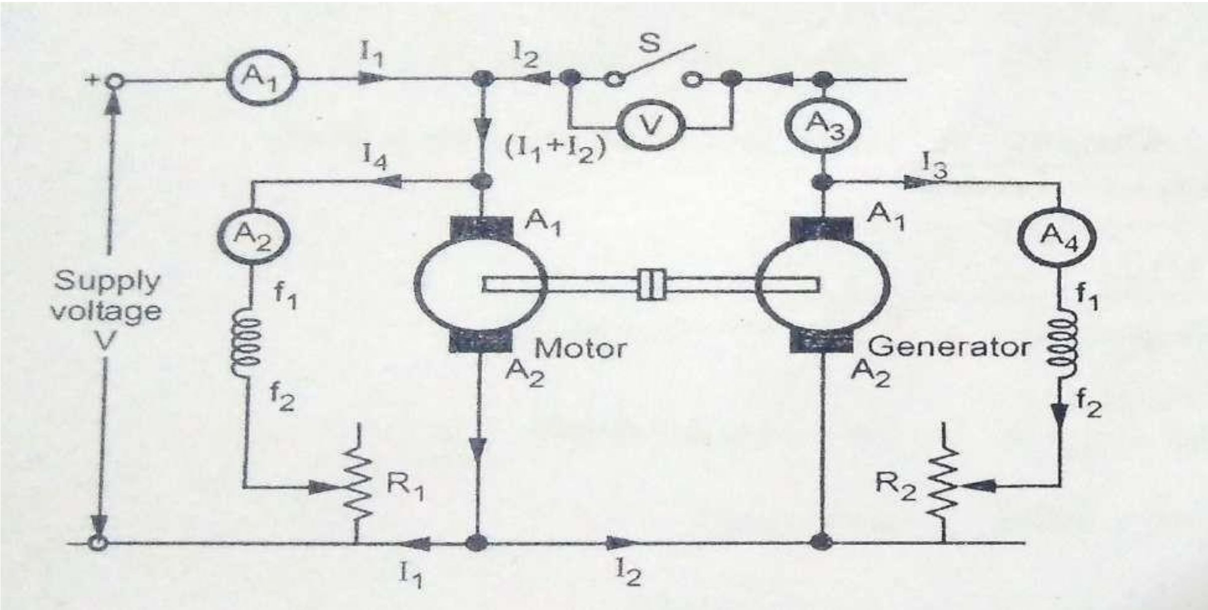


Figure: Hopkinson's Test Setup

(Ammeters are to be redesignated as 'A2 to A4' , 'A3 to A2 ' and 'A4 to A3' so that the current designations will match)

Initially the switch **S** is kept open and the Motor is run and brought to the rated speed by adjusting the field current using the field rheostat $\mathbf{R_1}$. The Generator Voltage is adjusted by adjusting it's field rheostat $\mathbf{R_2}$ until the voltmeter reads zero volts. (This indicates that the Generator voltage is same as that of the Motor both in amplitude and polarity) This prevents flow of any high circulating currents when the switch **S** is closed and the two machines are connected back to back.

Now the switch is closed thus bringing the machines back to back and the load on both the machines can be increased gradually by increasing the Generator excitation or decreasing the Motor excitation. The readings from all the meters are taken at every load setting for further evaluation. Let us spell out clearly all the parameters for ease of further derivations.

V = Terminal Voltage (Supply Voltage)

I_1 = Current drawn from mains supply

I_2 = Current supplied by the generator to the motor

I_3 = Excitation Current of Generator

I_4 = Excitation Current of Motor

R_a = Armature resistance of each machine

1. Equal efficiency : Let us now first find out the Efficiency 'η' assuming it to be same for both the machines:

Input to the motor = $V (I_1 + I_2)$

Output of the motor = η x Input to the motor = $\eta \times V (I_1 + I_2)$

This output of the motor is given as input to the generator. Hence

Input to the Generator = $\eta \times V (I_1 + I_2)$

Output of the generator = η x Input to the generator = $\eta \times \eta \times V (I_1 + I_2) = \eta^2 \times V (I_1 + I_2)$

But the output of the generator can also be given as = $V I_2$ and equating these two we get

$\eta^2 \times V (I_1 + I_2) = V I_2$ from which we get

$$\eta = \sqrt{[\, I_2 / (I_1 + I_2) \,]}$$

2. Un equal efficiency: Let us now find out the Efficiency 'η' assuming it to be unequal for the two machines:

In this analysis, the stray losses (Constant Losses) are assumed to be same for both the machines where as the field and armature copper losses are different (Since when the efficiencies are different it means that the currents are not the same and hence the copper losses also will not be same)

So first let us determine the Copper losses of the two machines independently and then the constant losses of both machines together can taken as the difference between the input power from the main supply and the total copper losses.

Armature copper losses in Generator = $(I_2 + I_3)^2 \times Ra$

Armature copper losses in Motor = $(I_1 + I_2 - I_4)^2 \times Ra$

Field copper losses in Generator = $V I_3$

Field copper losses in Motor = $V I_4$

We know that the total losses in both the machines put together are equal to the input power from the mains supply i.e $V I_1$

Hence we can take that the total stray losses for both the machines put together are the difference between the input power and all the copper losses put together. Hence

Stray losses of both machines together = $V I_1 - [(I_2 + I_3)^2 \times Ra + (I_1 + I_2 - I_4)^2 \times Ra + V I_3 + V I_4]$ = Say W_s and stray losses of each machine = $W_s/2$

Now that we know the variable losses (Armature and Field Copper losses) and the constant (stray) losses for both the machines we can easily find out the efficiencies of both Generator and Motor using the above data as shown below.

Efficiency of Generator:

Total losses = Generator's variable losses + Stray losses of one machine

$$= [(I_2 + I_3)^2 \times Ra + V I_3 + W_s/2]$$

Output of Generator = $V I_2$

Efficiency of Generator η_G = Output/ Input = Output/ (Output + Losses)

$$= V I_2 / V I_2 + [(I_2 + I_3)^2 \times Ra + V I_3 + W_s/2]$$

Efficiency of Motor:

Total losses = Motor's variable losses + Stray losses of one machine

$$= [(I_1 + I_2 - I_4)^2 \times Ra + V I_4 + W_s/2]$$

Input to Motor = $V (I_1 + I_2)$

Efficiency of Motor η_M = Output/ Input =(Input – losses) / Input

$$= [V (I_1 + I_2) - [(I_1 + I_2 - I_4)^2 \times Ra + V I_4 + W_s/2]] / V (I_1 + I_2)$$

Field's test:

Introduction and Methodology:

This test is for finding out the losses and efficiency of DC Series Motors by direct testing, since series motors cannot be tested on no load. The test is named after the inventor of test method 'MB Field '. Series motors which are normally used for Traction are available as pairs and hence this test is devised on two motors which are coupled mechanically. The test setup is shown in the figure below. One machine works as motor supplying power to the other one working as Generator. Their mechanical and iron (core) losses (put together called as *stray losses* or *constant losses*) are made equal by: (i) running them with equal speed and (ii) by connecting their both field windings in the motor armature circuit to the Motor input supply such that both the machines are equally excited . The load resistance is adjusted till the Motor draws the rated current as read by ammeter A1. In this condition all the other parameters are noted down from the respective meter readings as per the nomenclature given below.

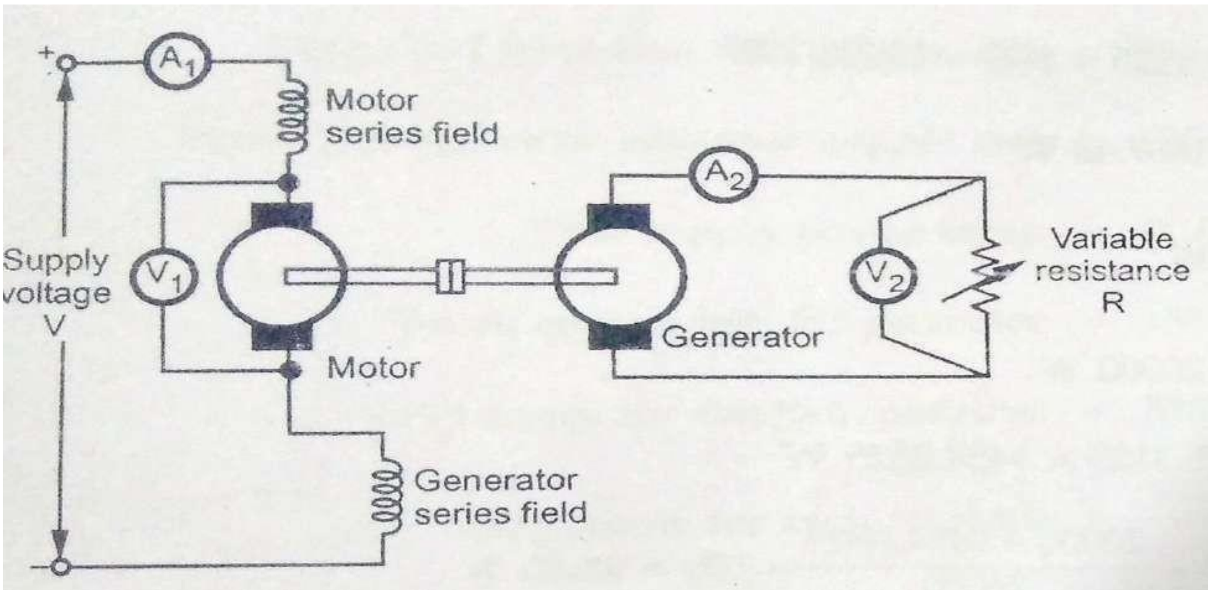


Figure: Field's Test Setup

Nomenclature:

V = Supply voltage

V_1 = Motor Supply voltage

V_2= Generator output voltage connected across the variable load resistance R

I_1 = Motor armature current and also the field current of both Motor and Generator

I_2 = Generator armature current

Also Let **R_A** = Motor armature resistance and **R_F** = Motor field resistance (which can be measured independently or can be taken from the machine data)

From this data obtained in this full load condition, the stray losses, copper losses and then efficiency of both the Motor and the Generator can be found out as below:

Stray Losses:

Input to the total Motor Generator test setup**: $V I_1$**

Output of Generator = **$V_2 I_2$**

Total losses of both motor and generator = **$W_T = V I_1 \bullet V_2 I_2$**

But total losses of both motor and generator **W_T** are also equal to (Armature Copper losses + Field Copper losses + Stray losses).Thus

$$W_T = V I_1 - V_2 I_2 = (I_1^2 + I_2^2) R_A + 2I_1^2 R_F + W_S$$

And total Stray Losses = Total Losses − Total Armature and Field copper losses

$$= VI_1 - V_2 I_2 - [(I_1^2 + I_2^2) R_A + 2I_1^2 R_F] \text{ and}$$

Stray losses per machine = Total Stray Losses/2 = **$W_S = [V I_1 - V_2 I_2 - [(I_1^2 + I_2^2) R_A + 2I_1^2 R_F]] / 2$**

Now using this value of stray losses of each machine, the efficiency of the machine as a Motor and as a Generator can be found out as below.

η as a Motor:

Input to Motor = $V_1 I_1$

Total losses = Armature and Field Copper losses + Stray losses = $I_1^2(R_A + R_F) + W_S$

Output of Motor = Input − Losses = $V_1 I_1 - \{I_1^2(R_A + R_F) + W_S\}$

Efficiency of the motor = η_M = **Output/Input** = $[V_1 I_1 - \{I_1^2(R_A + R_F) + W_S\}] / V_1 I_1$

η as a Generator:

This not very important because the machine is working in separately excited condition. However just for completion sake let us find it out.

Output of Generator = $V_2 I_2$

Total losses = Armature and Field Copper losses + Stray losses = $I_2^2 R_A + I_1^2 R_F + W_S$

Input to the Generator = Output + Losses = $V_2 I_2 + I_2^2 R_A + I_1^2 R_F + W_S$

Efficiency of the Generator = η_G = **Output/Input** = $V_2 I_2 / (V_2 I_2 + I_2^2 R_A + I_1^2 R_F + W_S)$

Retardation test or Running down test: This is an indirect test similar to the Swinburne's test where in the constant (Stray losses) losses are first determined and then the efficiency at any load when the machine is working both as a Generator and Motor are estimated on the same lines. However the constant losses in this test are determined by a different principle i.e. by finding out the kinetic energy spent by a rotating mass during the process of retardation from the rated speed to zero speed and then calculating the rate of change of kinetic energy which is equal to the Power Loss.

The setup to conduct this test is shown in the figure below.

Test Procedure: The motor is started and taken to a speed higher than the rated speed of the machine. Then the supply to the motor is cutoff by moving the Two Pole Two way switch from the supply side to the open terminals. The armature then slows down with its' own inertia and its stored energy is used up to supply the constant rotational losses (stray losses) like iron, friction, windage etc. This power loss is found out from the following principle.

If **'I'** is the moment of Inertia of the Armature, and **'ω'** is the angular velocity, then the kinetic energy of the armature is given by:

$$KE = \text{ъ}\ I\omega^2 \text{ and}$$

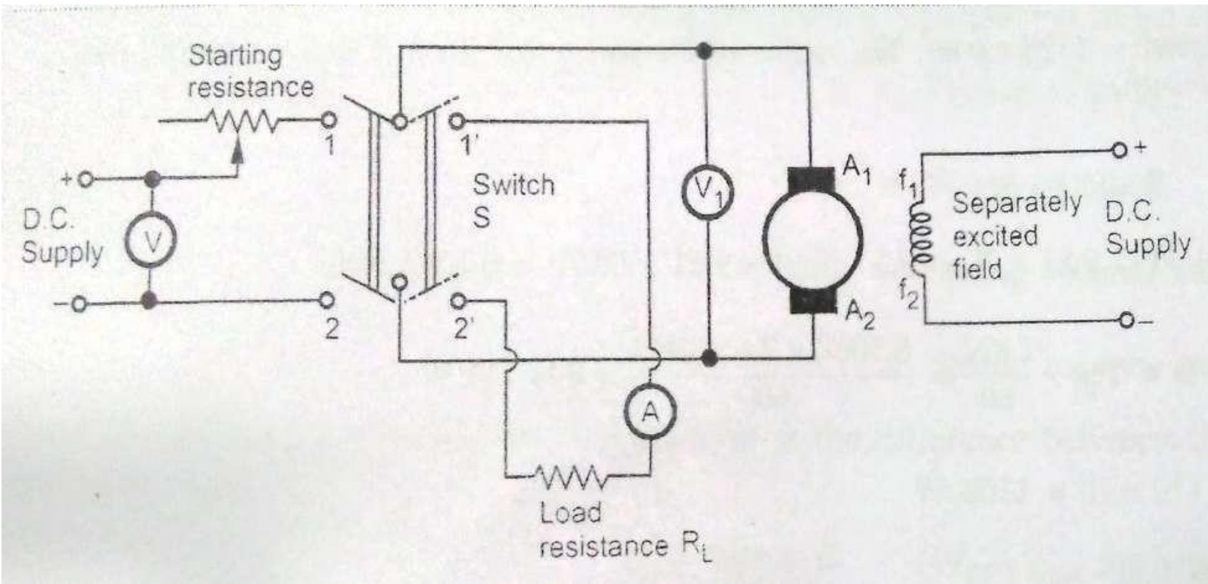


Figure: Retardation test setup

The power loss which is the rate of change of Energy is given by**:**

$$\mathbf{P_L = d/dt\ (Ъ\ I\omega^2\ )\ = I.\ \omega\ .\ d\omega/dt}$$

Substituting the value of **'ω'** in terms of the speed in RPM **'N'** (2uN/60) we get

$$\mathbf{P_L = (2u/60)^2\ I.\ N\ .\ dN/dt}$$

So, to find out the stray losses we must know **'I'**, the moment of inertia and **dN/dt**, the rate of change in speed. The method of finding out these quantities is given below.

*The method of finding out **dN/dt :***

When the motor is cutoff from the input supply, the speed starts falling down. The motor back e.m.f. as read by the Voltmeter $\mathbf{V_1}$ connected across the motor is noted down as a function of time. Since we know that the back e.m.f is proportional to the speed we can convert the $\mathbf{V_1}$ reading into speed in RPM and plot it as a function of time as shown in the figure below. In this figure, at the point **C** corresponding the rated speed a tangent AB is drawn whose slope gives the rate of change in speed dN/dt.

Thus: **dN/dt = OA(RPM)/OB(Time)**

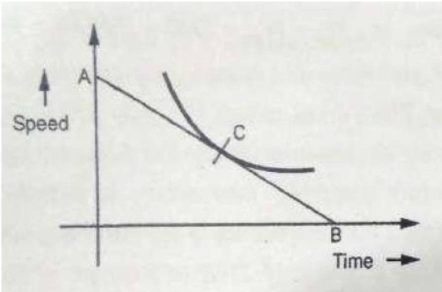


Figure: Speed fall during the retardation test

Determination of Moment of Inertia (I):

Method -1: Using Flywheel

The same test as earlier is repeated after adding a *flywheel* of known Moment of Inertia **'I_1'** to the armature shaft of the motor and the resulting rate of change of speed is obtained. Let us call the earlier rate of change as dN/dt_1 and the second rate of change with added inertia as dN/dt_2.

Then, since the losses can be assumed to be same with or without the new flywheel we have the following relations:

In the first case without flywheel**:** $\mathbf{P_L = (2u/60)^2\ I.\ N\ .\ dN/dt_1}$

In the second case with added flywheel: $\mathbf{P_L = (2u/60)^2\ (I + I_1).\ N\ .\ dN/dt}$

After equating the two equations and simplifying we get : $\mathbf{I = I_1\ (t_1)\ /(\ t_2 - t_1)}$

Method -2: Without using a Flywheel (Using Resistance Braking)

- First, the switch is taken from supply side to the open condition and then the time taken (say t_1) for the motor to slow down from a speed slightly higher than the rated speed to a speed slightly lower than the rated speed (say 6N) is noted down with just the Armature alone like in the earlier method step −1 (without any added external inertia)
- Then again the switch S is moved from the supply position to the Resistance position quickly and again the time taken(say t_2) for the same change in speed (same 6N)is noted down. By this effectively we are connecting an electrical load across the armature in which the stored electrical power is dissipated thus providing an additional retarding torque.
- This additional power loss due to the resistance is given by the product of the Average Voltage across the Armature (say V) and the average current (say I_A) that flows into the Braking Resistance R i.e. $\mathbf{I_A^2 (R_A + R) x V}$ = say **W'**
- Then the powers dissipated during the above two steps are given by

1. **W = $(2u/60)^2$ I. N . dN/dt_1 (Just due to the armature Inertia)**
2. **W + W' = $(2u/60)^2$ I. N . dN/dt_2 (Due to the armature Inertia and the braking resistance)**

Separation of Constant losses:

The theory required for the purpose of Separation of Constant losses is explained below.

At any given excitation:

- Friction losses and hysteresis losses are both proportional to speed .

- Windage losses and eddy current losses are both proportional to square of speed.
- Hence Friction losses = **AN** Watts, Windage losses = $\mathbf{BN^2}$ Watts, Hysteresis losses = **CN** Watts, and Eddy current losses = $\mathbf{DN^2}$ Watts where **N** = speed and **A,B,C** and **D** are constant coefficients
- Further the coefficient **C** of hysteresis losses is proportional to $\mathbf{B_{max}^{1.6}}$ and the coefficient **D** of Eddy current losses is proportional to $\mathbf{B_{max}^{2}}$

The other standard relation: For a motor on no load, power input to the armature is the sum of the armature copper losses and the above losses.

Hence from a no load test we can get the constant losses as usual and then equate them to the constant losses with the above categorization as shown below:

Power input to the armature = $\mathbf{V.I_a}$ watts.

Armature copper losses = $\mathbf{I_a^2.R_a}$ watts

Constant Losses = $\mathbf{W = V.I_a - I_a^2.R_a = (A + C)N + (B + D)N^2}$

$$\mathbf{W/N = (A+C) + (B+D)N.}$$

First the test is conducted with rated field current as per the following procedure:

1. The motor is started on no load with field current set to rated value by adjusting the field auto transformer.
2. The armature voltage is increased till the speed is about 200 rpm more than the rated value.
3. Now, the speed is gradually decreased by decreasing the armature voltage, the values of armature voltage, armature current and speed are noted down.

Auto Transformer

In Auto Transformer, one single winding is used as primary winding as well as secondary winding. A diagram of auto transformer is shown below, the winding AB of total turns N_1 is considered as primary winding. This winding is tapped from point ′C′ and the portion BC is considered as secondary. Let's assume the number of turns in between point's ′B′ and ′C′ is N_2. If V_1 voltage is applied across the winding i.e. in between ′A′ and ′C′.

So voltage per turn in this winding is $\frac{V_1}{N_1}$.

Hence, the voltage across the portion BC of the winding, will be

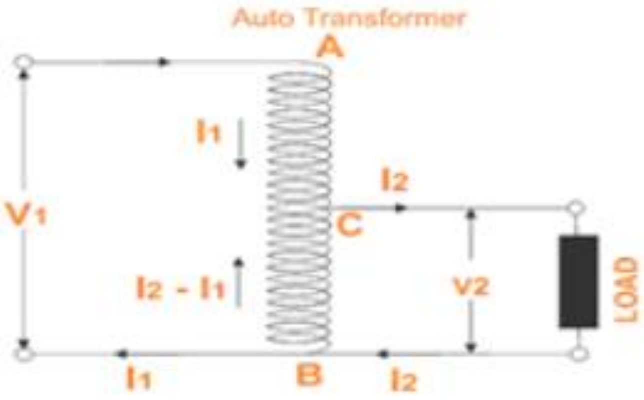
$\frac{V_1}{N_1} \times N_2$ and from the figure above, this voltage is V_2.

$$\text{Hence, } \frac{V_1}{N_1} \times N_2 = V_2$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{N_2}{N_1} = \text{Constant} = k$$

As BC portion of the winding is considered as secondary, it can easily be understood that value of constant ′k′ is nothing but turns ratio or voltage ratio of that Auto Transformer.

When load is connected between secondary terminals i.e. between ′B′ and ′C′, load current I_2 starts flowing .The current in the secondary winding or common winding is the difference of I_2 & I_1.



Disadvantages of using Auto Transformer

1. Because of electrical conductivity of the primary and secondary windings the lower voltage circuit is liable to be impressed upon by higher voltage. To avoid breakdown in the lower voltage circuit, it becomes necessary to design the low voltage circuit to e-withstand higher voltage.

2. The leakage flux between the primary and secondary windings is small and hence the impedance is low. This results into severer short circuit currents under fault conditions.

3. The connections on primary and secondary sides have necessarily to be same, except when using interconnected starring connections. This introduces complications due to changing primary and secondary phase angle particularly in the case-by-case of delta / delta connection.

4. Because of common neutral in a star/star connected auto transformer it is not possible to earth neutral of one side only. Both their sides have to have their neutrality either earth or isolated.

5. It is more difficult to preserve the electromagnetic balance of the winding when voltage adjustment tappings are provided. It should be known that the provision of adjusting tapping on an auto transformer increases considerably the frame size of the transformer. If the range of tapping is very large, the advantages gained in initial cost is lost to a great event

Copper savings in Auto Transformer

Now we will discuss the savings of copper in auto transformer compared to conventional two windings electrical power transformer. We know that weight of copper of any winding depends upon its length and cross - sectional area. Again length of conductor in winding is proportional to its number of turns and cross - sectional area varies with rated current. So weight of copper in winding is directly proportional to product of number of turns and rated current of the winding.
Therefore, weight of copper in the section AC proportional to $(N_1 - N_2)I_1$

And similarly, weight of copper in the section BC proportional to $N_2(I_2 - I_1)$

Hence, total weight of copper in the winding of Auto Transformer proportional to $(N_1 - N_2)I_1$ $N_2(I_2 - I_1)$

$\Rightarrow N_1I_1 \quad -N_2I_1 \quad N_2I_2 \quad -N_2I_1$

$\Rightarrow N_1I_1 \;\; N_2I_2 - 2N_2I_1$

$\Rightarrow 2N_1I_1 \quad -2N_2I_1 \quad (\text{Since}, N_1I_1 = N_2I_2)$

$\Rightarrow 2(N_1I_1 - N_2I_1)$

In similar way it can be proved, the weight of copper in two winding transformer is proportional to, $N_1I_1 \quad N_2I_2$

$\Rightarrow 2N_1I_1$ (Since, in a transformer $N_1I_1 = N_2I_2$)

Let's assume, W_a and W_{tw} are weight of copper in auto transformer and two winding transformer respectively,

Auto transformer employs only single winding per phase. Advantages of using auto transformer. For transformation ratio = 2, the size of the auto transformer would be approximately 50% of the corresponding size of two winding transformer. For transformation ratio say 20 however the size would be 95%. The saving in cost is of course not in the same proportion. The saving of cost is appreciable when the ratio of transformer is low, that is lower than 2.

Hence, $\dfrac{W_a}{W_{tw}} = \dfrac{2(N_1I_1 - N_2I_1)}{2(N_1I_1)}$

$= \dfrac{N_1I_1 - N_2I_1}{N_1I_1}$

$= 1 - \dfrac{N_2I_1}{N_1I_1}$

$= 1 - \dfrac{N_2}{N_1}$

$= 1 - k$

$\therefore W_a = W_{tw}(1-k)$

$\Rightarrow W_a = W_{tw} - kW_{tw}$

$\therefore$ Saving of copper in auto transformer compared to two winding transformer,

$\Rightarrow W_{tw} - W_a = kW_{tw}$

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Tertiary Winding of Transformer

Tertiary Winding of Transformer

Advantage of Tertiary Winding,

1. It reduces the unbalancing in the primary due to unbalancing in three phase load

2. It redistributed the flow of fault current

3. Sometime it is required to supply an auxiliary load in different voltage level in addition to its main secondary load. This secondary load can be taken from tertiary winding of three winding transformer.

4. As the tertiary winding is connected in delta formation in 3 winding transformer, it assists in limitation of fault current in the event of a short circuit from line to neutral.

Stabilization by tertiary winding of transformer:-

In star - star transformer comprising three single units or a single unit with 5 limb core offers high impedance to the flow of unbalanced load between the line and neutral. This is because, in both of these transformers, there is very low reluctance return path of unbalanced flux. If any transformer has N - turns in winding and reluctance of the magnetic path is R_L, then, mmf = N.I = ΦR_L .... (1) Where I and Φ are electric current and flux in the transformer. Again, induced voltage V = $4.44\Phi fN$ $\Rightarrow V \propto \Phi$
$\Rightarrow \Phi = K.V$ (Where K is constant)....... (2)

Now, from equation (1) & (2) , it can be rewritten as,

$N.I=K.V.R_L$
$\Rightarrow V/I=N/(K.R_L)$
$\Rightarrow Z=N/(K.R_L)$
$\Rightarrow Z \propto 1/R_L$

From, the above expression it is found that impedance is inversely proportional to reluctance. The impedance offered by the return path of unbalanced load current, is very high where very low reluctance return path is provided for unbalanced flux. In other words, very high impedance to the flow of unbalanced current in 3 phases system between line and neutral. Any unbalanced current in three phase system can be divided into three sets of components like wise positive sequence, negative sequence and zero sequence components. The zero sequence current actually co-phasial current in three lines. If value of co-phasial current in each line is I_o, then total current flows through the neutral of secondary side of transformer is $I_n = 3.I_o$. This current cannot be balanced by primary current as the zero sequence current cannot flow through the isolated neutral star connected primary. Hence the said electrical current in the secondary side set up a magnetic flux in the core. As we earlier in this chapter low reluctance path is available for the zero sequence flux in a bank of single phase units and in the 5 limb core consequently the impedance offered to the zero sequence current is very high. The delta connected tertiary winding of transformer permits the circulation of zero sequence current in it. This circulating current in this delta winding balance the zero sequence component of unbalance load, hence prevent unnecessary development of unbalance zero sequence flux in the transformer core. In few words it can be said that, placement of tertiary winding in star - star - neutral transformer considerably reduces the zero sequence impedance of transformer.

Rating of tertiary winding of transformer:-

Rating of tertiary winding of transformer depends upon its use. If it has to supply additional load, its winding cross - section and design philosophy is decided as per load and three phase dead short circuit on its terminal with power flow from both sides of HV & MV.

In case it is to be provided for stabilizing purpose only, its cross - section and design has to be decided from thermal and mechanical consideration for the short duration fault currents during various fault conditions single line - to - ground fault being the most onerous.

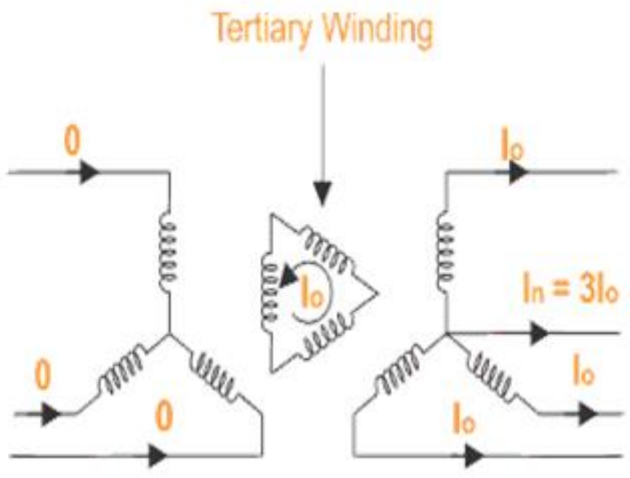

Diagram of Three Winding Transformer

Instrument Transformer

- In power system, the currents and voltages are very large
  - Therefore, their direct measurements are not possible.
- It might appear that the extension of range could be convenientlydone by the use of shunts for currents and multiplier for voltage measurement, as in DC.
  - But this method is suitable only for small values of current and voltage.
    - Difficult to achieve accuracy with a shunt on AC
    - Capability of having shunt of large range is not possible
    - The power consumed by multipliers become large as the voltage increases
    - The measuring circuit is not isolated electrically from the power circuit
- The solution is to step-down these currents/voltages with thehelp of Instrument Transformer
  - So that, they could be metered with instruments of moderate size

Current Transformer (C.T.):

- Transformers used for current measurement
- Steps down the current to the level of ammeter.

Voltage Transformer (V.T. or P.T.):

- Transformers used for voltage (Potential) measurement
- Steps down the voltage to the level of voltmeter.

- Used in AC system for the measurement of current, voltage,power and energy.
- Finds a wide application in protection circuits of power system
  - Ex. over current, under voltage, earth fault, etc.

Advantages of Instrument Transformer:

- Their reading do not depend upon circuit constant such as R, L & C
  - As in the case of shunts and multipliers
- Possible to standardize the instrument around their ratings.
  - This makes the replacement of instrument transformer very easy.
- The measuring circuit is isolated from the power circuit
- Low power consumption in the metering circuit
- Several instrument can be operated from a single instrumenttransformer

Ratios of Instrument Transformer:

Transformation Ratio, $R = I_{Pri} / I_{Sec}$ for a C.T. and $R = V_{Pri} / V_{Sec}$ for a P.T.

Nominal Ratio, K_n = Rated I_{Pri} / Rated I_{Sec} for a C.T.

= Rated V_{Pri} / Rated V_{Sec} for a P.T.

Turns Ratio, $n = N_{sec} / N_{Pri}$ for a C.T.

$= N_{Pri} / N_{Sec}$ for a P.T.

Ratio Correction Factor (RCF) = Transformation Ratio/Nominal Ratio$=R/K_n$

or $R = RCF \times K_n$

— The ratio marked on the transformer is their nominal ratio

— **What is the meaning of KVA reading on the Transformer**

Current Transformer:

- Primary winding is connected in series with line carrying the currentto be measured
  - Therefore, I_{Pri} α Load
- Primary winding consist of very few turns
  - Therefore, no appreciable voltage drop across it
- Secondary winding has larger number of turns
  - Exact number is being determined by the turn ratio

Relationship in a C.T.:

- r_s and x_s = resistance and reactance of secondary winding
- r_e and x_e = resistance and reactance of external burden
- E_p and E_s = primary and secondary winding induced voltage
- N_p and N_s = number of primary and secondary winding turns
- V_s = voltage at secondary winding terminals
- I_p and I_s = primary and secondary winding currents
- Φ = working flux of the transformer
- θ = phase angle of transformer (angle between I_s reversed and I_p)
- δ = angle between E_s and I_s
- Δ = phase angle of secondary winding load circuit
- I_o = exciting current
- I_m and I_e = magnetizing and loss component of I_o
- α = angle between I_o and Φ

– **Expression for Transformation Raito is derived on board**

Errors in C.T.:

- The secondary winding current is not a constant fraction of theprimary winding current
  - depend upon magnetizing and loss component of exciting current
  - this introduces considerable errors into current measurements

$$\text{ratio error} = (K_n - R)/R$$

- It is necessary that the phase of I_s shall be displaced exactly by180^0 from I_p.
  - but, it is displaced by an angle θ.

Characteristic of C.T.:

- *Effect of change of I_p*
  - If I_p changes, I_s also changes proportionally
  - At low values of I_p, the current I_m and I_e are a great portion of I_p
    - Therefore, errors are greater
  - As the I_p increases, I_s increases and results in decrement of R.

- *Effect of change of I_s*
  - Increment in I_s means increase in Volt-Ampere rating
  - This increases the secondary winding induced voltage
    - Therefore, I_m and I_e are increased
  - Thus, the errors will be increased.

- *Effect of change of Frequency*
  - Increase in frequency will result in proportionate decrease in fluxdensity

Means to reduce the error in C.T.:

- Ideally, R=n and θ=0
  - But, as a result of physical limitations inherent in electric and magneticcircuit, the ideality will be lost and errors are induced

- The expression of R and θ indicates that
  - Both depend upon the I_e and I_m respectively
- Thus, they are chosen small.
  - Specific design feature will help in minimization of the errors

Core:

- Core must have a low reluctance and low core loss
- Reduction of reluctance flux path can be done by
  - using materials of high permeability
  - short magnetic path
  - large cross section of core
  - Low value of flux density
- The number of joints in building up core should be minimum
  - because joints produce air gape
    - which offer path of high reluctance for the flux
- Core loss is reduced by choosing materials of low hysteresisand low eddy current losses

Effect of secondary winding open

- C.T. are always used with the secondary winding closed
  - Never open the secondary windings circuit of a C.T. while its primarywinding is energized
  - Failure to this may lead to serious consequences for both
- In case of P.T., the current flowing in the primary winding islargely the reflection of that flowing in the secondary circuit.
  - whereas, in case of a C.T., the primary winding is connected in serieswith the line whose current is being measured
  - This current is in no ways controlled or determined by the condition ofsecondary winding circuit
- Under normal operating conditions, both primary and secondary windings produces ***mmf***, which act against each other
  - The secondary *mmf* is slightly less than the primary *mmf*
    - Thus, the resultant *mmf* is small