Digital Signal Processing (Th- 03)

(As per the syllabus of the SCTE&VT, Bhubaneswar, Odisha)

<u>Sixth Semester</u>

Electrical & Electronics Engg.

Prepared By: *Er DHARMAPADA OJHA*

SL.NO.	TOPICS	EXPT.MARKS
01	INTRODUCTION OF SIGNALS, SYSTEMS, AND SIGNAL PROCESSING	15
02	DISCRETE TIME SIGNALS & SYSTEMS	25
03	THE Z -TRANSFORM & ITS APPLICATION TO THE ANALYSIS OF LTI SYSTEM	30
04	DISCUSS FOURIER TRANSFORM: ITS APPLICATION & PROPERTIES	20
05	FAST FOURIER TRANSFORM ALGORITHM & DIGITAL FILTERS	10
	TOTAL	100

CHAPTER: -1

INTRODUCTION OF SIGNALS, SYSTEMS, AND SIGNAL PROCESSING

INTRODUCTION TO DIGITAL SIGNAL PROCESSING: -

- Digital Signal Processing is the mathematical manipulation of an information signal, such as audio, temperature, voice, and video and modify or improve them in some manner.

BASICS OF SIGNAL, SYSTEM & SIGNAL PROCESSING: -

Signals: -

- In electrical engineering, the fundamental quantity of representing some information is called a signal. It does not matter what the information is i-e: Analog or digital information. In mathematics, a signal is a function that conveys some information. In fact, any quantity measurable through time over space or any higher dimension can be taken as a signal. A signal could be of any dimension and could be of any form.

Analog signals: -

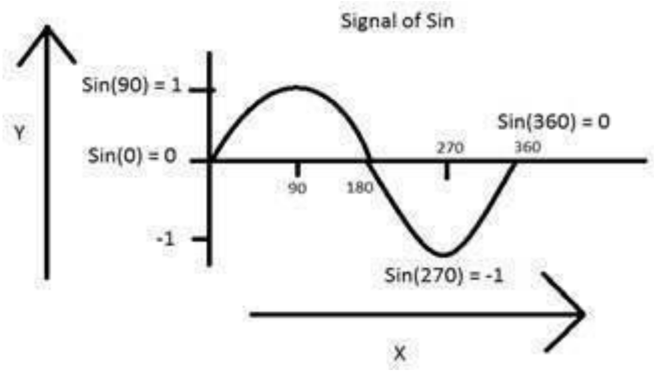
- A signal could be an analog quantity that means it is defined with respect to the time. It is a continuous signal.
- **For example:**

I. Human voice

Human voice is an example of analog signals. When you speak, the voice that is produced travel through air in the form of pressure waves and thus belongs to a mathematical function, having independent variables of space and time and a value corresponding to air pressure.

II. Another example is of sin wave which is shown in the figure below.

Y = sin(x) where x is independent.

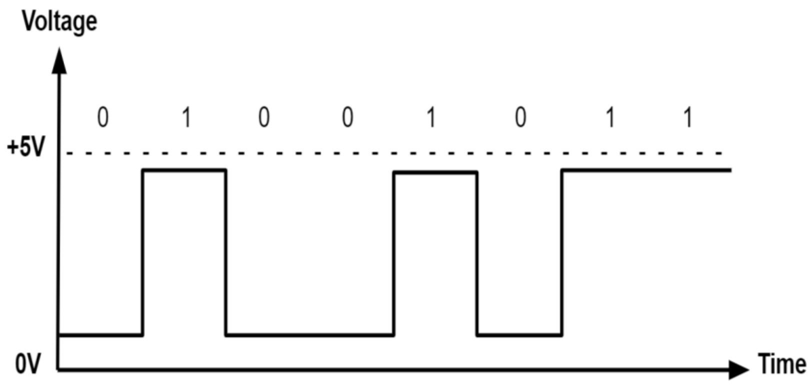


Digital signals

- The word digital stands for discrete values and hence it means that they use specific values to represent any information. In digital signal, only two values are used to represent something i-e: 1 and 0 (binary values). Digital signals are denoted by square waves. They are discontinuous signals.
- **For example:**

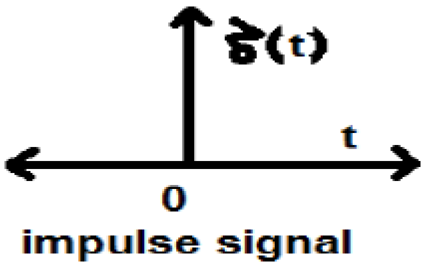
i. Computer keyboard

Whenever a key is pressed from the keyboard, the appropriate electrical signal is sent to keyboard controller containing the ASCII value that particular key. For example, the electrical signal that is generated when keyboard key a is pressed, carry information of digit 97 in the form of 0 and 1, which is the ASCII value of character a.



- Some other important signals are there. Such as:
- Unit Impulse or Delta Function

A signal, which satisfies the condition, δ(t)=limϵ→∞x(t) is known as unit impulse signal. This signal tends to infinity when t = 0 and tends to zero when t ≠ 0 such that the area under its curve is always equals to one. The delta function has zero amplitude everywhere excunit_impulse.jpgept at t = 0.



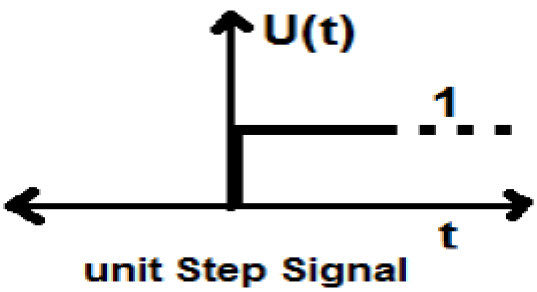
Unit Step Signal

A signal, which satisfies the following two conditions −

U(t)=1(whent≥0) and

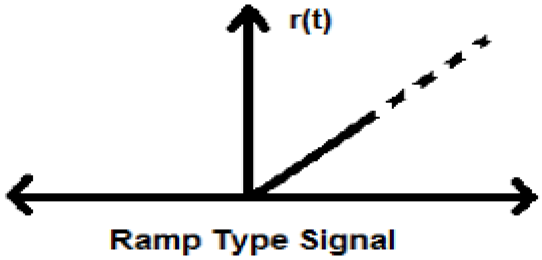
U(t)=0(when<0)

is known as a unit step signal.



- Ramp Signal

Integration of step signal results in a Ramp signal. It is represented by rt. Ramp signal also satisfies the condition r(t)=∫t−∞U(t)dt=tU(t). It is neither energy nor power NENP type signal.



➢ Systems

- A system is a defined by the type of input and output it deals with. Since we are dealing with signals, so in our case, our system would be a mathematical model, a piece of code/software, or a physical device, or a black box whose input is a signal and it performs some processing on that signal, and the output is a signal. The input is known as excitation and the output is known as response.

- In the above figure a system has been shown whose input and output both are signals but the input is an analog signal. And the output is a digital signal. It means our system is actually a conversion system that converts analog signals to digital signals.

➢ **Continuous systems vs discrete systems**

➢ Continuous systems

- The type of systems whose input and output both are continuous signals or analog signals are called continuous systems.

➢ Discrete systems

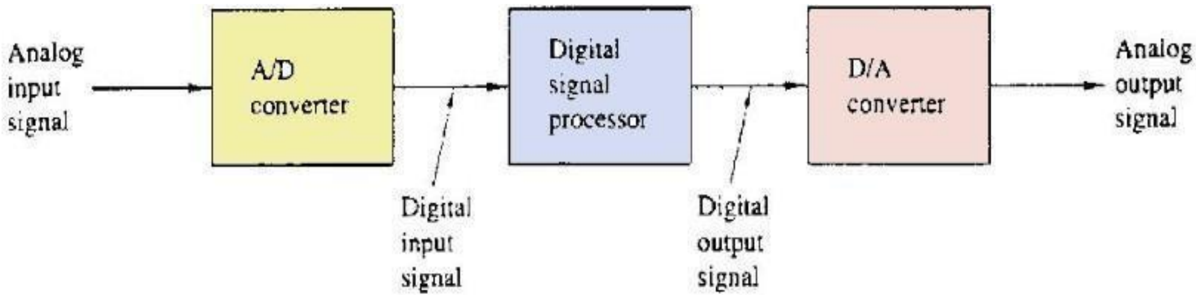
- The type of systems whose input and output both are discrete signals or digital signals are called digital systems.

➢ Signal processing

- Signal processing is an electrical engineering subfield that focuses on analysing, modifying, and synthesizing signals such as sound, images, and scientific measurements.
- Signal processing techniques can be used to improve transmission, storage efficiency and subjective quality and to also emphasize or detect components of interest in a measured signal.

➢ Basic elements of a Digital Signal Processing System: -

• In its most general form, a DSP system will consist of three main components, as illustrated in Figure.
• The analog-to-digital (A/D) converter transforms the analog signal $x_a(t)$ at the system input into a digital
signal $x_d[n]$. An A/D converter can be thought of as consisting of a sampler (creating a discrete time signal), followed by a quantizer (creating discrete levels).
• The digital system performs the desired operations on the digital signal $x_d[n]$ and produces a corresponding output $y_d[n]$ also in digital form.
• The digital-to-analog (D/A) converter transforms the digital output $y_d[n]$ into an analog signal $y_a(t)$ suitable for interfacing with the outside world.
• In some applications, the A/D or D/A converters may not be required; we extend the meaning of DSP systems to include such cases.



ANALOG I/P SIGNAL- $x_a(t)$
A/D O/P- $x_d[n]$
DIGITAL SYSTEM- $y_d[n]$
D/A O/P- $y_a(t)$

- Discrete-time signals are typically written as a function of an index n (for example, x(n) or xn may
- represent a discretisation of x(t) sampled every T seconds). In contrast to Continuous signal systems,

- where the behaviour of a system is often described by a set of linear differential equations, discrete-time systems are described in terms of difference equations.
- Transform-domain analysis of discrete-time systems often makes

use of the Z transform.

ADVANTAGES OF DSP OVER ASP: -

1. Physical size of analog systems is quite large while digital processors are more compact and lighter in weight.
2. Analog systems are less accurate because of component tolerance ex R, L, C and active components. Digital components are less sensitive to the environmental changes, noise and disturbances.
3. Digital system is most flexible as software programs & control programs can be easily modified.
4. Digital signal can be stores on digital hard disk, floppy disk or magnetic tapes. Hence becomes transportable. Thus, easy and lasting storage capacity.
5. Digital processing can be done offline.
6. Mathematical signal processing algorithm can be routinely implemented on digital signal processing systems. Digital controllers are capable of performing complex computation with constant accuracy at high speed.
7. Digital signal processing systems are upgradeable since that are software controlled.
8. Possibility of sharing DSP processor between several tasks.
9. The cost of microprocessors, controllers and DSP processors are continuously going down. For some complex control functions, it is not practically feasible to construct analog controllers.
10. Single chip microprocessors, controllers and DSP processors are more versatile and powerful.

1.2-CLASSIFY SIGNALS-MULTI CHANNEL & MULTI DIMENTIONAL SIGNALS: -

CLASSIFICATION OF SIGNALS: -

1. Single channel and Multi-channel signals
2. Single dimensional and Multi-dimensional signals
3. Continuous time and Discrete time signals.
4. Continuous valued and discrete valued signal
5. Analog and digital signals.

1. Single channel and Multi-channel signals: -

- If signal is generated from single sensor or source, it is called as single channel signal. If the signals are generated from multiple sensors or multiple sources or multiple signals are generated from same source called as Multi-channel signal. Example ECG signals. Multi-channel signal will be the vector sum of signals generated from multiple sources.

2. Single Dimensional (1-D) and Multi-Dimensional signals (M-D): -

- If signal is a function of one independent variable it is called as single dimensional signal like speech signal and if signal is function of M independent variables called as Multi - dimensional signals. Gray scale level of image or Intensity at particular pixel on black and white TV is examples of M-D signals.

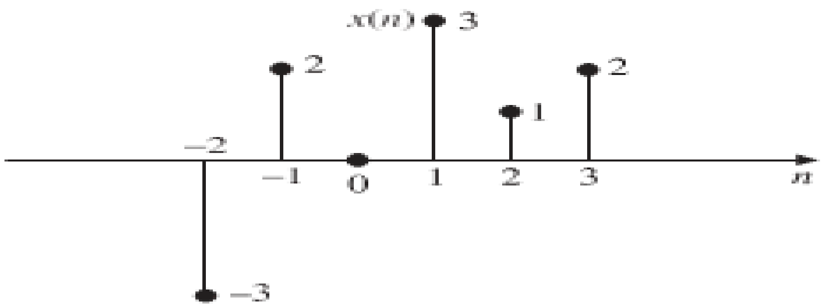
3. Continuous time and Discrete time signals: -

- **Continuous Time (CTS)**

1. This signal can be defined at any time instance & they can take all values in the continuous interval (a, b) where a can be -∞ & b can be ∞
2. These are described by differential equations.
3. This signal is denoted by x(t).
4. The speed control of a dc motor using a tacho generator feedback or Sine or exponential waveforms.

- **Discrete time (DTS)**

1. This signal can be defined only at certain specific values of time. These time instance need not be equidistant but in practice they are usually takes at equally spaced intervals.
2. These are described by difference equation.
3. These signals are denoted by x(n) or notation x(nT) can also be used.
4. Microprocessors and computer-based systems uses discrete time signals.



4. Continuous valued and Discrete Valued signals: -

- **Continuous Valued**

1. If a signal takes on all possible values on a finite or infinite range, it is said to be continuous valued signal.
2. Continuous Valued and continuous time signals are basically analog signals.

- **Discrete Valued**

1. If signal takes values from a finite set of possible values, it is said to be discrete valued signal.
2. Discrete time signal with set of discrete amplitude are called digital signal.

5. Analog and digital signal: -

- **Analog signal**

1. These are basically continuous time & continuous amplitude signals.
2. ECG signals, Speech signal, Television signal etc. All the signals generated from various sources in nature are analog.

- **Digital signal**

1. These are basically discrete time signals & discrete amplitude signals. These signals are basically obtained by sampling & quantization process.
2. All signal representation in computers and digital signal processors are digital.

Note: Digital signals (DISCRETE TIME & DISCRETE AMPLITUDE) are obtained by sampling the ANALOG signal at discrete instants of time, obtaining DISCRETE TIME signals and then by quantizing its values to a set of discrete values & thus generating DISCRETE AMPLITUDE signals.

1.3-<u>DISCUSS THE CONCEPT OF FREQUENCY IN CONTINUOUS TIME & DISCRETE TIME SIGNALS</u>: -

- **<u>continuous time and discrete time signal: -</u>**
  - The independent variable(s) for a signal may be continuous or discrete. A signal is considered to be a continuous time signal if it is defined over a continuum of the independent variable.
  - A signal is considered to be discrete time if the independent variable only has discrete values.
- **<u>Frequency of a signal</u>**

- **Frequency** is the rate at which current changes direction per second. It is measured in hertz (Hz), an international unit of measure where 1 hertz is equal to 1 cycle per second. Hertz (Hz) = One hertz is equal to one cycle per second. Cycle = One complete wave of alternating current or voltage.

- **CONCEPT OF FREQUENCY IN CONTINUOUS TIME & DISCRETE TIME SIGNALS: -**
  - Let us find out the representation of frequency for discrete time signals and also the relationship between sampling frequency, continuous time frequency and discrete time frequency. Let us consider the following analog signal:

$$x(t) = A\cos(\omega t + \phi),\ -\infty < t < \infty \qquad (1.35)$$

- The above represented analog signal is a continuous cosine wave having amplitude, frequency and phase A, ω and Φ respectively where $\omega = 2\pi f$. The above expression can also be written as

$$x(t) = A\cos(2\pi ft + \phi),\ -\infty < t < \infty \qquad (1.36)$$

$$= \frac{A}{2}e^{-j(\omega t+\phi)} + \frac{A}{2}e^{-j(\omega t+\phi)} \qquad (1.37)$$

- Equation (1.37) shows that the cosine wave can be expressed in terms of two equal ...

- **<u>CONTINEOUS TIME SINUSOIDAL SIGNAL:-</u>**

Sinusoidal Sequence

- The discrete-time sinusoidal sequence is given by

$$X(n) = A \sin(\omega n + \emptyset)$$

Where *A* is the amplitude, is angular frequency, is phase angle in radians and *n* is an integer.

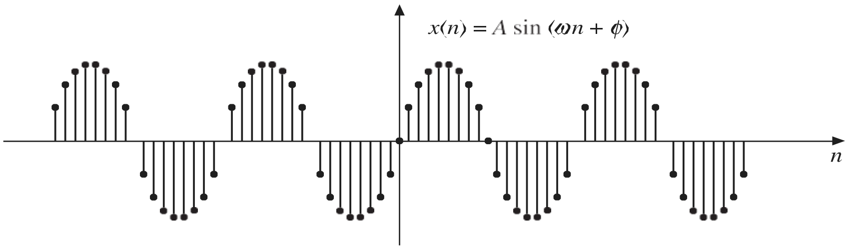
The period of the discrete-time sinusoidal sequence is:

$$N = \frac{2\pi}{\omega} m$$

Where *N* and *m* are integers.

- All continuous-time sinusoidal signals are periodic, but discrete-time sinusoidal sequences mayor may not be periodic depending on the value of.
- For a discrete-timesignal tobeperiodic, theangular frequency must bea rational multiple of 2.

The graphical representation of a discrete-time sinusoidal signal is shown in Figure 1.7.



<u>HARMONICALY RELATED COMPLEX EXPONENTIAL: -</u>

Considering periodic exponentials with common period N samples:

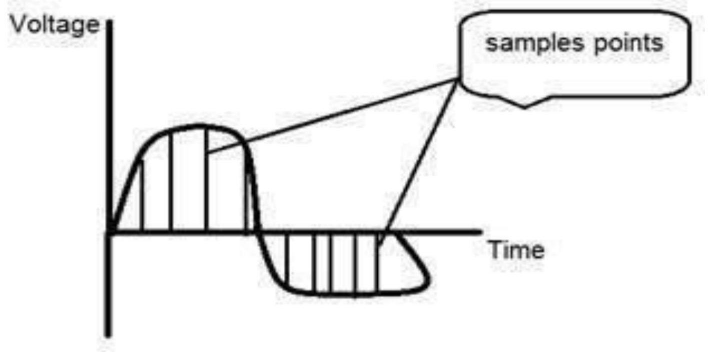
<u>Conversion of analog to digital signals: -</u>

Since there are lot of concepts related to this analog to digital conversion and vice-versa. We will only discuss those which are related to digital image processing. There are two main concepts that are involved in the conversion.

- Sampling
- Quantization

➢ **Sampling**

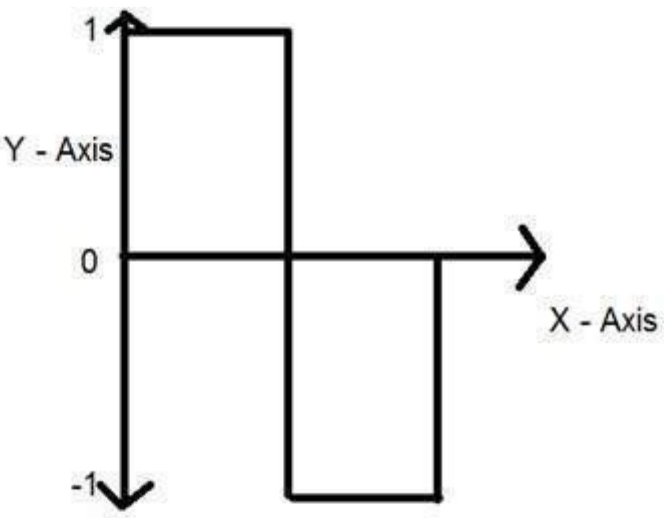
- Sampling as its name suggests can be defined as take samples. Take samples of a digital signal over x axis. Sampling is done on an independent variable. In case of this mathematical equation:



- Sampling is done on the x variable. We can also say that the conversion of x axis (infinite values) to digital is done under sampling.
- Sampling is further divide into up sampling and down sampling. If the range of values on x-axis are less then we will increase the sample of values. This is known as up sampling and its vice versa is known as down sampling.

➢ **Quantization**

- Quantization as its name suggest can be defined as dividing into quanta (partitions). Quantization is done on dependent variable. It is opposite to sampling.
- In case of this mathematical equation y = sin(x)
- Quantization is done on the Y variable. It is done on the y axis. The conversion of y axis infinite values to 1, 0, -1 (or any other level) is known as Quantization.
- These are the two basics steps that are involved while converting an analog signal to a digital signal.
- The quantization of a signal has been shown in the figure below.



<u>Need to convert an analog signal to digital signal: -</u>

- The first and obvious reason is that digital image processing deals with digital images, that are digital signals. So whenever the image is captured, it is converted into digital format and then it is processed.
- The second and important reason is, that in order to perform operations on an analog signal with a digital computer, you have to store that analog signal in the computer. And in order to store an

analog signal, infinite memory is required to store it. And since that's not possible, so that's why we convert that signal into digital format and then store it in digital computer and then performs operations on it.

Sampling of analog signal:-

- Sampling is defined as, "The process of measuring the instantaneous values of continuous-time signal in a discrete form." ... When a source generates an analog signal and if that has to be digitized, having 1s and 0s i.e., High or Low, the signal has to be discretized in time.

Sampling theorem: -

- The sampling theorem specifies the minimum-sampling rate at which a continuous-time signal needs to be uniformly sampled so that the original signal can be completely recovered or reconstructed by these samples alone. This is usually referred to as Shannon's sampling theorem in the literature.

➢ **Sampling theorem:**

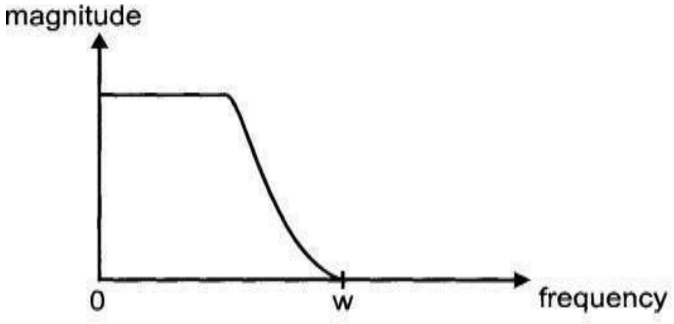
- If a continuous time signal contains no frequency components higher than W hz, then it can be completely determined by uniform samples taken at a rate f_s samples per second where

$$f_s \geq 2W$$

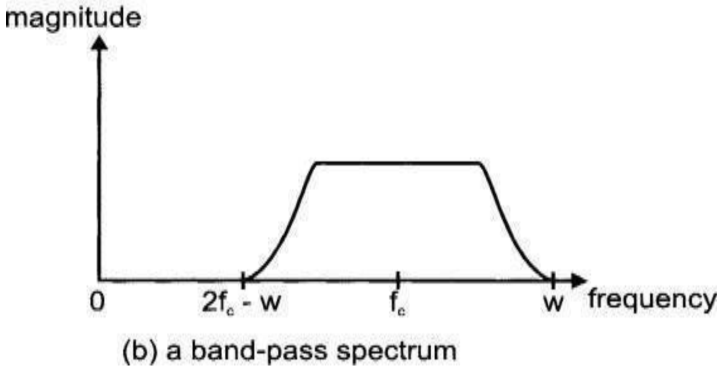
or, in term of the sampling period

$$T \leq 1/2W$$

- A signal with no frequency component above a certain maximum frequency is known as a bandlimited signal. Figure 2.4 shows two typical bandlimited signal spectra: one low-pass and one band-pass.



(a) a low-pass spectrum



(b) a band-pass spectrum

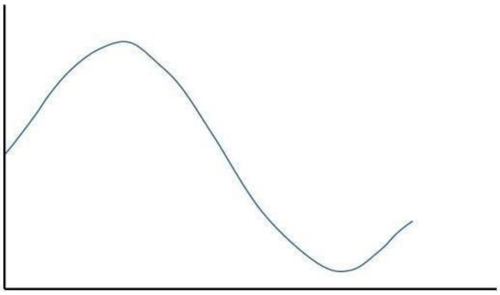
Quantization of continuous amplitude signals: -

➢ **Quantization: -**

The digitization of analog signals involves the rounding off of the values which are approximately equal to the analog values. The method of sampling chooses a few points on the analog signal and then these points are joined to round off the value to a near stabilized value. Such a process is called as **Quantization**.

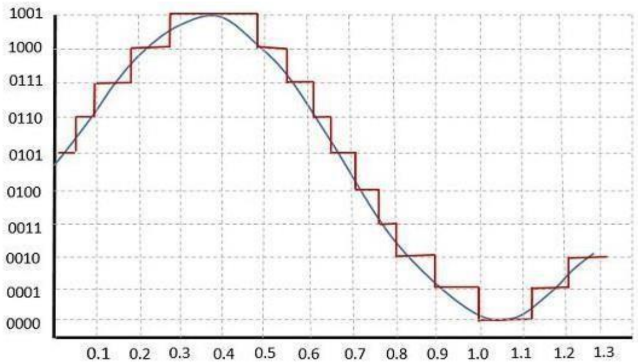
➢ Quantizing an Analog Signal

- The analog-to-digital converters perform this type of function to create a series of digital values out of the given analog signal. The following figure represents an analog signal. This signal to get converted into digital, has to undergo sampling and quantizing.



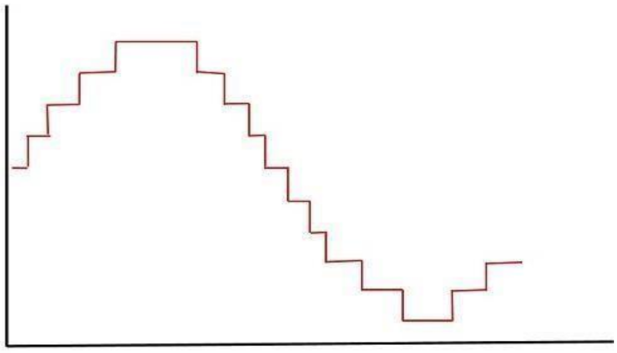
- The quantizing of an analog signal is done by discretizing the signal with a number of quantization levels. Quantization is representing the sampled values of the amplitude by a finite set of levels, which means converting a continuous-amplitude sample into a discrete-time signal.

- The following figure shows how an analog signal gets quantized. The blue line represents analog signal while the brown one represents the quantized signal.



- Both sampling and quantization result in the loss of information. The quality of a Quantizer output depends upon the number of quantization levels used. The discrete amplitudes of the quantized output are called as representation levels or reconstruction levels. The spacing between the two adjacent representation levels is called a quantum or step-size.

- The following figure shows the resultant quantized signal which is the digital form for the given analog signal.



Coding of quantised sample: -

Sampling:

• The process of converting continuous time in to discrete values is called sampling.

• The time axis is divided in to fixed intervals.

• The reading of each value of analog signal is taken for each time interval.

Quantization:

• The process of converting continuous sample values into discrete values is called quantization.

• In this process we divide the signal range into a fixed number of intervals.

• Each intervals is of same size and is assigned a number. These intervals are numbered between 0 to 7.

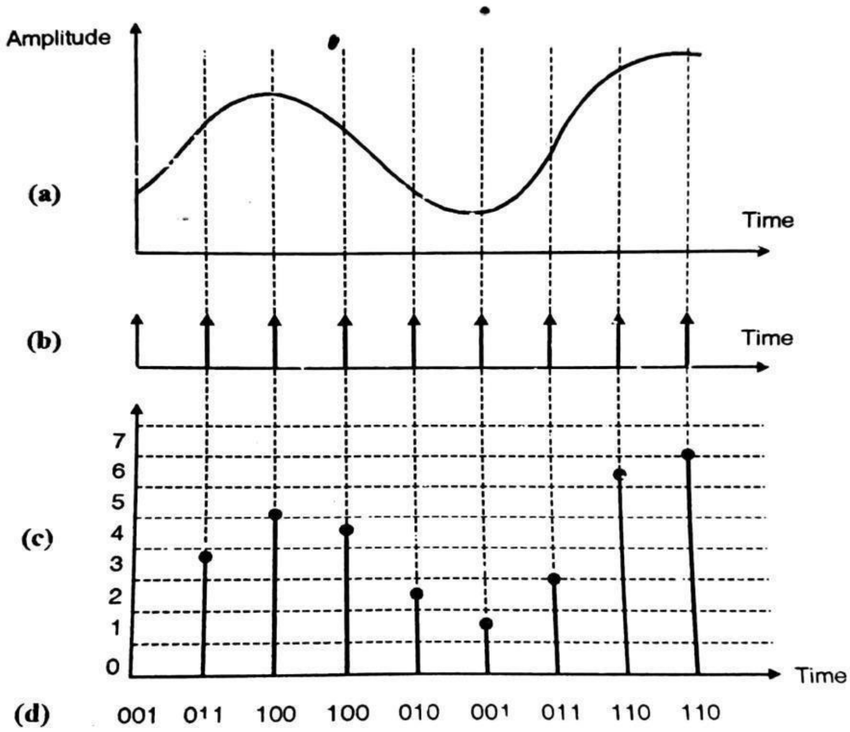
• Each sample falls in one of the intervals and is assigned that interval's number.

Coding:

• The process of representing quantized values digitally is called coding

• In our example, eight quantizing levels are used. These levels can be coded using 3 bits if the binary system is used, so each sample is represented by 3 bits.

• The analog signal is represented digitally by the following series of binary numbers: 001, 011, 100, 100, 010, 001, 011, 110, and 110.



<u>Digital to analog conversion: -</u>

- A **Digital to Analog Converter (DAC)** converts a digital input signal into an analog output signal. The digital signal is represented with a binary code, which is a combination of bits 0 and 1. This chapter deals with Digital to Analog Converters in detail.
- The **block diagram** of DAC is shown in the following figure −

- A Digital to Analog Converter (DAC) consists of a number of binary inputs and a single output. In general, the **number of binary inputs** of a DAC will be a power of two.

➢ Digital-to-Analog Conversion: -

- A digital-to-analog converter (DAC), as the name implies, is a data converter which generates an analog output from a digital input. A DAC converts a limited number of discrete digital codes to a corresponding number of discrete analog output values.
- Because of the finite precision of any digitized value, the finite word length is a source of error in the analog output. This is referred to as quantization error. Any digital value is really only an approximation of the real world analog signal. The more digital bits represented by the DAC, the more accurate the analog output signal.
- Basically, one LSB of the converter will represent the height of one step in the successive analog output. You can think of a DAC as a digital potentiometer that produces an analog output that is a fraction of the full scale analog voltage determined by the value of the digital code applied to the converter.
- Similar to ADCs, the performance of a DAC is determined by the number of samples it can process and the number of bits used in the conversion process.

- For example, a three bit converter as shown in fast fig. will have less performance than the four bit converter shown in 2nd Fig.

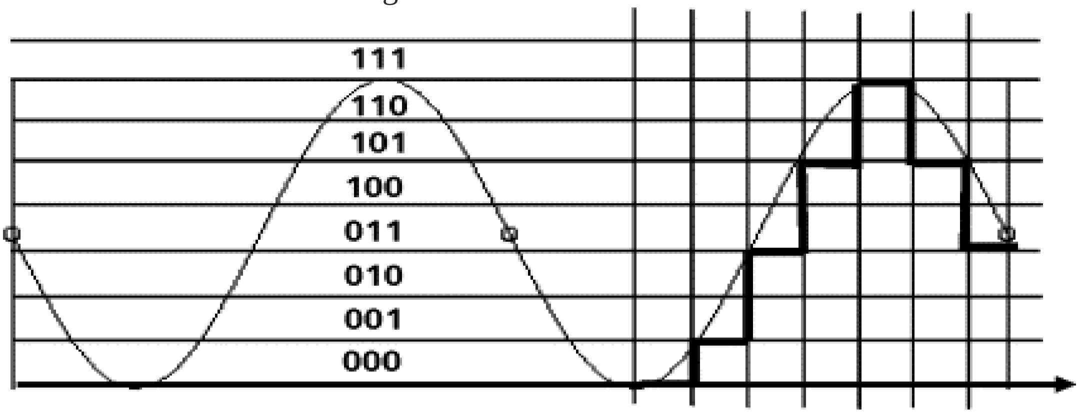


Fig.1

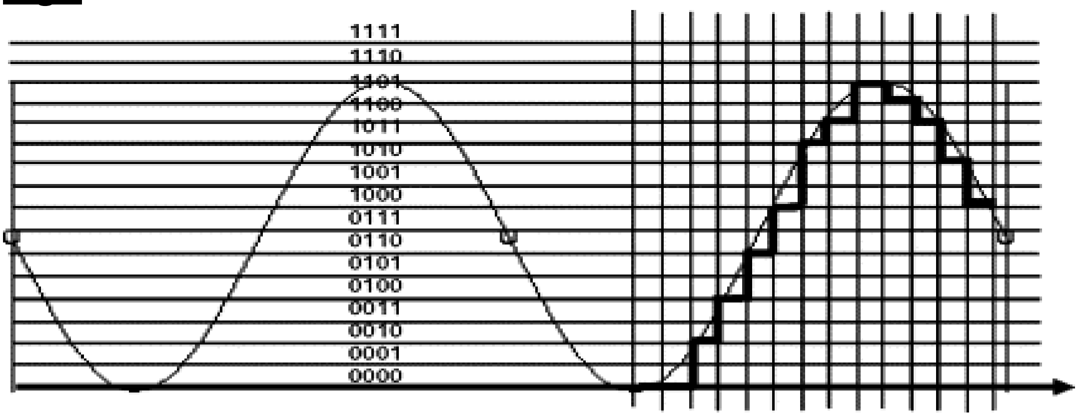
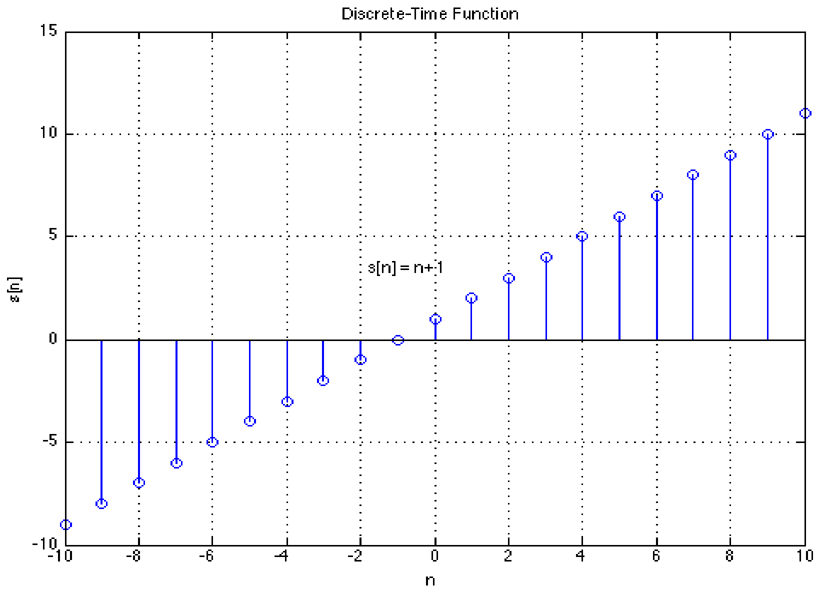


Fig.2

Digital system signal vs Discrete time signal system: -

➢ **Discrete-time (DT) Signal**

- A *discrete-time* signal is a bounded, continuous-valued sequence s[n]. Alternately, it may be viewed as a continuous-valued function of a discrete index n. We often refer to the index n as *time*, since discrete-time signals are frequently obtained by taking snapshots of a continuous-time signal as shown below. More correctly, though, n is merely an index that represents sequentially of the numbers in s[n].



If they DT signals are snapshots of real-world signals <u>realness</u> and <u>finiteness</u> apply.

Below are several characterizations of size for a DT signal

1. Energy

   $$E=\sum_n s^2[n]$$

2. Power

   $$P=\lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}s^2[n].$$

3. Amplitude = max|s[n]|

Smoothness is not applicable.

- **<u>Digital Signal</u>**

- We will work with digital signals but develop theory mainly around <u>discrete-</u>time signals.
- Digital computers deal with *digital* signals, rather than discrete-time signals. A *digital* signal is a sequence s[n], where index the values s[n] are not only finite, but can only take a finite set of values. For instance, in a digital signal, where the individual numbers s[n] are stored using 16 bits integers, s[n] can take one of only 2^{16} values.
- In the digital valued series s[n] the values *s* can only take a fixed set of values.
- Digital signals are discrete-time signals obtained after "digitalization." Digital signals too are usually obtained by taking measurements from real-world phenomena. However, unlike the accepted norm for analog signals, digital signals may take complex values.
-

Presented above are some criteria for real-world signals. ***Theoretical signals are not constrained***

- <u>real</u>- this is often violated; we work with complex numbers
- <u>finite/bounded</u>
- <u>energy</u> - violated ALL the time

- Signals that have infinite temporal extent, *i.e.*, which extend from $-\infty$ to ∞, can have infinite energy.
- power - almost never: nearly all the signals we will encounter have bounded power
- smoothness-- this is often violated by many of the continuous time signals we consider.

Short questions with answers: -

1. Define DSP?

Ans. Digital Signal Processing is the mathematical manipulation of an information signal, such as audio, temperature, voice, and video and modify or improve them in some manner.

2. Define signal?

Ans. In electrical engineering, the fundamental quantity of representing some information is called a signal.

3. Define system?

Ans. A system is a defined by the type of input and output it deals with. Since we are dealing with signals, so in our case, our system would be a mathematical model, a piece of code/software, or a physical device, or a black box whose input is a signal and it performs some processing on that signal, and the output is a signal.

4. Define discrite time signal?

Ans. The word digital stands for discrete values and hence it means that they use specific values to represent any information. In digital signal, only two values are used to represent something i-e: 1 and 0 (binary values). Digital signals are denoted by square waves. They are discontinuous signals.

5. Define Digital to Analog Converter (DAC)?

Ans. A **Digital to Analog Converter (DAC)** converts a digital input signal into an analog output signal. The digital signal is represented with a binary code, which is a combination of bits 0 and 1. This chapter deals with Digital to Analog Converters in detail.

Long questions: -

1. Explain basic elements of digital signal processing system?

2. Compare DSP over ASP?

3. Explain analog to digital convertion?

4. Explain digital to analog convertion?

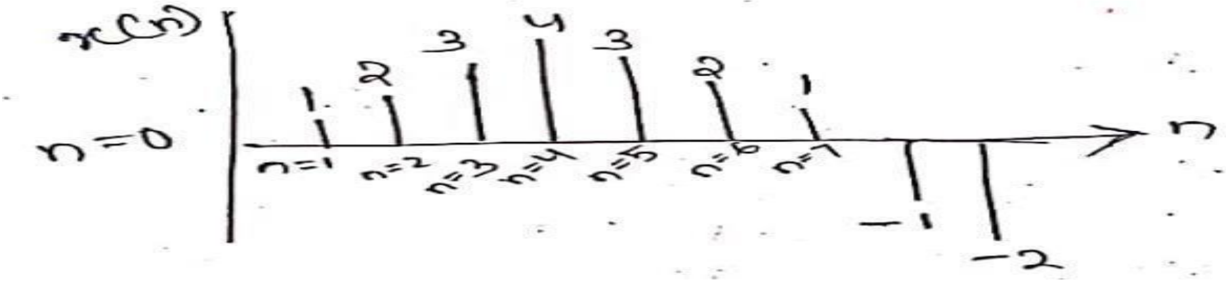
CHAPTER-02 : DISCRETE TIME SIGNALS & SYSTEMS.

Concept of Discrete time signals :

A discrete time signal S(n) is a function of an independent variable ‘n’.
Discrete Time signal can be represented in the following 4 ways,

1. Graphical representation
2. Functional representation
3. Tabular representation
4. Sequential representation

1. Graphical representation :



2. Functional representation :

$$x(n) \begin{cases} 1, & n = 1, 7 \\ 2, & n = 2, 6 \\ 3, & n = 3, 5 \\ 4, & n = 4 \\ 7, & n = 8 \\ 2, & n = 9 \end{cases}$$

3. Tabular representation :

n	1	2	3	4	5	6	7	8	9
x(n)	1	2	3	4	3.	2	1	-1	-2

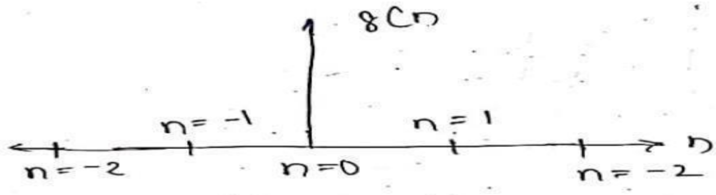
4. Sequential representation :

$$x(n) = \{\underset{\uparrow}{0}, 1, 2, 3, 4, 3, 2, 1, -1, -2$$

origin

Elementary Discrete time signals.

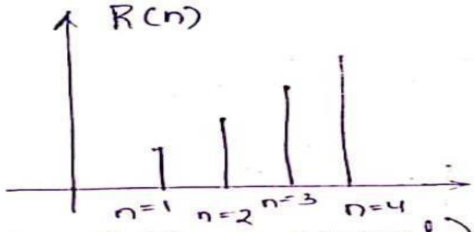
1. Unit Impulse Signal :



(Unit impulse signal)

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0; & n \neq 0 \end{cases}$$

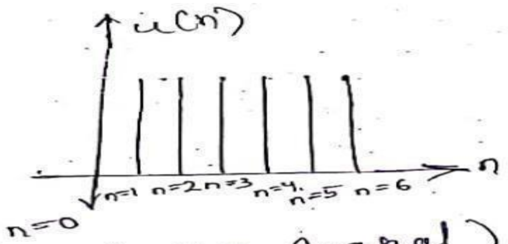
2. Unit Ramp Signal:



(unit Ramp signal)

$$R(n) = \begin{cases} n, & n \geq 0 \\ 0 & n < 0 \end{cases}$$

3. Unit Step Signal :

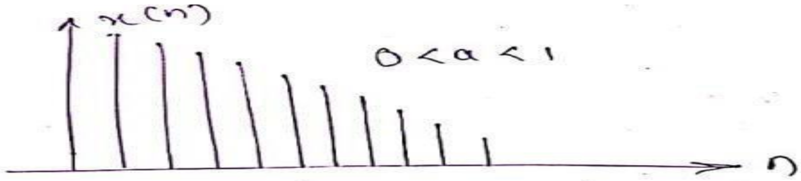


(Unit step signal)

$$u(n) = \begin{matrix} 1, & n \geq 0 \\ 0, & n < 0 \end{matrix}$$

4. Unit Exponential Signal :

- An Exponential signal is represented by x(n) = a^n for all values of ‘n’.
- If a is real, then x(n) is real.
- If a is complex, then x(n) is complex value function.



when ‘a’ is complex i.e.; $a = re^{J\theta}$

then $x(n) = r^n e^{J\theta n}$

$= r^n(\cos \theta n + J \sin \theta n)$

Classification Discrete time signal.

1. Energy Signal And Power Signal :

- The energy of a discrete time signal x(n) is defined as $\boxed{E = \sum_{n=-\infty}^{\infty} |x(n)|^2}$.
- A signal x(n) is called as energy signal if the energy is finite (0<E<infinity) and power equal to zero.
- The average power of a discrete time signal is defined as

$$\boxed{P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2}.$$

$$\text{where } \begin{cases} 0 < P < \infty \\ E = \infty \end{cases}$$

- The signal x(n) is called as power signal if and only if the power is finite and energy is infinite.

Infinite summetion formula :-

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \quad |a| < 1$$

$$\sum_{n=0}^{\infty} n a^n = \frac{a}{(1-a)^2}, \quad |a| < 1$$

$$\sum_{n=0}^{\infty} n^2 2^2 = \frac{a^2 + a}{(1-a)^3}$$

Finite summetion formula :-

① $$\sum_{n=0}^{N} a^n = \frac{1 - a^{n+1}}{1-a}, \quad a \neq 1$$

② $$\sum_{n=0}^{N} 1 = N+1$$

③ $$\sum_{n=N_1}^{N_2} 1 = N_2 - N_1 + 1$$

④ $$\sum_{n=0} n = \frac{N(N+1)}{2}$$

⑤ $$\sum_{n=0}^{N} n^2 = \frac{N(N+1)(2N+1)}{6}$$

Q. check whether the unit step signal is Energy or power signal.

Sol^n:- $x(n) = u(n)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$\Rightarrow \sum_{n=0}^{\infty} |u(n)|^2$$

$$= \sum_{n=0}^{\infty} (1)^2 = \sum_{n=0}^{\infty} 1 = \infty$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$\Rightarrow \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |u(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-\infty}^{N} 1$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \cdot N+1$$

$$\Rightarrow \lim_{N\to\infty} \frac{N+1}{2N+1}$$

$$\Rightarrow \lim_{N\to\infty} \frac{1+\frac{1}{N}}{2+\frac{1}{N}}$$

Q. Find the Energy and power of a signal.

$x(n) = \left(\frac{1}{3}\right)^n u(n)$

Sol^n:- $\sum_{n\to-\infty}^{\infty} |x(n)|^2$

$$\Rightarrow \sum_{n\to-\infty}^{\infty} \left(\frac{1}{3}\right)^n u(n)$$

$$= \sum_{n\to 0}^{\infty} \left(\frac{1}{3}\right)^n$$

$$\Rightarrow \frac{1}{1-a}$$

$$= \frac{1}{1-\frac{1}{3}}$$

$$\Rightarrow P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n\to-N}^{N} |x(n)|^2$$

2. Periodic Signal And Aperiodic Signal :

- A signal x(n) is periodic with period N when x(n+N)=x(n) for all vallu of n--------(1).
- The smallest value of N for which equation(1) is valid is known as fundamental period.
- If equation (1) is not satisfied for ay value of N, then the signal is Aperiodic signal.
- We Know that,

$$\omega_0 N = 2\pi K$$

$$\omega_0 = \frac{2\pi K}{N}$$

$$2\pi f_0 = \frac{2\pi K}{N}$$

$$\boxed{f_0 = \frac{K}{N}}$$

Q. Determine whether the following signals are periodic or not and also find the fundamental period.

(1) $x(n) = \underbrace{\cos 0.01\pi n}_{\omega_0}$

$$= \cos \omega_0 n$$

$$N = \frac{2\pi K}{\omega_0}$$

$$= \frac{2\pi K}{0.01\pi}$$

$$= \frac{2K}{0.01} \quad \boxed{N = 200K}$$

The signal is periodic because it is multiple of 2π

If the minimum value of K is taken then fundamental period is 200.

Example :-

(1). $x(n) = \sin\left(\frac{\pi}{4}\right)n = \sin \omega n$

$K, N = ?$

$\omega = \pi/4$

$2\pi f = \pi/4$

$f = \frac{1}{8} = \frac{K}{N}$

$K = 1$

$N = 8$

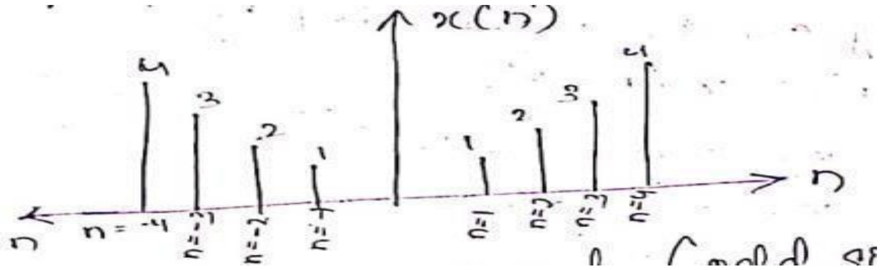
2. $x(n) = \cos 2\pi/3\, n + \sin \pi/2\, n$

$\cos 2\pi/3\, n$	$\sin \pi/2\, n$
$\omega = 2\pi/3$	$\omega = \pi/2$
$2\pi f = 2\pi/3$	$2\pi f = \pi/2$
$f = \frac{1}{3} = \frac{K}{N_1}$	$f = \frac{1}{4} = \frac{K}{N_2}$
$K = 1, N_1 = 3$	$K = 1$
	$N_2 = 4$

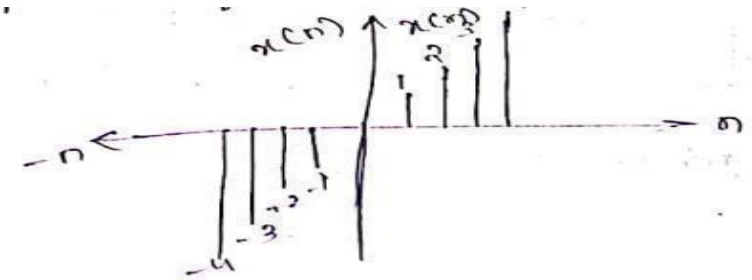
$N = \mathrm{Lcm}(N_1, N_2) \Rightarrow N = 12$

3. Symmetric Signal And Asymmetric Signal :

- A discrete time signal is said to be symmetric (Even) if it satisfies x(-n)=x(n) for all values of n.



- A discrete time signal is said to be Asymmetric (Odd) if it satisfies x(-n)=-x(n) for all values of n.



Simple manipulation of discrete time signal.

1. Shifting :

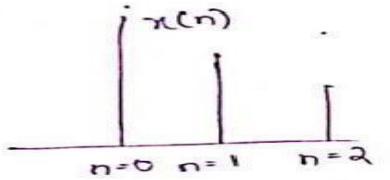
- The Shifting operation shifts the values of the sequence by an integer variable.
- Shifting can be either delay or advance.
- The Shifting can be represented by

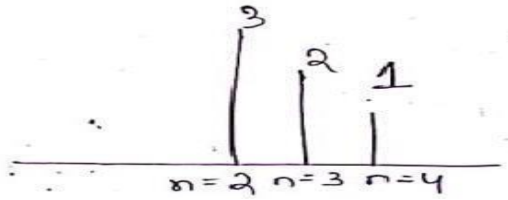
$$y(n) = x(n-k).$$

$K = +ve$ – delay

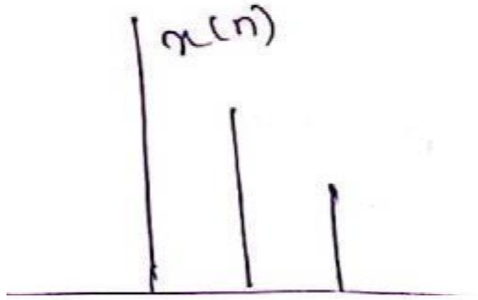
$K = -ve$ – advance

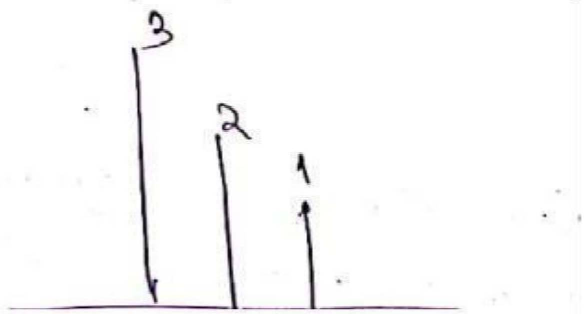
→ $y(n) = x(n-2$





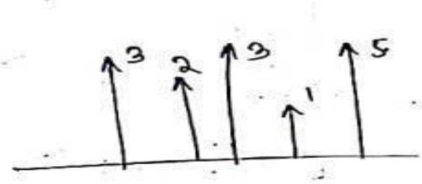
$y(n) = x(n+4)$



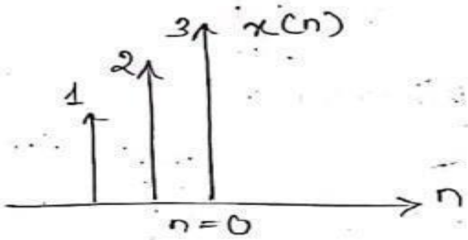


2. Time Reversal(Folding) :

Time Reversal of the sequence is done by holding the sequence about n=0.



OR

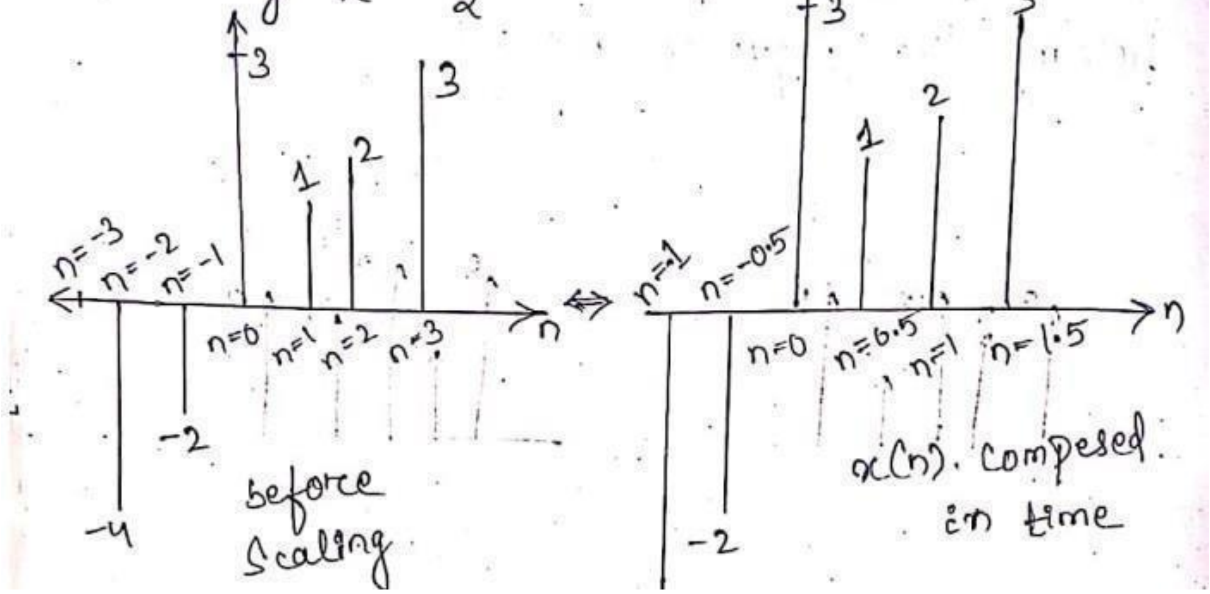


3. Time Scaling :

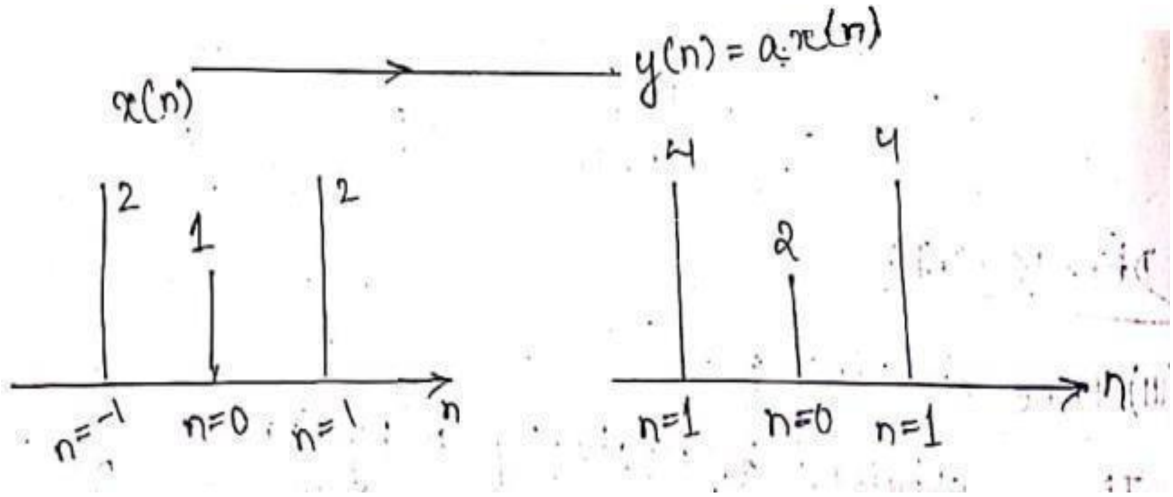
It is obtained by replacing n by λn.

→ Here $x(n)$ changed to $x(\lambda n)$

(b) Taking $\lambda = \frac{1}{2}$



4. Scalar Multiplication :



Discrete time system.

A **discrete**-**time system** is anything that takes a **discrete**-**time** signal as input and generates a **discrete**-**time** signal as output.

$$y(n) = T[x(n)]$$

→ Symbol T denotes the transformations.

Input-output of system :

→ There is a mathematical relations between the input and output of the discrete time system

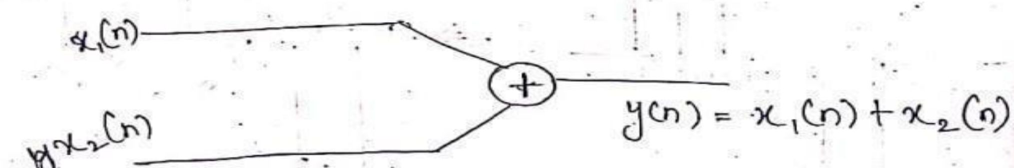
→ In order to give the input the input terminal is used and in order to get the output the output terminal is used.

→ The input and output both must be discrete in nature.

→ We can Express $x(n) \xrightarrow{T} y(n)$, which means $y(n)$ is the response of the system and $x(n)$ denotes the Excitations.

Block diagram of discrete- time systems:

→ Adder



Example :-

(a) $x_1(n) = \{\underset{\uparrow}{1}, 2, 1, 2\}$

$x_2(n) = \{\underset{\uparrow}{2}, 1, 3, 4\}$

$y(n) = \{\underset{\uparrow}{3}, 3, 4, 6\}$

(b) $x_1(n) = \{\underset{\uparrow}{1}, 2, 1, 2\}$

$x_2(n) = \{2, \underset{\uparrow}{1}, 3, 4\}$

$y(n) = \{2, \underset{\uparrow}{2.5}, 5, 2\}$

⇒ Constant Multiplier :-

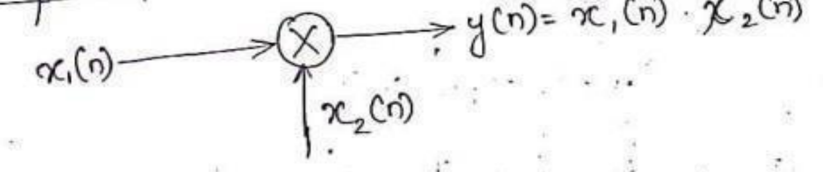
$$x(n) \xrightarrow{\quad a \quad} a\,x(n)$$

$x(n) = \{2, 4, 6\}$

$a = 2$

$a\,x(n) = \{4, 8, 12\}$

⇒ Multiplier :-

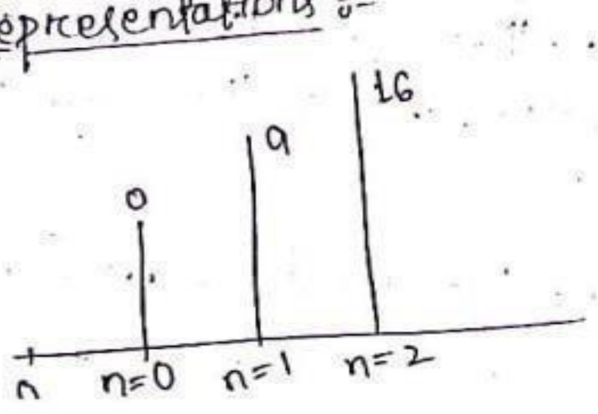


$x_1(n) = \{\underset{\uparrow}{2}, 3, 4, 5\}$

$x_2(n) = \{1, \underset{\uparrow}{2}, 3, 4\}$

$y(n) = \{0, \underset{\uparrow}{4}, 9, 16, 0\}$

Graphical Representations :-



Classify discrete time system. :

The discrete time systems can be classified as follows:

- Static/Dynamic
- Causal/Non-Causal
- Time invariant/Time variant
- Linear/Non-Linear
- Stable/Unstable
- FIR /IIR

Static/Dynamic :

The system is said to be static if its output depends only on the present input. On the other hand, if the output of the system depends on the past input, the system is said to be dynamic.

Example :- (i) $y(n) = a\,x(n) \rightarrow$ static system

(ii) $y(n) = 2x^3(n) \rightarrow$ Static system

(iii) $y(n) = 5x(n-1)+2x(n-2) \rightarrow$ Dynamic system.

Time invariant/Time variant :

Let $x(t)$ and $y(t)$ be the input and output signals, respectively, of a system shown in Figure 1. Then the transformation of $x(t)$ into $y(t)$ is represented by the mathematical notation

$$y(t) = \mathbf{T}x(t)$$

where $\mathbf{T}$ is the operator which defined rule by which $x(t)$ is transformed into $y(t)$.

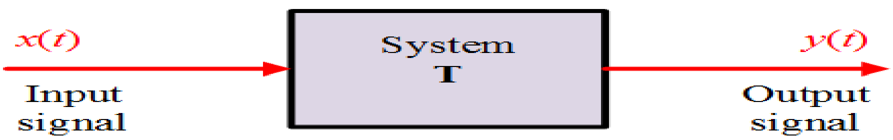


Figure 1: System with a single input and output signal.

A system is called time-invariant if a time shift in the input signal $x(t-t_0)$ causes the same time shift in the output signal $y(t-t_0)$, it is shown in Figure 2.

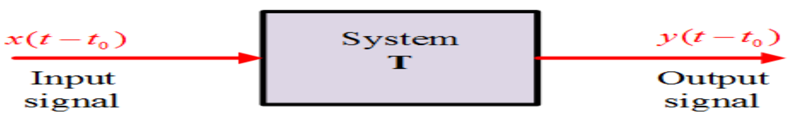


Figure 2: Time-Invariant System.

Q. Test wheathere the system is time varient or invarient

$y(n) = x(n) + x(n-1)$

Ans:- $y(n,k) = x(n-k) + x(n-k-1)$

$y(n,k) = x(n-k) + x(n-k-1)$

$\Rightarrow y(n,k) = y(n-k)$

So, the system is time invarient

Q. $y(n) = x(-n)$

$y(n,k) = x(-n-k)$

$y(n-k) = x[-n(n-k)]$

$= x(-n+k)$

$\Rightarrow y(n,k) \neq y(n-k)$

So, the system is time varient

Q. $y(n) = n\, x(n)$

$y(n,k) = n\, x(n-k)$

$y(n-k) = (n-k)\, x(n-k)$

$\Rightarrow y(n,k) \neq y(n-k)$

So, the system is time varient

Causal/Non-Causal :

- A **causal system** is one whose output depends only on the present and the past inputs.
- A **noncausal system's** output depends on the future inputs. In a sense, a **noncausal system** is just the opposite of one that has memory.
- It cannot because real **systems** cannot react to the future.

Q. $y(n) = x(n) + \frac{1}{x(n-1)}$

Ans:- Taking $n=0$

$\Rightarrow y(0) = x(0) + \frac{1}{x(0-1)} = x(0) + \frac{1}{x(-1)}$

Taking $n=1$

$\Rightarrow y(1) = x(1) + \frac{1}{x(1-1)} = x(1) + \frac{1}{x(0)}$

Taking $n=-1$

$y(-1) = x(-1) + \frac{1}{x(-1-1)} = x(-1) + \frac{1}{x(-2)}$

So, the system is causal as the output depends on past and present value.

Q. $y(n) = x(n^2)$

Ans:- Taking $n=0$

$\Rightarrow y(0) = x(0^2) = x(0)$

Taking $n=1$

$y(1) = x(1^2) = x(1)$.

Taking $n=2$

$y(2) = x(2^2) = x(4)$

Taking $n=-1$

$\Rightarrow y(-1) = x(-1^2) = x(1)$

Here, the output depends on present and future value to the system is non-causal.

Linear/Non-Linear :

- A system that satisfies the principle of superposition is said to be Linear System.
- Super position principle said that the response of the system to weighted sum of signal is equal to the corresponding weighted sum of output signal to each of individual output.

Ex:- $a_1 x_1(n) \longrightarrow a_1 y_1(n)$

$T[a_1 x_1(n) + a_2 x_2(n)] \longrightarrow T[a_1 x_1(n)] + T[a_2 x_2(n)]$

OR

Linear system

$$\Rightarrow \boxed{T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]}$$

The system which does not follow the principle of superposition is known as ~~superpo~~ Non-linear system.

Q. Test whether the following system is linear or non-linear

$y(n) = x^2(n)$

$y_1(n) = x_1^2(n)$

$y_2(n) = x_2^2(n)$

$a_1 y_1(n) = a_1 x_1^2(n)$

$a_2 y_2(n) = a_2 x_2^2(n)$

$$y(n) = a_1 x_1^2(n) + a_2 x_2^2(n) \quad \text{------} \quad ①$$

$$y(n) = [a_1 x_1(n) + a_2 x_2(n)]^2 \quad \text{------} \quad ②$$

Equation ① $\neq$ ② → Non-linear

Q. $y(n) = n\, x(n)$

$y_1(n) = n\, x_1(n)$

$y_2(n) = n\, x_2(n)$

$y_1(n) = a_1 n x_1(n) + a_2 n x_2(n) \quad \text{------} \quad ①$

$T[a_1 x_1(n) + a_2 x_2(n)]$

$\Rightarrow$

$= a_1 n x_1(n) + a_2 n x_2(n) \quad \text{----} \quad ②$

Equation ① = ② → linear

Stable/Unstable :

- LDI system is said to be stable if it produces bounded output for AB for bounded.
- The necessary condition for stability is , $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$

Q Test of stability of system whose impulse system $h(n) = \left(\frac{1}{2}\right)^n u(n)$

$$= \sum_{n=-\infty}^{\infty} h(n)$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u(n)$$

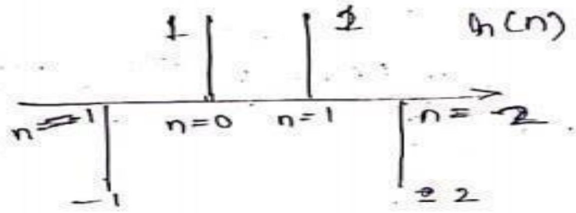
$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n \cdot 1$$

$$= \frac{1}{1-\frac{1}{2}} - \frac{1}{\frac{2-1}{2}} = 2$$

As $\sum_{n=-\infty}^{\infty} h(n) < \infty = 2$, so, the system is stable

FIR(FINITE IMPULSE RESPONSE) /IIR(INFINITE IMPULSE RESPONSE) :

→ If the impulse response is of finite duration the system is said to be finite impulse response system.



$$h(n) = \begin{cases} 1, & n=0,1 \\ -2, & n=2 \\ -1, & n=-1 \end{cases}$$

An infinite impulse response system have infinite duration sequence.

Inter connection of discrete -time system. :

consider to LDI system with impulse response $h_1(n)$ and $h_2(n)$ connected in cascade

$$x_1(n) \longrightarrow \boxed{h_1(n) * h_2(n)} \longrightarrow y_2(n)$$

$$y_2(n) = x_1(n) * h(n)$$

$$h(n) = \sum_{n=0}^{\infty} h_1(K)\, h_2(n-K)$$

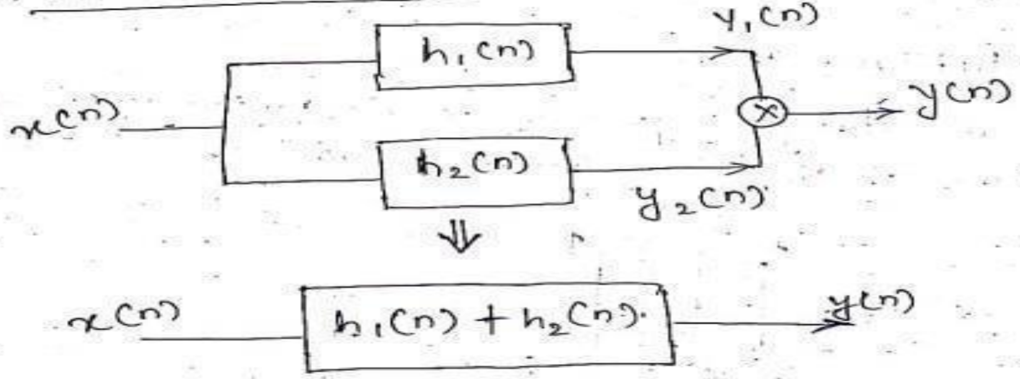
$$= h_1(n) * h_2(n)$$

↓

convolutions

→ Here, the impulse response of 2 LTI system connected in cascade is the convolution of individual impulse response.

II. PARALLEL INTERCONNECTION :-



$$y_1(n) = x(n) * h_1(n)$$

$$y_2(n) = x(n) * h_2(n)$$

$$y(n) = y_1(n) + y_2(n)$$

$$y(n) = \sum_{n=-\infty}^{\infty} x(K)\, h_1(n-K) + \sum_{n=-\infty}^{\infty} x(K)\, h_2(n-K)$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} x(K)\, h(n-K)$$

$$y(n) = x(n) * h(n)$$

→ So, when system are connected in parallel the overall Impulse response is the summation of individual Impulse response.

Discrete time time-invariant system :

A system is called time-invariant if a time shift in the input signal $x(t-t_0)$ causes the same time shift in the output signal $y(t-t_0)$, it is shown in Figure 2.

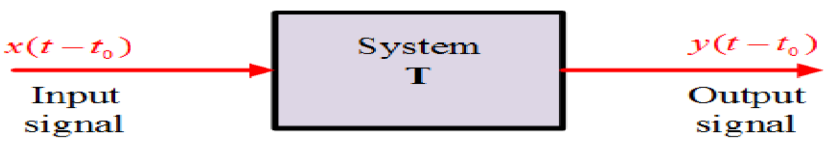


Figure 2: Time-Invariant System.

Different techniques for the Analysis of linearsystem :

Time-varying impulse response [edit]

The **time-varying impulse response** $h(t_2, t_1)$ of a linear system is defined as the response of the system at time $t = t_2$ to a single impulse applied at time $t = t_1$. In other words, if the input $x(t)$ to a linear system is

$$x(t) = \delta(t - t_1)$$

where $\delta(t)$ represents the Dirac delta function, and the corresponding response $y(t)$ of the system is

$$y(t)|_{t=t_2} = h(t_2, t_1)$$

then the function $h(t_2, t_1)$ is the time-varying impulse response of the system. Since the system cannot respond before the input is applied the following **causality condition** must be satisfied:

$$h(t_2, t_1) = 0, t_2 < t_1$$

The convolution integral [edit]

The output of any general continuous-time linear system is related to the input by an integral which may be written over a doubly infinite range because of the causality condition:

$$y(t) = \int_{-\infty}^{t} h(t, t')x(t')dt' = \int_{-\infty}^{\infty} h(t, t')x(t')dt'$$

If the properties of the system do not depend on the time at which it is operated then it is said to be **time-invariant** and h is a function only of the time difference $\tau = t - t'$ which is zero for $\tau < 0$ (namely $t < t'$). By redefinition of h it is then possible to write the input-output relation equivalently in any of the ways,

$$y(t) = \int_{-\infty}^{t} h(t - t')x(t')dt' = \int_{-\infty}^{\infty} h(t - t')x(t')dt' = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau = \int_{0}^{\infty} h(\tau)x(t - \tau)d\tau$$

Linear time-invariant systems are most commonly characterized by the Laplace transform of the impulse response function called the *transfer function* which is:

$$H(s) = \int_{0}^{\infty} h(t)e^{-st}\, dt.$$

In applications this is usually a rational algebraic function of s. Because $h(t)$ is zero for negative t, the integral may equally be written over the doubly infinite range and putting $s = i\omega$ follows the formula for the *frequency response function*:

$$H(i\omega) = \int_{-\infty}^{\infty} h(t)e^{-i\omega t} dt$$

Discrete time systems [edit]

The output of any discrete time linear system is related to the input by the time-varying convolution sum:

$$y[n] = \sum_{m=-\infty}^{n} h[n, m]x[m] = \sum_{m=-\infty}^{\infty} h[n, m]x[m]$$

or equivalently for a time-invariant system on redefining h(),

$$y[n] = \sum_{k=0}^{\infty} h[k]x[n - k] = \sum_{k=-\infty}^{\infty} h[k]x[n - k]$$

where

$$k = n - m$$

represents the lag time between the stimulus at time m and the response at time n.

Resolution of a discrete time signal in to impulse. :

$$x(n) = \sum_k c_k x_k(n)$$

Suppose:

$$x_k(n) = \delta(n-k)$$

then:

$$x(n)\delta(n-k) = x(k)\delta(n-k)$$

is zero everywhere except at $n = k$.

This means we can write $x(n)$ as:

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$$

Example:

$$x(n) = \{2, \mathbf{4}, 0, 3\}$$

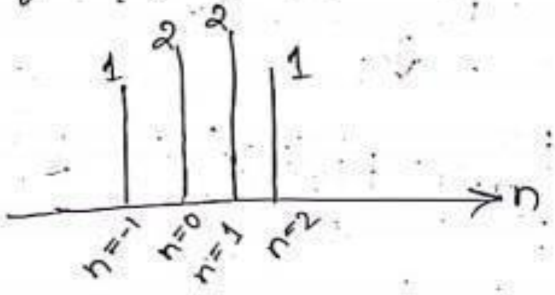
$$x(n) = 2\delta(n+1) + 4\delta(n) + 3\delta(n-2)$$

Response of LTI system to arbitrary inputs using convolution sum :

Convolution Sum :-

Any arbitary sequence $x(n)$ can be represente

in terms of Impulse sequence $\delta(n)$.



$$x(n) = 2 \;,\; n = 0, 1 \\ \quad = 1 \;,\; n = -1, 2$$

$$2\,\delta(n) + 1\,\delta(n+1) + 2\,\delta(n-1) + 1\,\delta(n-2)$$

→ A discrete time system performs an operations on the input signal based on some predefined criteria to produce a modified output.

→ If the input of a system is unit impulse then the output of the system is denoted by h(n).

$$h(n) = T[\delta(n)] \quad \text{----- (I)}$$

→ we can write the arbitary sequence as a weighted sum of discrete impulse.

$$y(n) = \sum_{n=-\infty}^{\infty} x(K)\,\delta(n-K) \quad \text{----- (II)}$$

→ For a Linear system, we can write

$$\boxed{y(n) = \sum_{n=\infty}^{\infty} x(n) + T[\delta(n-K)]}$$

→ For time invarient system.

$$\boxed{h(n, K) = h(n-K)}$$

→ The convolution sum can be represented as $\boxed{y(n) = x(n) * h(n)}$

STEPS INCLUDED IN CONVOLUTION :-

The process of convolution between the sequence x(K) and h(K) involves the following steps :-

I. **Folding :-**
   h(K) is folded and we obtain h(−K).

II. **Shifting :-**
   Shift h(−K) by n.

III. **Multiplication :-**
   Multiply x(K) by h(n−K).

IV. **Summation :-**
   Sum all the value of product sequence

Q. Determine the convolution sum of two sequence. (matrix method).

$x(n) = \{ \underset{\uparrow\, n=0}{3}, 2, 1, 2 \}$

$h(n) = \{ 1, \underset{\uparrow\, n=0}{2}, 1, 2 \}$

Starting point of $x(n) = 0$

$h(n) = 0$

$y(n) = 0 + (-1) = -1$

N_1 = Number of Elements of $x(n) = 4$

N_2 = Number of Elements of $h(n) = 4$

N = Number of Elements of $y(n)$

$N = N_1 + N_2 - 1$

$\Rightarrow N = 4 + 4 - 1 = 8 - 1 = 7$

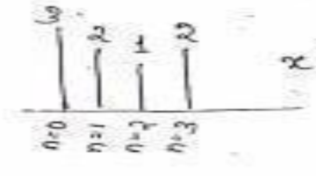
h(n) \ x(n)	3	2	1	2
1	3	2	1	2
2	6	4	2	4
1	3	2	1	2
2	6	4	2	4

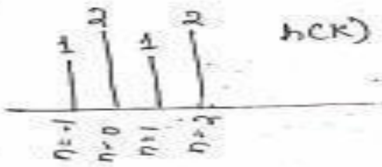
$\{3, \underset{\uparrow}{8}, 8, 12, 9, 4, 4\}$

$\{3, \underset{\uparrow}{8}, 8, 12, 9, 4, 4\}$

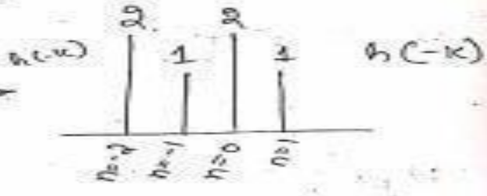
Graphical method :-

(constant)





folding



(ii) Shifting $n = 0$

$h(n-K)$

$h(0-K)$

$h(-K)$

$y(n) = \sum x(K) * h(n-K)$

$y(0) = 0$

(iii) Multiplications

$y(0) = x(K)\, h(-K)$

$= 2 + 6 = 8$

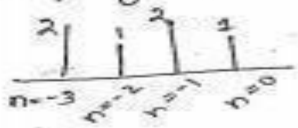
$h(n-K)$

$h(-1-K)$

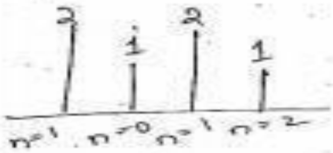
$-1-K=0$

$K=1$

Shifting of $h(-k)$



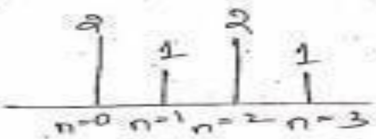
$y(-1) = x(k)\, h(-1-k)$
$= 3 \cdot 1$
$= 3$



$\underline{n=1}$
$h(n-k)$
$h(1-k)$
$k=1$

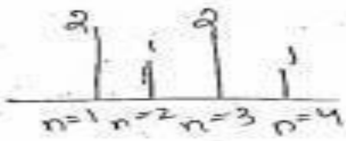
$y(1) = x(k)\, h(-k)$
$= 3+4+1$
$= 8$

$\underline{n=2}$
$h(n-k)$
$h(2-k)$
$k=2$



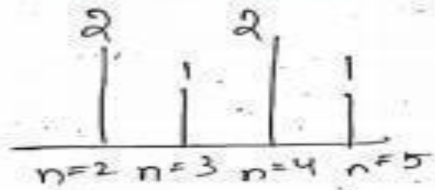
$y(2) = x(k)\, h(2-k)$
$= 6+2+2+2$
$= 12$

$\underline{n=3}$
$h(n-k)$
$h(3-k)$
$k=3$



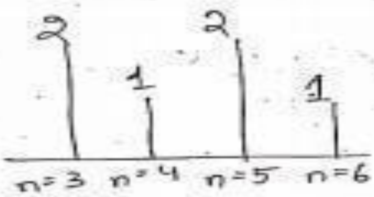
$y(3) = x(k)\, h(n-k)$
$= 4+1+4$
$= 9$

$\underline{n=4}$
$h(n-k)$
$h(4-k)$
$k=4$



$y(4) = x(k)\, h(n-k)$
$= 2+2$
$= 4$

$\underline{n=5}$
$h(n-k)$
$h(5-k)$
$k=5$



$y(5) = x(k)\, h(n-k)$
$= 4$

Convolution & interconnection of LTI system - properties. :

PROPERTIES OF LINEAR CONVOLUTION

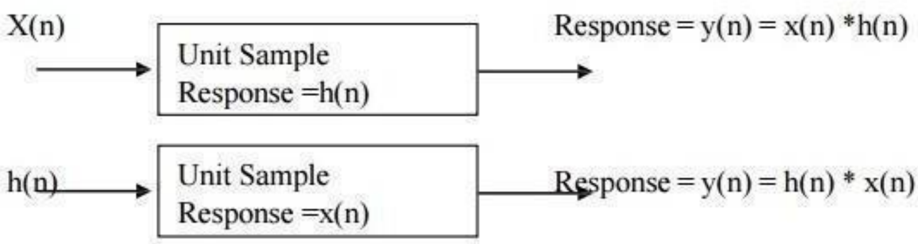
$x(n)$ = Excitation Input signal $y(n)$

= Output Response

$h(n)$ = Unit sample response

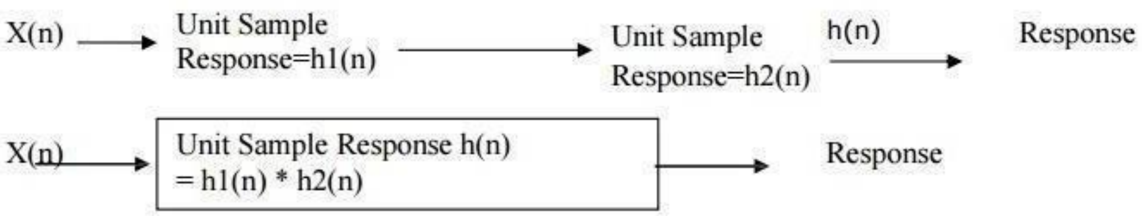
1. Commutative Law: (Commutative Property of Convolution)

$$x(n) * h(n) = h(n) * x(n)$$



2. Associate Law: (Associative Property of Convolution)

$$[\, x(n) * h1(n) \,] * h2(n) = x(n) * [\, h1(n) * h2(n) \,]$$



3. Distribute Law: (Distributive property of convolution)

$$x(n) * [\, h1(n) + h2(n) \,] = x(n) * h1(n) + x(n) * h2(n)$$

interconnection of LTI system :

1. Cascade Interconnection
2. Parallel Interconnection
3. Series- parallel Interconnection

→ According to the duration of Impulse response LTI System can be classified into two types.

1. Finite impulse Response System

— If the impulse response sequence is of finite durations.

Example :-

$$h(n) = \begin{cases} 1, & n=0 \\ 2, & n=3 \end{cases}$$

→ For causal FIR system $h(n) = 0$ for $n < 0$

The system whose Impulse response is infinite is known as infinite impulse.

Discrete time system described by difference equation. :

Recursive & non-recursive discrete time system. :

Recursive discrete time system

A **recursive system** is a **system** in which current output depends on previous output(s) and input(s).

Example :- $y(n) = F[x(n), y(n-1), y(n-2)]$

non-recursive discrete time system

A **non-recursive system** is a **system** in which current output does not depend on previous output(s).

Determine the impulse response of linear time invariant recursive system.

&

Correlation of Discrete Time signal

SOLUTION OF LINEAR CONSTANT COEFFICIENT DIFFERENCE EQUATION :-

we can solve it by two methods :-

1. Direct method.
2. Indirect method.

DIRECT METHOD :-

In this method, the solution of difference equations consists of two parts.

1. Homogeneous part - $y_h(n)$
2. particular Integral part - $y_p(n)$

$$\boxed{y(n) = y_h(n) + y_p(n)}$$

INDIRECT METHOD :-

In this method, z transform is used to solve the difference equations.

HOMOGENEOUS SOLUTION OF A DIFFERENCE EQUATION :-

Homogeneous solutions is obtained by putting $x(n) = 0$ so, that,

$$\sum_{K=0}^{N} a_K\, y(n-K) = 0 \quad \text{——— (I)}$$

Assume that the solutions of this equations is in Exponential form

$$y_h(n) = \lambda^n \quad \text{——— (II)}$$

putting equation (II) in equation (I)

$$\sum_{K=0}^{N} a_K\, \lambda^{(n-K)} = 0$$

$$= a_0\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} \ldots a_N\lambda^{n-N}$$

$$= \lambda^{n-N}\left[a_0\lambda^N + a_1\lambda^{N-1} + a_2\lambda^{N-2} \ldots + a_N\right] = 0$$

The polynomial in the bracket is known as characteristic equations

Case-1 :- ROOTS ARE DISTINCT :

If roots are $\lambda_1, \lambda_2, \lambda_3 ---- \lambda_n$. The general solutions is in the form $y_h(n)$

$$y_h(n) = C_1 \lambda_1^n + C_2 \lambda_2^n + ---- C_n \lambda_N^n$$

where, C_1, C_2, C_3 .... are weighting coefficient

Case-2 :- REPEATED ROOTS

let λ_1 is repeated for m times so, the general solutions $y_h(n) = \lambda_1 [C_1 + C_2 n$

Case-3 :- COMPLEX ROOTS

If the roots are complex conjugate

$$\lambda_1 = a + ib$$

$$\lambda_2 = a - ib$$

Then, the solutions $y_h(n) = r^n (\lambda_1^n \cos n\theta + \lambda_2^n \sin n\theta)$

here $r = \sqrt{a^2+b^2}$

$$\boxed{\theta = \tan^{-1} b/a}$$

PARTICULAR SOLUTIONS:-

This can be obtained by assuming a form that depends on the input $x(n)$. There are some general structure given below.

$x(n)$

$A \longrightarrow K$

$AM^n \longrightarrow KM^n$

$An^m \longrightarrow K_0 n^m + K_1 n^{m-1} + ...$

$A^n n^m \longrightarrow A^n [K_0 n^m + K_1 n^{m-1} + ----]$

$A \cos \omega_0 n + A \sin \omega_0 n \longrightarrow K_1 \cos \omega_0 n + K_2 \sin \omega_0 n$

Q. Determine the homogeneous solutions of the system described by the first order differential equation is $y(n) + a_1 y(n-1) = x(n)$

$$y_h(n) = y(n) + a_1 y(n-1) = 0 \quad \text{---- (1)}$$

$\Rightarrow \lambda^n + a_1 \lambda^{n-1} = 0$

$\Rightarrow \lambda^{n-1} (\lambda^n + a_1) = 0$

$\Rightarrow \lambda = -a_1$

$$y_h(n) = C_1 \lambda^n$$

$$\Rightarrow y_h(n) = C_1(-a_1)^n \text{ ——— (ii)}$$

value of C_1 in $n=0$ in equation (ii)

$$\boxed{y(0) = C_1 \cancel{\lambda} 1 = C_1} \text{ ——— (iii)}$$

put $n=0$ in equations (i),

$$y(0) + a_1 y(-1) = 0$$

$$y(0) = -a_1 y(-1)$$

Since, $y_0 = C_1$ (from eq^n (iii))

$$\boxed{C_1 = -a_1 y(-1)}$$

$$y_h(n) = C_1(\lambda)^n$$

$$y_h(n) = C_1(-a_1)^n$$

$$y_h(n) = (-a_1)^1 y(-1)(-a_1)^n$$

$$\boxed{y_h(n) = (-a_1)^{n+1} y(-1)}$$

Q. Determine the impulse response $h(n)$ for the system described by the difference equations

$$y(n) - 2y(n-1) = x(n) + x(n-1)$$

$$y_h(n) = y(n) - 2y(n-1) = 0 \text{ ——— (i)}$$

putting $\lambda^n = y(n)$ $\lambda^n - 2\lambda^{n-1} = 0$

$$\Rightarrow \lambda^{n-1}(\lambda - 2) = 0$$

$$\boxed{\lambda = 2}$$

$$y_h(n) = C_1 \lambda^n$$

$$\Rightarrow y_h(n) = C_1 2^n \text{ ——— (ii)}$$

putting $n=0$ in equations (iii)

$$\boxed{y(0) = C_1}$$

putting $n=0$ in equations (i)

$$y(n) - 2y(n-1) = 0$$

$$y(0) = 2y(-1).$$

$$\boxed{C_1 = y(0) = 2y(-1)}$$

$$\Rightarrow \boxed{y_h(n) = C_1 2^n = 2y(-1)2^n}$$

Q. Consider a causal stable system whose input is $x(n)$ and output is $y(n)$ are related by the difference equations.

$$y(n) - \frac{1}{6} y(n-1) - \frac{1}{6} y(n-2) = x(n) \qquad \text{——— (i)}$$

Then find the step response of the systems-

The total solution $y(n)$

$= y_p(n) + y_h(n)$

$\boxed{\text{Step response of the system } y(n) = y_p(n) + y_h(n)}$

As the input is step-input $y_p(n) = K$.

so, that the difference equation is written as,

$$K - \frac{1}{6}(K-1) - \frac{1}{6}(K-2)$$

$$\Rightarrow K - \frac{1}{6}(K) - \frac{1}{6}(K) = 1$$

$$\Rightarrow \frac{6K - K - K}{6} = 1$$

$$\Rightarrow \frac{4K}{6} = 1$$

$$\Rightarrow K = \frac{6}{4} = \frac{3}{2} = 1.5$$

For homogeneous solutions

$$y(n) - \frac{1}{6} y(n-1) - \frac{1}{6} y(n-2) = 0$$

$$\Rightarrow \lambda^n - \frac{1}{6}\lambda^{n-1} - \frac{1}{6}\lambda^{n-2} = 0$$

$$\Rightarrow \lambda^{n-2}\left[\lambda^2 - \frac{1}{6}\lambda' - \frac{1}{6}\right] = 0$$

$$\Rightarrow \lambda^2 - \frac{1}{6}\lambda - \frac{1}{6} = 0$$

$$\Rightarrow \frac{6\lambda^2 - \lambda - 1}{6} = 0$$

$$\Rightarrow 6\lambda^2 - \lambda - 1 = 0$$

$$\Rightarrow 6\lambda^2 - 3\lambda + 2\lambda - 1 = 0$$

$$\Rightarrow 3\lambda(2\lambda - 1) + 1(2\lambda - 1) = 0$$

$$\Rightarrow (3\lambda + 1)(2\lambda - 1) = 0$$

$$\Rightarrow \boxed{\lambda_2 = -\frac{1}{3}} \quad \text{or} \quad \boxed{\lambda_1 = \frac{1}{2}}$$

$$y_h(n) = C_1 \lambda_1^n + C_2 \lambda_2^n$$

$$= C_1\left(\frac{1}{2}\right)^n + C_2\left(-\frac{1}{3}\right)^n \quad \text{——(ii)}$$

According to case-I of homogeneous solution

---

putting $n=0$ in equation (ii)

$$y_h(n) = C_1\left(\frac{1}{2}\right)^n + C_2\left(-\frac{1}{3}\right)^n$$

$$\Rightarrow y(0) = C_1\left(\frac{1}{2}\right)^0 + C_2\left(-\frac{1}{3}\right)^0$$

$$\Rightarrow y(0) = C_1 + C_2 \quad \text{——(iii)}$$

putting $n=0$ in equation (i),

$$y(n) - \frac{1}{6} y(\cancel{n}^{0}-1) - \frac{1}{6} y(\cancel{n}^{0}-2)$$

$$y(0) - \frac{1}{6} y(-1) - \frac{1}{6} y(-2) = 1$$

$$y(0) = 1 \quad \text{——(iv)}$$

comparing equation (iii) & (iv), we get,

$$C_1 + C_2 = 1 \quad \text{——(v)}$$

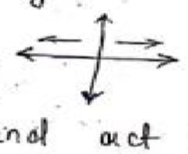
putting $n=1$ in equation (i),

$$y(n) - \frac{1}{6} y(n-1) - \frac{1}{6} y(n-2)$$

$$\Rightarrow y(1) - \frac{1}{6} y(0) - \frac{1}{6} y(\cancel{-1}^{0}) = 1$$

$$\Rightarrow y(1) - \frac{1}{6} \times 1 = 1$$

causal system presence of only +ve value



and act the -ve values will be zero

$$\Rightarrow y(1) = \frac{1}{6} + 1 = \frac{7}{6} \quad \text{——(vi)}$$

putting $n=1$ in equation (ii),

$$y_h(n) = C_1\left(\frac{1}{2}\right)^n + C_2\left(-\frac{1}{3}\right)^n$$

$$\Rightarrow y(1) = C_1\left(\frac{1}{2}\right)^1 + C_2\left(-\frac{1}{3}\right)^1$$

$$\Rightarrow y(1) = c_1\left(\frac{1}{2}\right)^1 \phi - \frac{1}{3}c_2 \quad \text{———} (vii).$$

comparing equations (vi) & (vii),

$$c_1\left(\frac{1}{2}\right) - c_2\left(\frac{1}{3}\right) = 7/6$$

comparing equation (v) and (vii), we get,

$$c_1 + c_2 = 1 \quad \text{———} (v)$$

$$\frac{1}{2}c_1 - \frac{1}{3}c_2 = 7/6 \quad \text{———} (viii)$$

Multiplying 1/2 in equation (v), we get

$$\frac{1}{2}\cancel{c_1} + \frac{1}{2}c_2 = \frac{1}{2}$$

$$\frac{1}{2}\cancel{c_1} - \frac{1}{3}c_2 = \frac{7}{6}$$

$$\frac{1}{2}\cancel{c_2} - \frac{1}{3}\cancel{c_2} = \quad c_1 = 9/5$$

$$c_2 = -4/5$$

putting the value of c_1 and c_2 in equation (ii),

$$y_h(n) = c_1\left(\frac{1}{2}\right)^n + c_2\left(-\frac{1}{3}\right)^n$$

$$= \frac{9}{5}\left(\frac{1}{2}\right)^n - \frac{4}{5}\left(\frac{1}{3}\right)^n$$

$$y(n) = y_p(n) + y_h(n)$$

$$= \frac{3}{2} + \left(\frac{9}{5}\right)\left(\frac{1}{2}\right)^n - \frac{4}{5}\left(-\frac{1}{3}\right)^n$$

Short Questions with Answers

1. **What is a Discrete Signal ?**

Ans -A discrete time signal S(n) is a function of an independent variable ‘n’.

2. **How Discrete Time signal can be represented ?**

Ans-Discrete Time signal can be represented in the following 4 ways,
1. Graphical representation
2. Functional representation
3. Tabular representation
4. Sequential representation

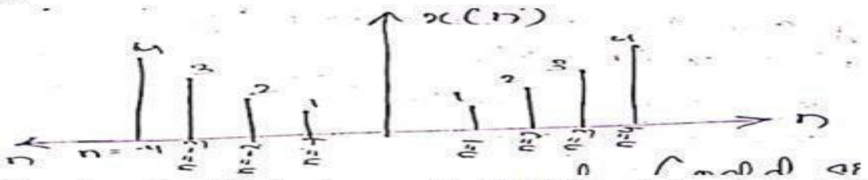
3. **Define fundamental period .**

Ans - A signal x(n) is periodic with period N when x(n+N)=x(n) for all vallu of n. The smallest value of N for which equation(1) is valid is known as fundamental period.

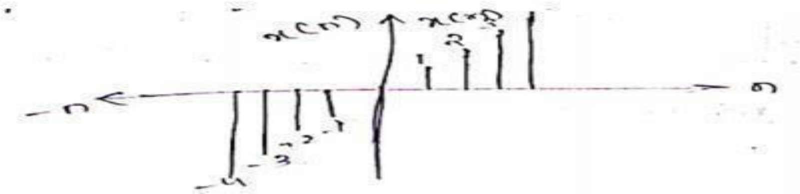
4. **What is Symmetric and Assymetric Signal ?**

Ans -

- A discrete time signal is said to be symmetric (Even) if it satisfies x(-n)=x(n) for all values of n.



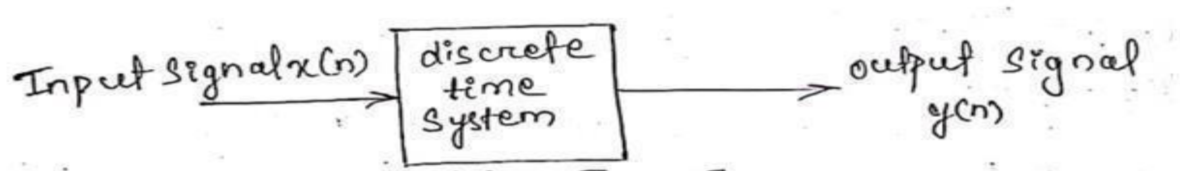
- A discrete time signal is said to be Asymmetric (Odd) if it satisfies x(-n)=-x(n) for all values of n.



5. **What do you mean by a Discrete Time Sysyem** ?

Ans -

A **discrete-time system** is anything that takes a **discrete-time** signal as input and generates a **discrete-time** signal as output.



y(n)= T[x(n)]

→ Symbol T denotes the transformations.

6. **Classify Discrete Time System .**

- Ans- Static/Dynamic
- Causal/Non-Causal
- Time invariant/Time variant
- Linear/Non-Linear
- Stable/Unstable
- FIR /IIR

7. **What do you mean by Causal And NonCausal System ?**

Ans –

- A **causal system** is one whose output depends only on the present and the past inputs.
- A **noncausal system's** output depends on the future inputs. In a sense, a **noncausal system** is just the opposite of one that has memory.
- It cannot because real **systems** cannot react to the future.

8. **What do you mean by Linear And Non-Linear System ?**

Ans-

- A system that satisfies the principle of superposition is said to be Linear System.
- Super position principle said that the response of the system to weighted sum of signal is equal to the corresponding weighted sum of output signal to each of individual output.

Long Questions

1. Classify Discrete Time Signal And Explain In detail.
2. Classify Discrete Time System And Explain In detail.
3. Write Convolution Properties.
4. Describe the steps for finding the homogeneous solution of a Difference equation.
5.

Determine the convolution sum of two sequence. (matrix method).

$$x(n) = \{\underset{\underset{n=0}{\uparrow}}{3}, 2, 1, 2\}$$

$$h(n) = \{1, \underset{\underset{n=0}{\uparrow}}{2}, 1, 2\}$$

6.

Consider a causal stable system whose input is $x(n)$ and output is $y(n)$ are related by the difference equations.

$$y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = x(n) \quad \text{--- (i)}$$

Then find the step response of the systems:-

CHAPTER-03 : THE Z-TRANSFORM & ITS APPLICATION TO THE ANALYSIS OF LTI SYSTEM.

Z-transform & its application to LTI system

- Z-Transform is used to analyse the LTI discrete time signal.
- Z-Transform converts the discrete Time signal into frequency domain.
- It is used for both stable and unstable systems.
- In Z-domain, the convolution of 2 sequences is equivalent to multiplications of their corresponding Z-Trasform.

Direct Z-transform.

-- Z- Transform of a discrete time signal x(n) ca be written as,

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad \text{——— (i)}$$

Where, Z is a complex variable.

- Z-Transform of a signal x(n) is deoted by,

$$x(z) = Z[x(n)]$$
$$x(z) \xleftrightarrow{z} x(n)$$

Equation (1) refers to two sided Z-Transform.

- If x(n) is causal, then x(n)=0 for n<0 and the Z- Transform can be written as,

$$\boxed{x(z) = \sum_{n=0}^{\infty} x(n) z^{-n}} \quad \text{——— (ii)}$$

- If x(n) is causal, then the Z- Transform exists for those values of n for which series converse.

Example :

Q. Find the z- transform of the given sequence

$$x(n) = \{1, 2, 3, \underset{\uparrow n=0}{4}, 1, 2\}$$

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-3}^{2} x(n) z^{-n}$$

$$= x(-3) z^{-(-3)} + x(-2) z^{-(-2)} + x(-1) z^{-(-1)}$$
$$+ x(0) z^{0} + x(1) z^{-1} + x(2) z^{-2}$$

$$\boxed{x(z) = 1z^3 + 2z^2 + 3z^1 + 4 + 1z^{-1} + 2z^{-2}}$$

ROC(Region Of Covergence)

The region of convergence of x(z) is the set of all values of Z for which x(z) attains a finite value.

Properties :

- **ROC** is a concentric ring or a circular disc in a Z-Plane centered at origin.
- ROC doesnot contain any pole.
- If x(n) is causal sequence, then the ROC is entire Z-plane except z=0.
- If x(n) is two sided sequence, then ROC is entire Z-plane except z=0 ad infinite.
- If x(n) is on-causal sequence, then ROC is entire z-plane except z=infinite.
- If x() is infinite duration two sided sequence, the ROC consists of a circular ring in the Z-plane bounded on the interior or exterior by a pole, but doesnot contain any pole.
- The ROC of LTI stable system contains the unit circle.

Inverse Z-transform.

The process used for transforming the Z-domain signal into time domain signal is known as inverse Z-Transform.

V.V.I. PROBLEMS ON INFINITE DURATION SEQUENCE:

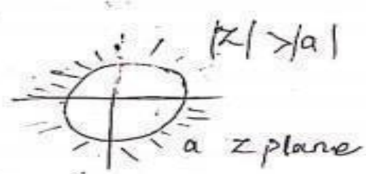
Q. Find the z-transform and ROC of $x(n) = a^n u(n)$

Sol^n:-

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} a^n u(n) z^{-n}$$

$$\Rightarrow \sum_{n=0}^{\infty} a^n . 1 . z^{-n}$$



$$\Rightarrow \sum_{n=0}^{\infty} a^n z^{-n}$$

$$\Rightarrow \sum_{n=0}^{\infty} (a z^{-1})^n$$

$$\Rightarrow \sum_{n=0}^{\infty} \frac{1}{1 - a z^{-1}}$$

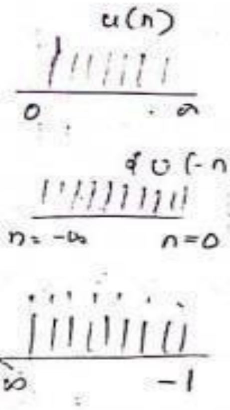
$$\Rightarrow \sum_{n=0}^{\infty} \frac{1}{1 - \frac{a}{z}} \Rightarrow \frac{1}{\frac{z-a}{z}} \Rightarrow \frac{z}{z-a}$$

$$\therefore \boxed{x(z) = \frac{z}{z-a}} \quad \boxed{ROC : |z| > |a|}$$

Q. Find the z-transform and ROC of the signal

$$x(n) = -b^n u(-n-1)$$

Sol^n: $x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$



$$= \sum_{n=-\infty}^{\infty} -b^n u(-n-1) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty-1} -b^n \, 1 \, z^{-n}$$

$$\Rightarrow \sum_{n=-\infty}^{-1} -b^n z^{-n}$$

$$\Rightarrow \sum_{n=-\infty}^{-1} (-bz^{-1})^n \Rightarrow \sum_{n=1}^{\infty} (b^{-n} z^n)$$

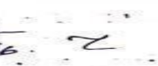
$$\Rightarrow \sum_{n=1}^{\infty} (b^{-1} z)^n$$

$$\Rightarrow \sum_{n=0}^{\infty} (b^{-1} z)^n - 1 \Rightarrow \frac{1}{1 - b^{-1} z} - 1$$

$$\Rightarrow \frac{1}{1 - \frac{z}{b}} - 1 \Rightarrow \frac{1}{\frac{b-z}{b}} - 1$$

$$\Rightarrow \frac{b}{b-z} - 1 \Rightarrow \frac{b - b + z}{b - z} = \frac{z}{b-z}$$

$$x(z) = \frac{z}{b-z} \qquad \boxed{ROC: |z| < |b|}$$



find the z-transform of ROC $x(n)$ =

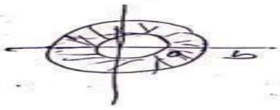
$$x(n) = a^n u(n) + [-b^n u(-n-1)]$$

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$x(z) = \sum_{n=0}^{\infty} a^n u(n) +$$

$$x(z) = \frac{z}{z-a} + \frac{z}{b-z}$$

$$= \frac{z}{z-a} - \frac{z}{z-b}$$



$a < |z| < b$, $a < b$

Various properties of Z-transform.

1) Differentiation in z-Domain :-

$$x(n) \xleftrightarrow{z} X(z)$$

then, $$n\,x(n) \xleftrightarrow{z} -z\frac{dX(z)}{dz}$$

Proof :-

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

Differentiating above on both side with respect to z.

$$\frac{dX(z)}{dz} = \frac{d}{dz}\left\{\sum_{n=-\infty}^{\infty} x(n) z^{-n}\right\}$$

$$= \sum_{n=-\infty}^{\infty} x(n) \frac{d}{dz}\left\{z^{-n}\right\}$$

$$= \sum_{n=-\infty}^{\infty} -n\,x(n)\, z^{-n-1}$$

$$-z\frac{dX(z)}{dz} = \sum_{n=-\infty}^{\infty} n\,x(n)\, z^{-n}$$

comparing both equations $-z\frac{dX(z)}{dz}$ is the z-transform of $n x(n)$

II. parseval's Theorem :-

$$\sum_{n=-\infty}^{\infty} x(n)\, x^{*}(n) = \int_{-\pi}^{\pi} f(z)\, f^{R}(z)\, dz$$

parseval's relation tells us that the Energy of a signal is equal to the Energy of its fourier transform

TIME SHIFTING:-

If $x(n) \xleftrightarrow{Z} X(z)$ with ROC $= R$ then

$$x(n-m) \longleftrightarrow Z^{-m} X(z) \text{ with ROC} = R$$

proof :- $$Z[x(n-m)] = \sum_{n=-\infty}^{\infty} x(n-m) z^{-n}$$

Let $n-m = P$

$$= \sum_{n=-\infty}^{\infty} x(P) z^{-(P+m)}$$

$$= Z^{-m} \sum_{n=-\infty}^{\infty} x(P) Z^{P} = Z^{-m} X(z)$$

IV. convolution property:-

$$x_1(n) \xleftrightarrow{Z} Y_1(z) \text{ with ROC} - R_1$$

$$x_2(n) \xleftrightarrow{Z} X_2(z) \text{ with ROC} = R_2$$

then $x_1(n) * x_2(n) \xleftrightarrow{Z} X_1(z) \cdot X_2(z)$ with ROC containing $R_1 R_2$

Proof:-

$$Z[x_1(n) * x_2(n)] = \sum_{n=-\infty}^{\infty} \{x_1(n) * x_2(n)\} Z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left\{ \sum_{m=-\infty}^{\infty} x_1(m) \cdot x_2(n-m) \right\} Z^{-m}$$

Interchanging the order of Σ

$$Z[x_1(n) * x_2(n)] = \sum_{n=-\infty}^{\infty} x_1(m) \left\{ \sum_{n=-\infty}^{\infty} x(n-m) Z^{-n} \right\}$$

$$= \sum_{m=-\infty}^{\infty} x_1(m) \left\{ Z^{-m} X_2(z) \right\}$$

Since the time Shifting property

$$X_2(z) \left\{ \sum_{m=-\infty}^{\infty} x_1(m) Z^{-m} \right\} = X_1(z) \cdot X_2(z)$$

V. Initial value theorem :-

$$x_t(z) = Z[x(n)]$$

then $x(0) = \lim_{z \to \infty} x_t(z)$

VI Final value theorem:-

$$x_t(z) = Z[x(n)]$$

then $x(0) = \lim_{z \to 1} (z^{-1})\, x_t(z)$

Rational Z-transform.

Poles & zeros

Pole : The value of Z forwhich x(z) becomes infinite is known as pole.

Zero : The value of Z for which x(z) becomes zero is known as zero.

example :-

$$x(z) = \frac{(z-2)}{(z+3)(z+2)}$$

Pole = $z+3=0$
$z=-3$

$z-2=0$
$z=2$

Zero = $z-2=0$
$\Rightarrow z=2$

Sequence	Transform	ROC
$\delta(n)$	1	all value of z
$u(n)$	$\frac{z}{z-1}$	$\lvert z\rvert > 1$
$-u(-n-1)$	$\frac{z}{z-1}$	$\lvert z\rvert < 1$
$a^n u(n)$	$\frac{1}{1-az^{-1}}$ or $\frac{z}{z-a}$	$\lvert z\rvert > a$
$-a^n u(n)$	$\frac{1}{1-az^{-1}}$ or $\frac{z}{z-a}$	$\lvert z\rvert < a$

Pole location time domain behaviour for casual signals.

(a) Find the inverse z-transform of

$$x(z) = \frac{z(z^2-4z+5)}{(z-3)(z-1)(z-2)}$$

ROC: $2 < |z| < 3$

$|z| > 3$

$|z| < 1$

Solutions:- By partial fraction method

$$x(z) = \frac{z(z^2-4z+5)}{(z-3)(z-1)(z-2)}$$

$$\frac{x(z)}{z} = \frac{z^2-4z+5}{(z-3)(z-1)(z-2)} = \frac{A}{z+3} + \frac{B}{z-1} + \frac{C}{z}$$

$$A = \frac{z^2-4z+5}{\cancel{(z-3)}(z-1)(z-2)} \times \cancel{(z-3)} \Big|_{z=3}$$

$$= \frac{9-12+5}{2 \cdot 1} = \frac{2}{2} = 1$$

$$B = \frac{z^2-4z+5}{(z-3)\cancel{(z-1)}(z-2)} \times \cancel{(z-1)} \Big|_{z-1}$$

$$= \frac{1-4+5}{-2 \cdot -1} = \frac{2}{2} = 1$$

$$C = \frac{z^2-4z+5}{(z-3)(z-1)(z-2)} \times (z-2) \Big|_{z=2}$$

$$= \frac{4-8+5}{-1 \cdot 1} = \frac{1}{-1} = -1$$

$$\frac{x(z)}{z} = \frac{1}{z-3} + \frac{1}{z-1} - \frac{1}{z-2}$$

$$x(z) = \frac{z}{z-3} + \frac{z}{z-1} - \frac{1}{z-2}$$

$$z\left[a^n u(n)\right] = \frac{z}{z-a}$$

$3^n u(n)$

$1^n u(n) = 1 \cdot u(n) = u(n)$

$2^n u(n)$

Case-1:- for ROC : $2 < |z| < 3$

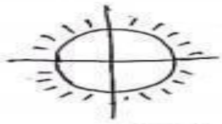
$$X(Z) = \frac{Z}{Z-3} + \frac{Z}{Z-1} - \frac{Z}{Z-2}$$

$$x(n) = u(n) - 2^n u(n) + 3^n u(-n)$$

Case-2:- for ROC: $|z| > 3$

$$x(n) = u(n) - 2^n u(n) + 3^n u(-n)$$

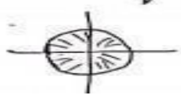
∴ causal sequence



Case-3:- For ROC : $|z| < 1$

$$x(n) = -u(0-n-1) - 2^n u(n) + 3^n u(n)$$

∴ Non - causal sequence



System function of a linear time invariant system.

- Linearity

$$x_1(n) \xleftrightarrow{z} x_1(z)$$

$$x_2(n) \xleftrightarrow{z} x_2(z)$$

$$[a_1 x_1(n) + a_2 x_2(n)] \longleftrightarrow a_1 x_1(z) + a_2 x_2(z)$$

Proof:-

$$\sum_{n=-\infty}^{\infty} x(n) Z^{-n} = \sum_{n=-\infty}^{\infty} [a_1 x_1(n) + a_2 x_2(n)] Z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} [a_1 x_1(n) Z^{-n} + a_2 x_2(n) Z^{-n}]$$

$$= \sum_{n=-\infty}^{\infty} a_1 x_1(n) Z^{-n} + \sum_{n=-\infty}^{\infty} a_2 x_2(n) Z^{-n}$$

$$= a_1 x_1(z) + a_2 x_2(z) \text{ proved}$$

Multiplication by an Exponential Sequence:-

$$x(n) \xleftrightarrow{z} x(z)$$

$$a^n x(n) \xleftrightarrow{z} x(a^{-1}z)$$

proof:- $x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$$= \sum_{n=-\infty}^{\infty} a^n x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x(n) (a^{-1} z)^{-n} = x(a^{-1} z) \text{ proved}$$

Time reversal:-

$$x(n) \xleftrightarrow{z} x(z)$$

$$x(-n) \longleftrightarrow x(z^{-1})$$

proof:- $\sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$$= \sum_{n=-\infty}^{\infty} x(-n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x(l) z^{l}$$

$$= \sum_{n=-\infty}^{\infty} x(l) (z^{-1})^{l}$$

$$= x(z^{-1}) \text{ proved}$$

Discuss inverse Z-transform.

The process through which x(z) is converted back to x(n) is known as Inverse Z-transform. This can be done by the following methods,

1. Partial fraction Method
2. contour Integration

Inverse Z-transform by partial fraction expansion.

Find the inverse z-transform of

$$x(z) = \frac{1+3z^{-1}}{1+3z^{-1}+2z^{-2}}, \text{ ROC} = |z| > 2$$

proof:-

multiplying z^2 up and down

$$x(z) = \frac{z^2+3z}{z^2+3z+2}$$

$$x(z) = \frac{z(z+3)}{z^2+3z+2}$$

$$\Rightarrow \frac{x(z)}{z} = \frac{z+3}{z^2+3z+2}$$

$$\Rightarrow \frac{x(z)}{z} = \frac{z+3}{z^2+2z+z+2}$$

$$= \frac{z+3}{z(z+2)+1(z+2)}$$

$$\Rightarrow \frac{x(z)}{z} = = \frac{z+3}{(z+2)(z+1)}$$

$$\frac{z+3}{(z+2)(z+1)} = \frac{A}{z+2} + \frac{B}{z+1}$$

$$A = \frac{z+3}{(z+2)(z+1)} \times (z+2) \Big|_{z=-2}$$

$$= \frac{-2+3}{-2+1} = \frac{1}{-1} = -1$$

$$B = \frac{z+3}{(z+2)(z+1)} \times (z+1) \Big|_{z=-1}$$

$$= \frac{-1+3}{-1+2} = \frac{2}{1} = 2$$

$$\frac{x(z)}{z} = \frac{-1}{z+2} + \frac{2}{z+1}$$

$$\Rightarrow x(z) = \frac{-z}{z+2} + \frac{2}{z+1}$$

$$= \frac{2z}{z+1} - \frac{z}{z+2}$$

$$\boxed{a^n u(n) = \frac{z}{z-a} \qquad \frac{z}{z+a} = (-a)^n u(n)}$$

$$\boxed{2(-1)^n u(n) - (-2)^n u(n) = x(n)}$$

Inverse Z-transform by contour Integration

Find inverse Z-Transform of $x(n) = \delta(n)$ by using contour Itegration method.

$$x(n) = \frac{1}{2\pi j}\oint_c \frac{1}{1+\frac{1}{2}z^{-1}} z^{n-1}\,dz$$

$$= \frac{1}{2\pi j}\oint_c \frac{z^n}{z+\frac{1}{2}}\,dz$$

Since the contour of integration must lie inside the region of convergence, i.e. for $|z| > \frac{1}{2}$, it encloses the pole at $z = -\frac{1}{2}$. For $n \geq 0$ there are no poles at $z = 0$. Thus, for $n \geq 0$

$$x(n) = \text{Residue of } \frac{z^n}{z+\frac{1}{2}} \quad \text{at} \quad z = -\frac{1}{2} \quad \text{or}$$

$$x(n) = (-\tfrac{1}{2})^n \quad n \geq 0$$

For $n < 0$ we use the substitution of variables. Thus

$$x(n) = \frac{1}{2\pi j}\oint X(1/p)\, p^{-n-1}\,dp$$

where now the contour of integration must lie inside the region of convergence of $X(1/p)$ i.e. for $|p| < 2$. Then

$$x(n) = \frac{1}{2\pi j}\oint_{c'} \frac{2p^{-n-1}}{p+2}\,dp$$

For n negative the only pole is at $p = -2$ which is outside the contour of integration. Thus for $n < 0$ $x(n) = 0$. Combining these two results, then

$$x(n) = (-\tfrac{1}{2})^n\, u(n)$$

Since for this example $X(z)$ has only a single pole, the partial fractions expansion method wouldn't apply. The inspection method would and in fact corresponds to problem 6.1 (iii) for $a = -\frac{1}{2}$.

Short Questions with Answers

1. Define Z-Transform.

Ans :

- Z-Transform is used to analyse the LTI discrete time signal.
- Z-Transform converts the discrete Time signal into frequency domain.

2. Define ROC.

Ans : The region of convergence of x(z) is the set of all values of Z for which x(z) attains a finite value.

3. Write Properties of Z-Transform.

Ans : Properties are,

i. Differentiation in Z-domain

ii. Parseval's Theorem

iii. Time Shifting

iv. Convolution Property

v. Initial Value Theorem

vi. Final Value Theorem

4. What are the methods of Inverse Z-transform ?

Ans : Inverse Z-Transform can be done by the following methods,

1. Partial fraction Method

2. contour Integration

5. Define Inverse z-transform .

Ans : The process through which x(z) is converted back to x(n) is known as Inverse Z-transform.

6. What are the applications of Z-Transform ?

Ans :

- Z-Transform converts the discrete Time signal into frequency domain.
- In Z-domain, the convolution of 2 sequences is equivalent to multiplications of their corresponding Z-Trasform.

Long Questions

1. Find Inverse Z-Transform of

$$x(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}} \quad \begin{array}{|l} \text{ROC } |z| > 1 \\ |z| < 0.5 \\ 0.5 < |z| < 1 \end{array}$$

2. Find Z-Transform and ROC of

$$x(n) = (n + 0.5)\left(\frac{1}{3}\right)^n u(n)$$

3. Find Z- Transform and ROC of $x(n) = a^n u(n)$.

4. Find inverse Z-Transform of

$$x(z) = \frac{1 + 3z^{-1}}{1 + 3z^{-1} + 2z^{-2}}$$

5. Find inverse Z-Transform of the sequence by using Long Division Method

$$x(z) = \frac{1 + 2z^{-1}}{1 - 2z^{-1} + z^{-2}} \text{. If } x(n) \text{ is causal}$$

CH.04

DISCUSS FOURIER TRANSFORM: ITS APPLICATIONS PROPERTIES

: -DISCUSS DISCRETE FOURIER TRANSFORM(DFT): -

- The DFT of a sequence $X_P(n)$ with a period of N samples so that

$X_P(n)= X_P(n+l.N)$.Since $X_P(n)$ is a periodic, it can be represented as a weighted some of complex exponentials whose frequencies are integer multiples of the fundamental frequencies $2\pi/N$.

- These periodic complex exponentials are offer from
  $e^{j2\pi kn/N}=e^{j2\pi k(n+NL)}$

  where,

  N is an integer.

- The discrete Fourier transform eq^n is
  $X_p(K)=\sum^{x1}_{n=0} X_P(n)e^{-j2\pi kn/N}$

: -DETERMINE FREQUENCY DOMAIN SAMPLING AND RECONSTRUCTION OF DISCRETE TIME SIGNALS: -

- Frequency domain Sampling & Reconstruction of Analog Signals

We know that continuous-time finite energy signals have continuous spectra. Here we will consider sampling of such signals periodically and reconstruction of signals from samples of their spectra.

- Consider an analog signal x(t) with a spectrum X(F). Then suppose we obtain samples of X(F) every δF hertz as shown below in Fig. 1.

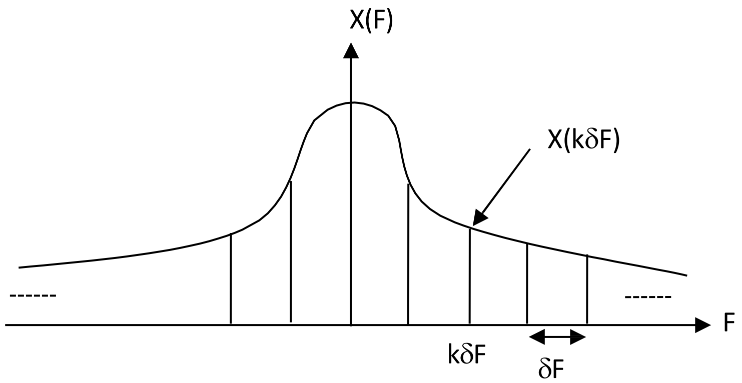


Fig .1

- Then the question is given these samples $X(k\delta F)$ where $-\infty<k<\infty$ can the signal x(t) or spectrum X(F) be recovered?

❖ Note: This problem is mathematically the dual to that of sampling a continuous time signal in the time-domain.

- If we sample the spectrum every δF then

$$X(k\delta F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi k\delta F t} dt$$

- If we define the reciprocal of δF as below:

$$T_s = \frac{1}{\delta F}$$

- above expression can be written as:

$$X(k\delta F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi kt/T_s} dt$$

- We can then subdivide the integration range of the integral into an infinite number of intervals of width T_s and change the variable of integration to translate each integral into fundamental range $\frac{-T_s}{2} \le t \le \frac{T_s}{2}$.

- **<u>Frequency Domain Sampling & Reconstruction of Discrete Time Signals</u>**
  - An aperiodic finite energy signal has continuous spectra.
  - For an aperiodic signal x[n] the spectrum is :

$$X[w] = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn} \qquad (1)$$

- Suppose we sample X[w] periodically in frequency at a sampling of δw radians between successive samples. We know that DTFT is periodic with 2π, therefore only samples in the fundamental frequency range will be necessary. For convenience we take N equidistant samples in the interval 0<=w<2π. The spacing between samples will be $\delta w = \frac{2\pi}{N}$ as shown below in Fig.

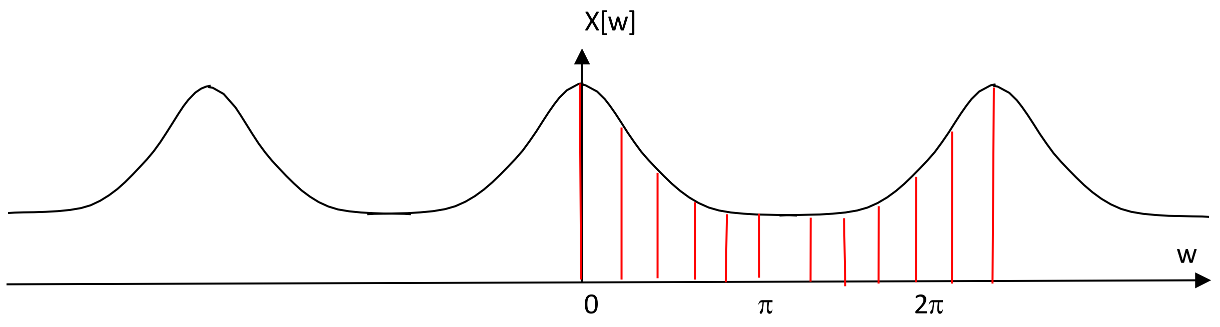


Fig. 1

- Let us first consider selection of N, or the number of samples in the frequency domain.

- If we evaluate equation (1) at $w = \frac{2\pi k}{N}$

$$X\left[\frac{2\pi k}{N}\right] = \sum_{n=-\infty}^{\infty} x[n]e^{-j2\pi kn/N} \qquad k = 0,1,2,......,(N-1)$$

- We can divide the summation in (1) into infinite number of summations where each sum contains N terms.

$$X\left[\frac{2\pi k}{N}\right] = ...+ \sum_{n=-N}^{-1} x[n]e^{-j2\pi kn/N} + \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N} + \sum_{n=N}^{2N-1} x[n]e^{-j2\pi kn/N}$$

$$= \sum_{l=-\infty}^{\infty} \sum_{n=lN}^{lN+N-1} x[n]e^{-j2\pi kn/N}$$

- If we then change the index in the summation from n to n-lN and interchange the order of summations we get:

$$X\left[\frac{2\pi k}{N}\right] = \sum_{n=0}^{N-1}\left[\sum_{l=-\infty}^{\infty} x[n-lN]\right]e^{-j2\pi kn/N} \qquad for\ k = 0,1,2,.....,(N-1) \qquad (3)$$

- denote the quantity inside the bracket as $x_p[n]$. This is the signal that is a repeating version of x[n] every N samples. Since it is a periodic signal it can be represented by the Fourier Series.

$$x_p[n] = \sum_{k=0}^{N-1} c_k e^{j2\pi kn/N} \qquad n = 0,1,2,.......,(N-1)$$

with FS coefficients:

$$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x_p[n]e^{-j2\pi kn/N} \qquad k = 0,1,2,......,(N-1) \qquad (4)$$

- Comparing the expressions in equations (4) and (3) we conclude the following:

$$c_k = \frac{1}{N} X\left[\frac{2\pi}{N}k\right] \qquad k = 0,1,......,(N-1)$$

- Therefore it is possible to write the expression $x_p[n]$ as below:

$$x_p[n] = \frac{1}{N}\sum_{k=0}^{N-1} X\left[\frac{2\pi}{N}k\right]e^{j2\pi kn/N} \qquad n = 0,1,....,(N-1) \qquad (7)$$

- The above formula shows the reconstruction of the periodic signal $x_p[n]$ from the samples of the spectrum X[w]. But it does not say if X[w] or x[n] can be recovered from the samples.

- Let us have a look at that :

- Since $x_p[n]$ is the periodic extension of x[n] it is clear that x[n] can be recovered from $x_p[n]$ if there is no aliasing in the time domain. That is if x[n] is time-limited to less than the period N of $x_p[n]$.This is depicted in Fig. 3 below:

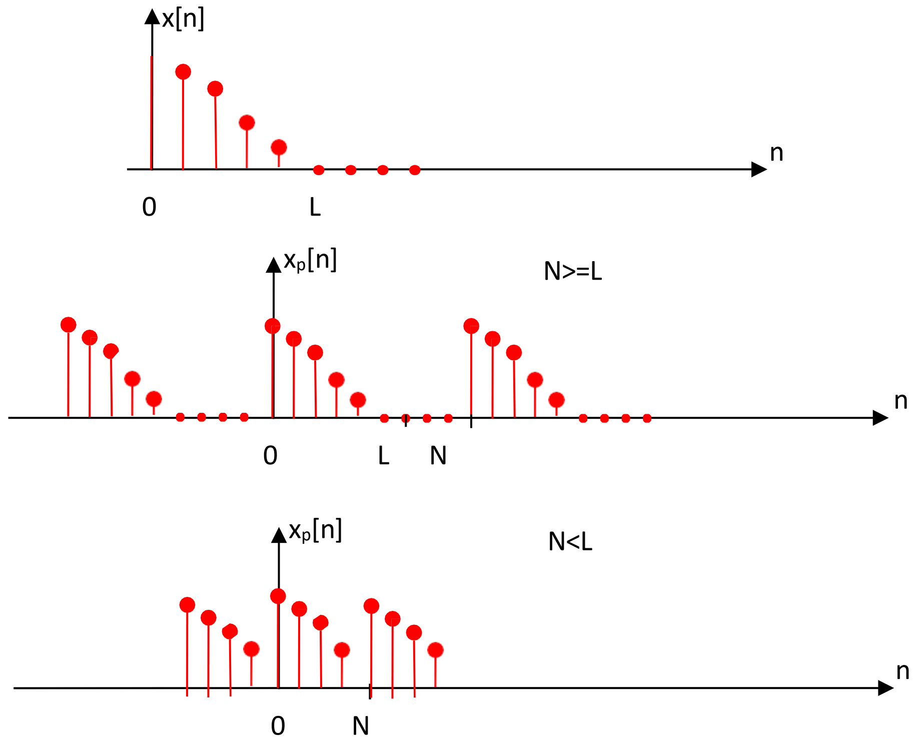


Fig . 3

- Hence we conclude :
- The spectrum of an aperiodic discrete-time signal with finite duration L can be exactly recovered from its samples at frequencies $w_k = \frac{2\pi k}{N}$ if N >= L.

- We compute $x_p[n]$ for n=0,1,.....,N-1 using equation (7)
- Then X[w] can be computed using equation (1).

: -STATE & EXPLAIN DISCRETE TIME FOURIER TRANSFORMATION(DTFT): -

- Like continuous time signal Fourier transform, discrete time Fourier Transform can be used to represent a discrete sequence into its equivalent frequency domain representation and LTI discrete time system and develop various computational algorithms.

- X jω in continuous F.T, is a continuous function of xn. However, DFT deals with representing xn with samples of its spectrum Xω. Hence, this mathematical tool carries much importance computationally in convenient representation. Both, periodic and non-periodic sequences can be processed through this tool. The periodic sequences need to be sampled by extending the period to infinity.

: - STATE & EXPLAIN DISCRETE FOURIER TRANSFORMATION(DFT): -

- DFT

- The discrete Fourier transform (DFT) is the primary transform used for numerical computation in digital signal processing. It is very widely used for spectrum analysis, fast convolution, and many other applications. The DFT transforms NN discrete-time samples to the same number of discrete frequency samples, and is defined as

$$X(k)=\sum_{n=0}^{N-1}x(n)e^{-\left(\frac{i2\pi nk}{N}\right)}$$

- The DFT is widely used in part because it can be computed very efficiently using fast Fourier transform (FFT) algorithms.

- IDFT

- The inverse DFT (IDFT) transforms NN discrete-frequency samples to the same number of discrete-time samples. The IDFT has a form very similar to the DFT,

$$x(n)=\frac{1}{N}\sum_{k=0}^{N-1}X(k)e^{\frac{i2\pi nk}{N}}$$

and can thus also be computed efficiently using FFTs.

: -COMPUTE DFT AS A LINEAR TRANSFERMATION: -

- Linear transformation in DSP: -
- DEFINITION (Linear Transformation): A transformation (or mapping) T from a vector space V1 to a vector space V2, T: V1 → V2 is a linear transformation (or a linear operator, a linear map, etc.), if: ... If T is a linear transformation, then T0 must be 0.

- DFT as linear transformation: -
- With these definitions, the N-point DFT can be expressed as, XN=WN×N. where, WN is the matrix of the linear transformation and WN is symmetric matrix. If we assume that inverse of the WN is exists then above eqn can be inverted by premultiplying both sides by W−1N Therefore we get.

: - RELATE DFT TO OTHER TRANSFORMS: -

- RELATIONSHIP BETWEEN DFT AND Z-TRANSFORM
- Let us develop the relationship between the DFT and *z*-transform. The *z*-transform of a discrete time sequence of finite duration is given by

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n} \qquad (11.89)$$

- Let us consider a finite duration sequence $x(n)$, $0 \le n \le N-1$.
- The above equation reduces to

$$X(z) = \sum_{n=0}^{N-1} x(n) z^{-n} \qquad (11.90)$$

- DFT of $x(n)$ is given by

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi kn}{N}} \qquad (11.91)$$

- Let us put $z = e^{-j\frac{2\pi k}{N}}$ in Eq. (11.90), we ...

: -**DISCUSS THE PROPERTY OF THE DFT: -**

Properties of DFT

1. Linearity
   - It states that the DFT of a combination of signals is equal to the sum of DFT of individual signals. Let us take two signals $x_1 n$ and $x_2 n$, whose DFT s are $X_1\omega$ and $X_2\omega$ respect ively. So, if

$x_1(n)\rightarrow X_1(\omega)$x1(n)→X1(ω)and$x_2(n)\rightarrow X_2(\omega)$x2(n)→X2(ω)

Then $ax_1(n)+bx_2(n)\rightarrow aX_1(\omega)+bX_2(\omega)$ax1(n)+bx2(n)→aX1(ω)+bX2(ω)

where **a** and **b** are constants.

2. Symmetry
   - The symmetry properties of DFT can be derived in a similar way as we derived DTFT symmetry properties. We know that DFT of sequence xn is denoted by XK. Now, if xn and XK are complex valued sequence, then it can be represented as under

$x(n)=x_R(n)+jx_1(n), 0\le n\le N-1$x(n)=xR(n)+jx1(n),0≤n≤N−1

And $X(K)=X_R(K)+jX_1(K), 0\le K\le N-1$X(K)=XR(K)+jX1(K),0≤K≤N−1

3. Duality Property
   - Let us consider a signal xn, whose DFT is given as XK. Let the finite duration sequence be XN. Then according to duality theorem,

If, $x(n)\longleftrightarrow X(K)$x(n)↔X(K)

Then, $X(N) \leftrightarrow N x[((-k))_N]$ $X(N)\leftrightarrow Nx[((-k))N]$

So, by using this theorem if we know DFT, we can easily find the finite duration sequence.

4. Complex Conjugate Properties

- Suppose, there is a signal xn, whose DFT is also knownto us as XK. Now, if the complex conjugate of the signal is given as x*n, then we can easily find the DFT without doing much calculation by using the theorem shown below.

If, $x(n) \leftrightarrow X(K)$ $x(n)\leftrightarrow X(K)$

Then, $x*(n) \leftrightarrow X*((K))_N = X*(N-K)$ $x*(n)\leftrightarrow X*((K))N=X*(N-K)$

5. Circular Frequency Shift

- The multiplication of the sequence xn with the complex exponential sequence $e_{j2\Pi kn/N}$ $ej2\Pi kn/N$ is equivalent to the circular shift of the DFT by L units in frequency. This is the dual to the circular time shifting property.

If, $x(n) \leftrightarrow X(K)$ $x(n)\leftrightarrow X(K)$

Then, $x(n)e_{j2\Pi Kn/N} \leftrightarrow X((K-L))_N$ $x(n)ej2\Pi Kn/N\leftrightarrow X((K-L))N$

6. Multiplication of Two Sequence

- If there are two signal $x_1$$n$n and $x_2$$n$n and their respective DFTs are $X_1$$k$ and $X_2$$K$, then multiplication of signals in time sequence corresponds to circular convolution of their DFTs.

If, $x_1(n) \leftrightarrow X_1(K) \& x_2(n) \leftrightarrow X_2(K)$ $x1(n)\leftrightarrow X1(K)\&x2(n)\leftrightarrow X2(K)$

Then, $x_1(n) \times x_2(n) \leftrightarrow X_1(K) © X_2(K)$ $x1(n)\times x2(n)\leftrightarrow X1(K)©X2(K)$

7. Parseval's Theorem

- For complex valued sequences xn and yn, in general

If, $x(n) \leftrightarrow X(K) \& y(n) \leftrightarrow Y(K)$ $x(n)\leftrightarrow X(K)\&y(n)\leftrightarrow Y(K)$

Then, $\sum_{N-1n=0} x(n) y_*(n) = 1N \sum_{N-1k=0} X(K) Y_*(K)$

: - **EXPLAIN MULTIPLICATION OF TWO DFT & CIRCULAR CONVOLUTION: -**

Let us take two finite duration sequences $x_1$$n$n and $x_2$$n$n, having integer length as N. Their DFTs are $X_1$$K$and $X_2$$K$ respectively, which is shown below −

$$X_1(K) = \sum_{n=0N-1} x_1(n) e_{j2\Pi \quad knN} \quad k=0,1,2...N-1$$ $X1(K)=\sum n=0N-1x1(n)ej2\Pi knNk=0,1,2...N-1$

$$X_2(K) = \sum_{n=0N-1} x_2(n) e_{j2\Pi \quad knN} \quad k=0,1,2...N-1$$ $X2(K)=\sum n=0N-1x2(n)ej2\Pi knNk=0,1,2...N-1$

Now, we will try to find the DFT of another sequence $x_3$$n$n, which is given as $X_3$$K$

$X_3(K) = X_1(K) \times X_2(K)$ $X3(K)=X1(K)\times X2(K)$

By taking the IDFT of the above we get

$x_3(n) = 1N \sum_{n=0N-1} X_3(K) e_{j\ 2\Pi\ knN}$ $x3(n)=1N\sum n=0N-1X3(K)ej2\Pi knN$

After solving the above equation, finally, we get

$x_3(n) = \sum_{m=0N-1} x_1(m) x_2[((n-m))_N] m=0,1,2...N-1$ $x3(n)=\sum m=0N-1x1(m)x2[((n-m))N]m=0,1,2...N-1$

Comparison points	Linear Convolution	Circular Convolution
Shifting	Linear shifting	Circular shifting
Samples in the convolution result	N_1+N_2-1 N1+N2−1	$Max(N_1,N_2)$ Max(N1,N2)
Finding response of a filter	Possible	Possible with zero padding

Methods of Circular Convolution

Generally, there are two methods, which are adopted to perform circular convolution and they are −

- Concentric circle method,
- Matrix multiplication method.

Concentric Circle Method

Let $x_1(n)$x1(n) and $x_2(n)$x2(n) be two given sequences. The steps followed for circular convolutio of $x_1(n)$x1(n) and $x_2(n)$x2(n) are

- Take two concentric circles. Plot N samples of $x_1(n)$x1(n) on the circumference of the oute circle maintainingequaldistancesuccessivepointsmaintainingequaldistancesuccessivepoin s in anti-clockwise direction.
- For plotting $x_2(n)$x2(n), plot N samples of $x_2(n)$x2(n) in clockwise direction on the inner circle starting sample placed at the same point as 0^{th} sample of $x_1(n)$x1(n)
- Multiply corresponding samples on the two circles and add them to get output.
- Rotate the inner circle anti-clockwise with one sample at a time.

Matrix Multiplication Method

Matrix method represents the two-given sequence $x_1(n)$x1(n) and $x_2(n)$x2(n) in matrix form.

- One of the given sequences is repeated via circular shift of one sample at a time to form a N X N matrix.
- The other sequence is represented as column matrix.
- The multiplication of two matrices gives the result of circular convolution.

<u>SHORT QUESTIONS WITH ANSWERS: -</u>

1.WRITE DIFFERENT PROPERTIES OF DFT?

ANS: Different properties of DFT are

A. Periodicity

B. Linearity

C. Time reversal

D. Circular time shift

E. Multiplication of two sequence

F. Circular co-relation

2. Define Linearity

ANS. Linearity

- It states that the DFT of a combination of signals is equal to the sum of DFT of individual signals. Let us take two signals x_1nn and x_2nn, whose DFT s are $X_1\omega$ and $X_2\omega$ respectively. So, if

$x1(n)\rightarrow X1(\omega)x1(n)\rightarrow X1(\omega)andx2(n)\rightarrow X2(\omega)x2(n)\rightarrow X2(\omega)$

Then $ax1(n)+bx2(n)\rightarrow aX1(\omega)+bX2(\omega)ax1(n)+bx2(n)\rightarrow aX1(\omega)+bX2(\omega)$

where **a** and **b** are constants.

3. Define periodicity?

ANS: IF X(K) is an N-point DFT at X(n), then X(n+N) =X(n) for all value of n.

X(K+N) =X(K) For all value of K.

4. Define Parseval's Theorem?

ANS: For complex valued sequences xn and yn, in general

If, $x(n)\leftrightarrow X(K)\&y(n)\leftrightarrow Y(K)x(n)\leftrightarrow X(K)\&y(n)\leftrightarrow Y(K)$

Then, $\sum N-1n=0x(n)y_*(n)=1N\sum N-1k=0X(K)Y_*(K)$

Q-05 What is N-point DFT? (2012,11)

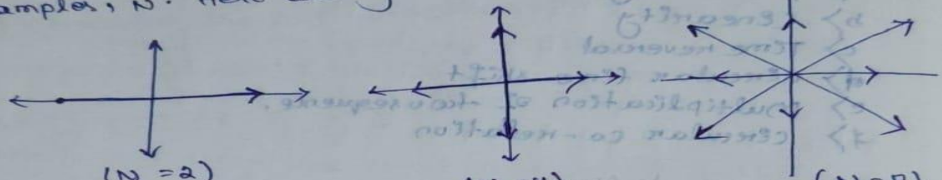
A finite sequence x(n) of length N has fourier transform.

$$X(K) = \sum_{n=0}^{n-1} x(n) . e^{-j2\pi nK/N}$$

for K = 0, 1, 2 - - - (N-1).

Q:-06 What are the twiddle factors of the DFT? (2014(S),13(?)

The twiddle factor, W, describes a "rotating vector", which rotates in increments according to the number of samples, N. Here are graphs where N = 2, 4 & 8 samples.



Q:-07 Define fourier transform pairs?

Fourier transform.

$$x(f) = \int_{-\infty}^{\infty} x(t)\, e^{-i2\pi ft}\, dt$$ for every real no.

Reverse fourier transform.

$$x(t) = \int_{-\infty}^{\infty} x(t)\, e^{i2\pi ft}\, dt$$ for every real no. t.

Q:-08 Name the number of complex multiplications and additions required to compute an N-point DFT? (2014(S)

Basically its number of multiplication and addition of N-point DFT. such as based on their porperetion of N-point DFT. such as periodicity, linearity, circulator convolation & multiplication of two sequence etc.

LONG QUESTIONS: -

1. DISCUSS DISCRETE FOURIER TRANSFORM(DFT)?
2. DISCUSS THE PROPERTY OF THE DFT?
3. DISCUSS METHODS OF CIRCULAR CONVOLUTION?

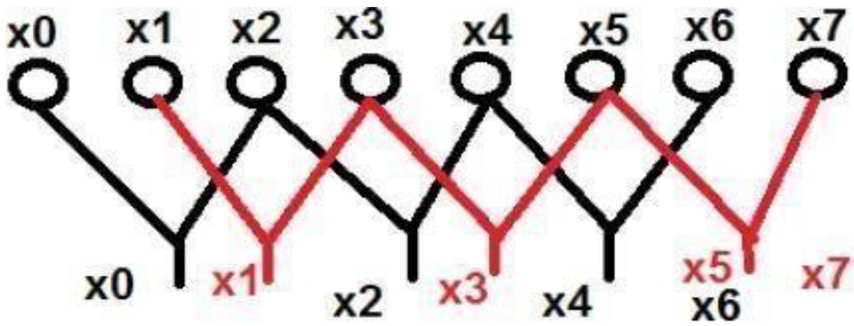
CHAPTER-05 : FAST FOURIER TRANSFORM ALGORITHM & DIGITAL FILTERS.

Compute DFT & FFT algorithm.

- In earlier DFT methods, we have seen that the computational part is too long.
- This can be reduced through FFT or fast Fourier transform.
- So, we can say FFT is nothing but computation of discrete Fourier transform in an algorithmic format, where the computational part will be reduced.
- The main advantage of having FFT is that through it, we can design the FIR filters.
- Mathematically, the FFT can be written as follows;

$$x[K] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$$

Let us take an example to understand it better. We have considered eight points named from $x_0 to x_7$. We will choose the even terms in one group and the odd terms in the other. Diagrammatic view of the above said has been shown below –



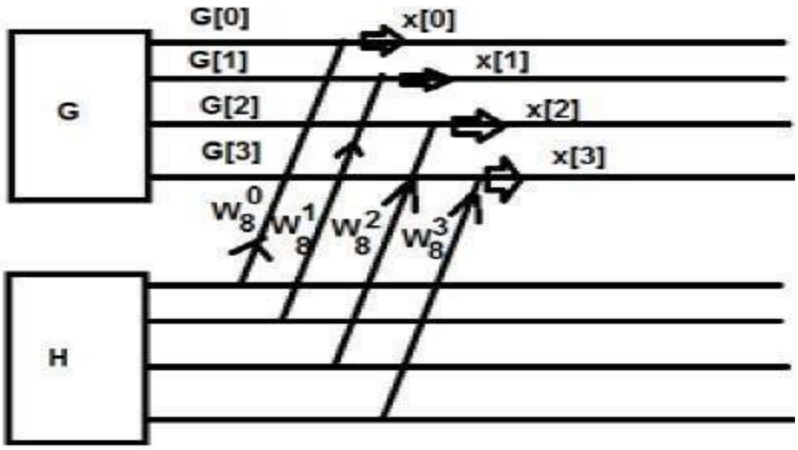
Here, points x_0, x_2, x_4 and x_6 have been grouped into one category and similarly, points x_1, x_3,x_5 and x_7 has been put into another category. Now, we can further make them in a group of two and can proceed with the computation. Now, let us see how these breaking into further two is helping in computation.

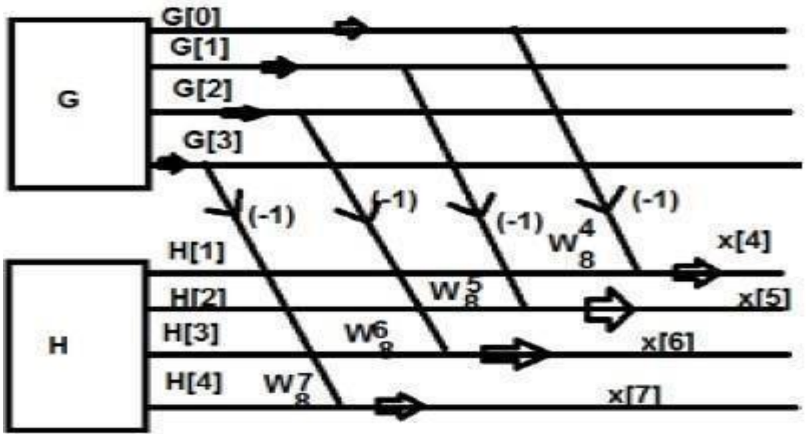
$$x[k] = \sum_{r=0}^{\frac{N}{2}-1} x[2r] W_N^{2rk} + \sum_{r=0}^{\frac{N}{2}-1} x[2r+1] W_N^{(2r+1)k}$$

$$= \sum_{r=0}^{\frac{N}{2}-1} x[2r] W_{N/2}^{rk} + \sum_{r=0}^{\frac{N}{2}-1} x[2r+1] W_{N/2}^{rk} \times W_N^k$$

$$= G[k] + H[k] \times W_N^k$$

Initially, we took an eight-point sequence, but later we broke that one into two parts G[k] and H[k]. G[k] stands for the even part whereas H[k] stands for the odd part. If we want to realize it through a diagram, then it can be shown as below –





Fromthe above figure, we can see that

$W_{48}=-1$

$W_{58}=-W_{18}$

$W_{68}=-W_{28}$

$W_{78}=-W_{38}$

Similarly, the final values can be written as follows −

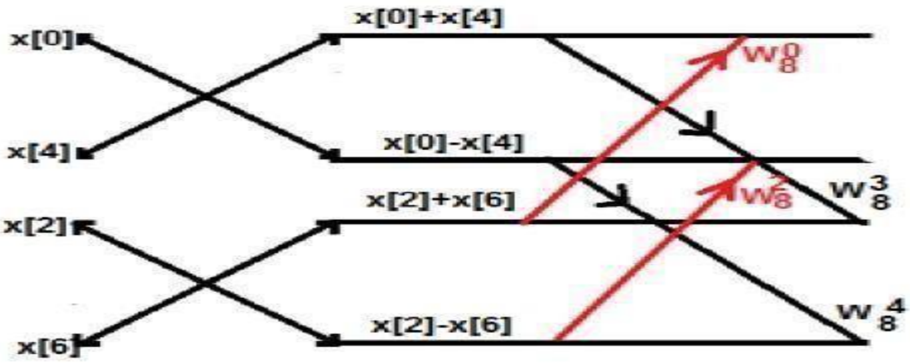
$G[0]-H[0]=x[4]$

$G[1]-W_{18}H[1]=x[5]$

$G[2]-W_{28}H[2]=x[6]$

$G[1]-W_{38}H[3]=x[7]$

The above one is a periodic series. The disadvantage of this system is that K cannot be broken beyond 4 point. Now Let us break down the above into further. We will get the structures something like this.

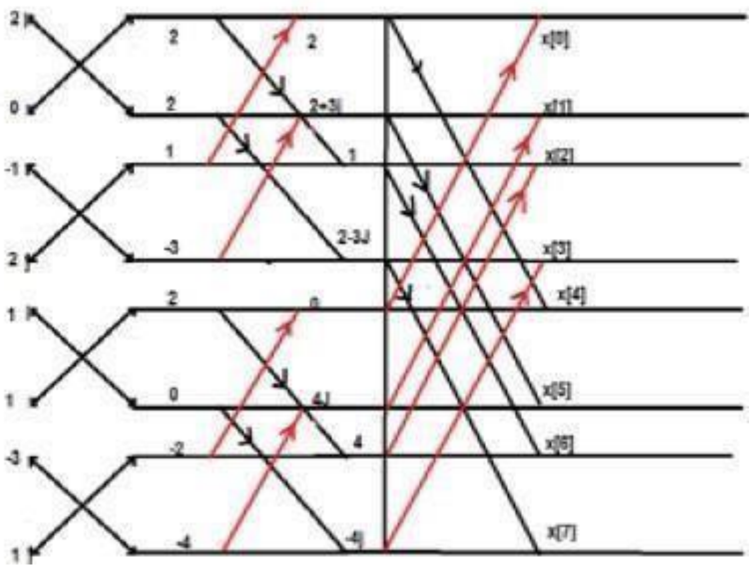


Example

Consider the sequence x[n]={ 2,1,-1,-3,0,1,2,1}. Calculate the FFT.

Solution – The given sequence is x[n]={ 2,1,-1,-3,0,1,2,1}

Arrange the terms as shown below;



❖ **FFT Algorithms can be of two types, such as**:

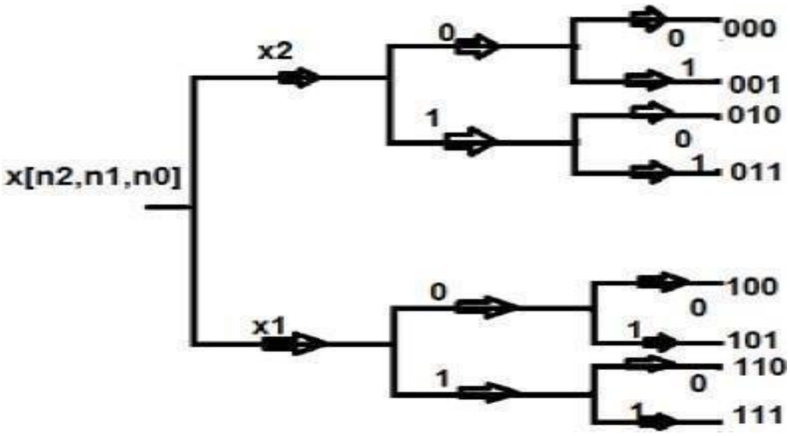
1. Decimation in Time Sequence

2. Decimation in Frequency Sequence

Decimation in Time Sequence

In this structure, we represent all the points in binary format i.e. in 0 and 1. Then, we reverse those structures. The sequence we get after that is known as bit reversal sequence. This is also known as decimation in time sequence. In-place computation of an eight-point DFT is shown in a tabular format as shown below –

POINTS	BINARY FORMAT	REVERSAL	EQUIVALENT POINTS
0	000	000	0
1	001	100	4
2	010	010	2
3	011	110	6
4	100	001	1
5	101	101	5
6	110	011	3
7	111	111	7



Decimation in Frequency Sequence

Apart from time sequence, an N-point sequence can also be represented in frequency. Let us take a four-point sequence to understand it better.

Let the sequence be $x[0],x[1],x[2],x[3],x[4],x[5],x[6],x[7]$

. We will group two points into one group, initially. Mathematically, this sequence can be written as;

$x[k]=\sum_{n=0}^{N-1} x[n]W_N^{n-k}$

Now let us make one group of sequence number 0 to 3 and another group of sequence 4 to 7. Now, mathematically this can be shown as;

$\sum_{n=0}^{\frac{N}{2}-1} x[n]W_N^{nk}+\sum_{n=N/2}^{N-1} x[n]W_N^{nk}$

Let us replace n by r, where r = 0, 1 , 2….$N/2-1$

. Mathematically,

$\sum_{n=0}^{\frac{N}{2}-1} x[r]W_{N/2}^{nr}$

We take the first four points $x[0],x[1],x[2],x[3]$

initially, and tryto represent them mathematically as follows −

$$\sum_{n=0}^{3} x[n] W_8^{nk} + \sum_{n=0}^{3} x[n+4] W_8^{(n+4)k}$$

$$= \left\{ \sum_{n=0}^{3} x[n] + \sum_{n=0}^{3} x[n+4] W_8^{(4)k} \right\} \times W_8^{nk}$$

Now

$$X[0] = \sum_{n=0}^{3} (X[n] + X[n+4])$$

$$X[1] = \sum_{n=0}^{3} (X[n] + X[n+4]) W_8^{nk}$$

$$= [X[0] - X[4] + (X[1] - X[5]) W_8^{1} + (X[2] - X[6]) W_8^{2} + (X[3] - X[7]) W_8^{3}$$

We can further break it into two more parts, which means instead of breaking them as 4-point sequence, we can break them into 2-point sequence.

Direct computation of DFT.

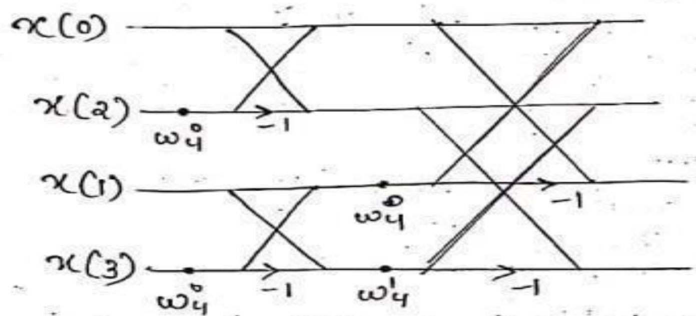
For 4 - point sequence

$x(n) = \{1, 2, 3, 4\}$

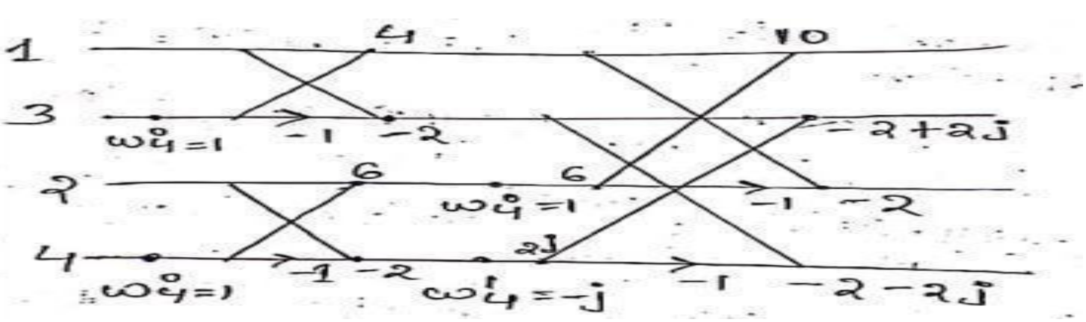
$= \{x(0), x(1), x(2), x(3)\}$

$x_e(n) = \{x(0), x(2)\} = \{1, 3\}$

$x_o(n) = \{x(1), x(3)\} = \{2, 4\}$



Putting the value in the above sequence /digram.

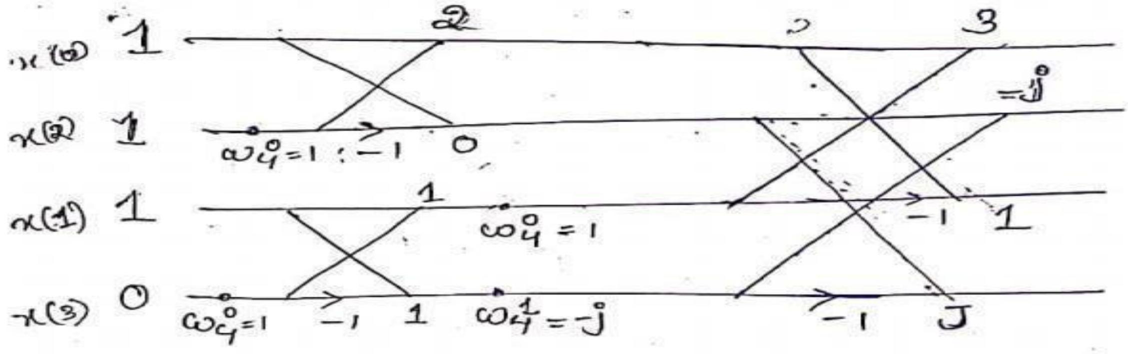


Q. $x(n) = \{1, 1, 1, 0\}$

$\{x(0), x(1), x(2), x(3)\}$

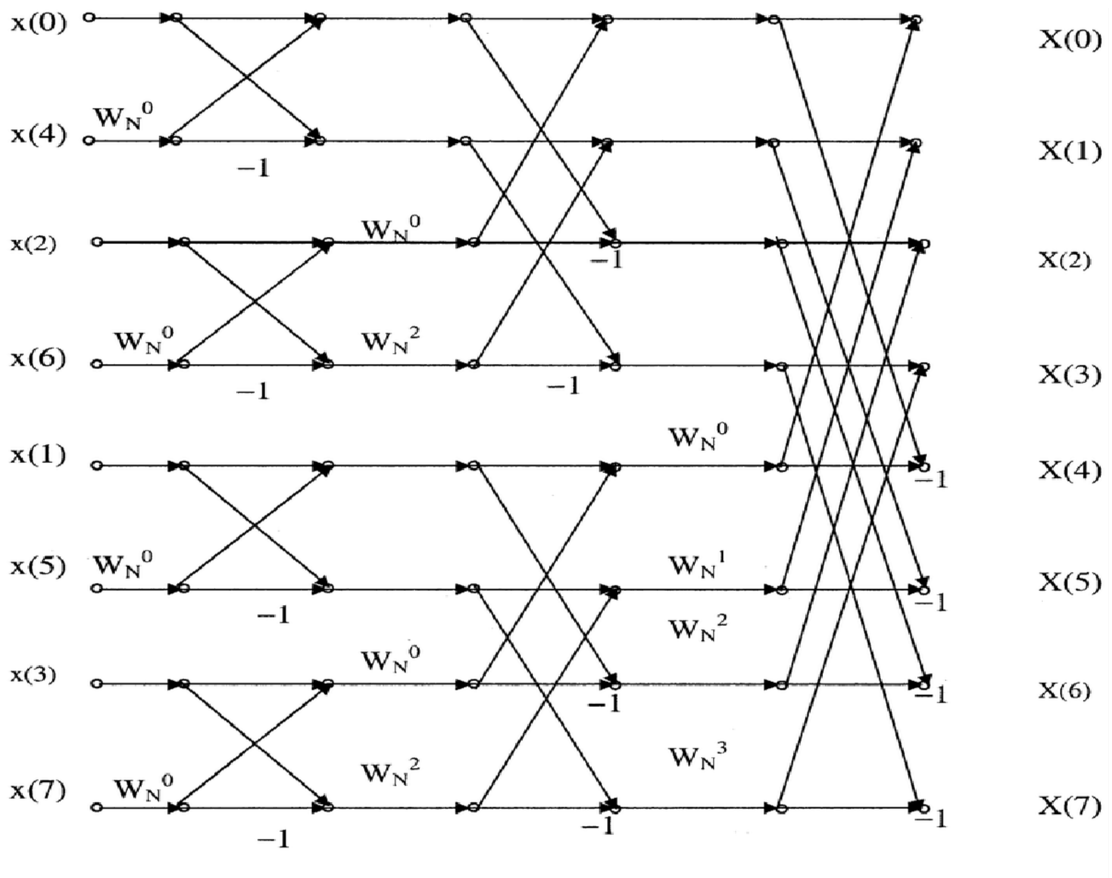
$x_e(n) = \{x(0), x(2)\} = \{1, 1\}$

$x_o(n) = \{x(1), x(3)\} = \{1, 0\}$



8-point sequence

8-Poit DFT-FFT Algorithm



$W_N^0 = 1$, $W_N^1 = (1-j)/\sqrt{2}$, $W_N^2 = -j$, $W_N^3 = -(1+j)/\sqrt{2}$

Divide and Conquer Approach to computation of DFT

To start, we will define the DFT as,

$$X_N[k]=\sum_{n=0}^{N-1}x[n]e^{-j2\pi kn/N} \quad (Eq.1)$$

It is fairly easy to visualize this 1 point DFT, but how does it look when x[n] has 8 points, 256 points, 1024 points, etc. That's where matrices come in. For an N point DFT, we will define our input as x[n] where n = 0, 1, 2, ... N-1. Similarly, the output will be defined as X[k] where k = 0, 1, 2, ... N-1. Referring to our definition of the Discrete Fourier Transform above, to compute an N point DFT, all we need to do is simply repeat Eq. 1, N times. For every value of x[n] in the discrete time domain, there is a corresponding value, X[k], in the frequency domain.

Input **x**[n] =

$$\begin{bmatrix} x[0] & x[1] & \cdots & x[N-1] \end{bmatrix}$$

Output X[k] =

$$\begin{bmatrix} X[0] \\ X[1] \\ \vdots \\ X[N-1] \end{bmatrix}$$

To solve for X[K], means simply repeating Eq. 1, N times, where **x**[n] is a real scalar value for each entry. We represent this in the matrices below.

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ \vdots \\ X[N-1] \end{bmatrix} = x[0]\begin{bmatrix} e^{-j2\pi(0)/N} \\ e^{-j2\pi(0)/N} \\ e^{-j2\pi(0)/N} \\ \vdots \\ e^{-j2\pi(0)/N} \end{bmatrix} + x[1]\begin{bmatrix} e^{-j2\pi(0)/N} \\ e^{-j2\pi(1)/N} \\ e^{-j2\pi(2)/N} \\ \vdots \\ e^{-j2\pi(N-1)/N} \end{bmatrix} + x[2]\begin{bmatrix} e^{-j2\pi(0)/N} \\ e^{-j2\pi(2)/N} \\ e^{-j2\pi(4)/N} \\ \vdots \\ e^{-j2\pi 2(N-1)/N} \end{bmatrix} + \cdots + x[N-1]\begin{bmatrix} e^{-j2\pi(0)/N} \\ e^{-j2\pi(N-1)/N} \\ e^{-j2\pi 2(N-1)/N} \\ \vdots \\ e^{-j2\pi(N-1)^2/N} \end{bmatrix}$$

From this matrix representation of the DFT, you can see that N^2 complex multiplications and N^2 - N complex additions are necessary to fully compute the discrete fourier transform. There are a few simplifications that can be made right away. The first column vector of complex exponentials can be reduced to a vector of 1's. This is possible because, e^(0*anything) will always equal 1. This also applies for the first entry in each vector of complex exponentials because n always begins at 0.

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ \vdots \\ X[N-1] \end{bmatrix} = x[0]\begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} + x[1]\begin{bmatrix} 1 \\ e^{-j2\pi(1)/N} \\ e^{-j2\pi(2)/N} \\ \vdots \\ e^{-j2\pi(N-1)/N} \end{bmatrix} + x[2]\begin{bmatrix} 1 \\ e^{-j2\pi(2)/N} \\ e^{-j2\pi(4)/N} \\ \vdots \\ e^{-j2\pi 2(N-1)/N} \end{bmatrix} + \cdots + x[N-1]\begin{bmatrix} 1 \\ e^{-j2\pi(N-1)/N} \\ e^{-j2\pi 2(N-1)/N} \\ \vdots \\ e^{-j2\pi(N-1)^2/N} \end{bmatrix} \quad (Eq.3)$$

One of the key building blocks of the Fast Fourier Transform, is the Divide and Conquer DFT. As the name implies, we will divide Eq. 1 into two separate summations. The first summation processes the even components of x[n] while the second summation processes theodd components of x[n]. This produces,

$$X_N[k]=\sum_{m=0}^{N/2-1}x[2m]e^{-j2\pi k(m)/(N/2)}+e^{-j2\pi k/N}\sum_{m=0}^{N/2-1}x[2m+1]e^{-j2\pi k(m)/(N/2)}(Eq.4)$$

Our DFT has now been successfully split into two N/2 pt DFT's. For simplification purposes,the first summation will be defined as X0[k] and the second summation as X1[k]. We can now simplify Eq. 4 to the following form.

$$X[k]=X_0[k]+e^{-j2\pi k/n}X_1[k](Eq.5)$$

where

$$X_0[k]=\sum_{m=0}^{N/2-1}x[2m]e^{-j2\pi k(m)/(N/2)}$$

and

$$X_1[k]=\sum_{m=0}^{N/2-1}x[2m+1]e^{-j2\pi k(m)/(N/2)}$$

The complex exponential preceding X1[k] in Eq. 4 is generally called the "twiddle factor" and represented by

$$W_N^k=e^{-j2\pi k/N}$$

By definition of the discrete fourier transform, X0[k] and X1[k] are periodic with period N/2. Therefore we can split Eq. 4 into two separate equations.

$$X[k]=X_0[k]+W_N^kX_1[k]$$

$$X[k+(N/2)]=X_0[k]-W_N^kX_1[k]$$

Once again were are left with a number of 1 pt. DFT's. So how do represent this in matrix form? First, note that we have two separate equations and therefore need two separate equations of matrices. Similar to Eq. 2, we will repeat the DFT for the entire length of the input signal. However, since we split x[n] into even and odd components, we will only repeatthe DFT (N/2) times for X0 and X1. The first equation solves for the first half of X[k].

Eq. 6 and 7,

$$\begin{bmatrix} X_0[0] \\ X_0[1] \\ X_0[2] \\ \vdots \\ X_0[N/2-1] \end{bmatrix} + \begin{bmatrix} {W_N}^0 & 0 & \cdots & 0 \\ 0 & {W_N}^1 & & \vdots \\ \vdots & & {W_N}^2 & \\ & & \ddots & 0 \\ 0 & \cdots & & {W_N}^{(N/2)-1} \end{bmatrix} \begin{bmatrix} X_1[0] \\ X_1[1] \\ X_1[2] \\ \vdots \\ X_1[N/2-1] \end{bmatrix} = \begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ \vdots \\ X[N/2-1] \end{bmatrix}$$

The second equation solves for the second half of X[k].

$$\begin{bmatrix} X_0[0] \\ X_0[1] \\ X_0[2] \\ \vdots \\ X_0[N/2-1] \end{bmatrix} - \begin{bmatrix} {W_N}^0 & 0 & \cdots & 0 \\ 0 & {W_N}^1 & & \vdots \\ \vdots & & {W_N}^2 & \\ & & \ddots & 0 \\ 0 & \cdots & & {W_N}^{(N/2)-1} \end{bmatrix} \begin{bmatrix} X_1[0] \\ X_1[1] \\ X_1[2] \\ \vdots \\ X_1[N/2-1] \end{bmatrix} = \begin{bmatrix} X[N/2] \\ X[(N/2)+1] \\ X[(N/2)+2] \\ \vdots \\ X[N-1] \end{bmatrix}$$

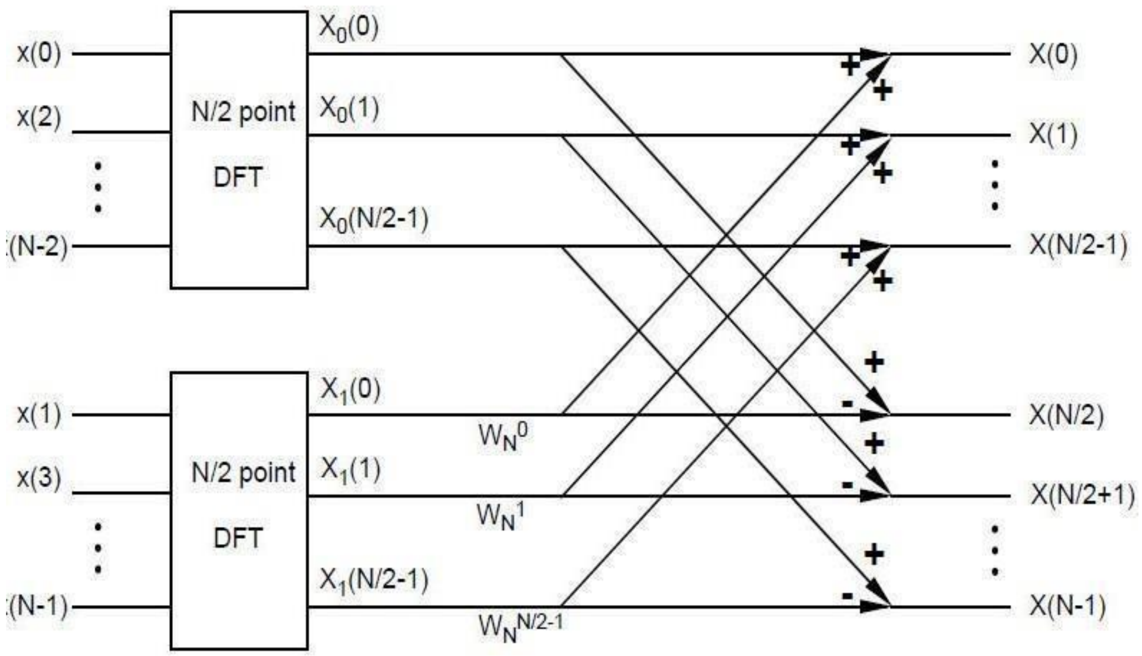
where

$$k = 0, 1, 2, \ldots (N/2) - 1$$

After analyzing the two matrices above, there are a few concepts you should understand.

1) The matrix X0 in each equation is just the condensed form of Eq. 2 and then cut in half. It is simply the DFT repeated (N/2) times where the input is the even indices of x[n].

2) The matrix X1 in each equation, is also the condensed form of Eq. 2 cut in half. However, the input is now the odd indices of x[n].



Radix-2 algorithm.

The radix-2 decimation-in-time and decimation-in-frequency fast Fourier transforms (FFTs) are the simplest FFT algorithms. Like all FFTs, they gain their speed by reusing the results of smaller, intermediate computations to compute multiple DFT frequency outputs.

Decimation in time

The radix-2 decimation-in-time algorithm rearranges the discrete Fourier transform (DFT) equation into two parts: a sum over the even-numbered discrete-time indices $n=[0,2,4,\ldots,N-2]$

and a sum over the odd-numbered indices $n=[1,3,5,\ldots,N-1]$

as in Equation:

$$X(k)=\sum_{n=0}^{N-1} x(n)e^{-\left(i\frac{2\pi nk}{N}\right)} = \sum_{n=0}^{\frac{N}{2}-1} x(2n)e^{-\left(i\frac{2\pi\times(2n)k}{N}\right)}+\sum_{n=0}^{\frac{N}{2}-1} x(2n+1)e^{-\left(i\frac{2\pi(2n+1)k}{N}\right)} = \sum_{n=0}^{\frac{N}{2}-1} x(2n)e^{-\left(i\frac{2\pi nk}{\frac{N}{2}}\right)}+e^{-\left(i\frac{2\pi k}{N}\right)}\sum_{n=0}^{\frac{N}{2}-1} x(2n+1)e^{-\left(i\frac{2\pi nk}{\frac{N}{2}}\right)} = \mathrm{DFT}_{\frac{N}{2}}[[x(0),x(2),\ldots,x(N-2)]]+W_N^k\,\mathrm{DFT}_{\frac{N}{2}}[[x(1),x(3),\ldots,x(N-1)]]$$

The mathematical simplifications in Equation reveal that all DFT frequency outputs $X(k)$

can be computed as the sum of the outputs of two length-$\frac{N}{2}$ DFTs, of the even-indexed and odd-indexed discrete-time samples, respectively, where the odd-indexed short DFT is multiplied by a so-called twiddle factor term $W_N^k=e^{-\left(i\frac{2\pi k}{N}\right)}$. This is called a decimation in time because the time samples are rearranged in alternating groups, and a radix-2 algorithm because there are two groups. Figure graphically illustrates this form of the DFT computation, where for convenience the frequency outputs of the length-$\frac{N}{2}$ DFT of the even-indexed time samples are denoted $G(k)$ and those of the odd-indexed samples as $H(k)$. Because of the periodicity with $\frac{N}{2}$ frequency samples of these length-$\frac{N}{2}$ DFTs, $G(k)$ and $H(k)$ can be used to compute **two** of the length-N DFT frequencies, namely $X(k)$ and $X\left(k+\frac{N}{2}\right)$, but with a different twiddle factor. This reuse of these short-length DFT outputs gives the FFT its computational savings.

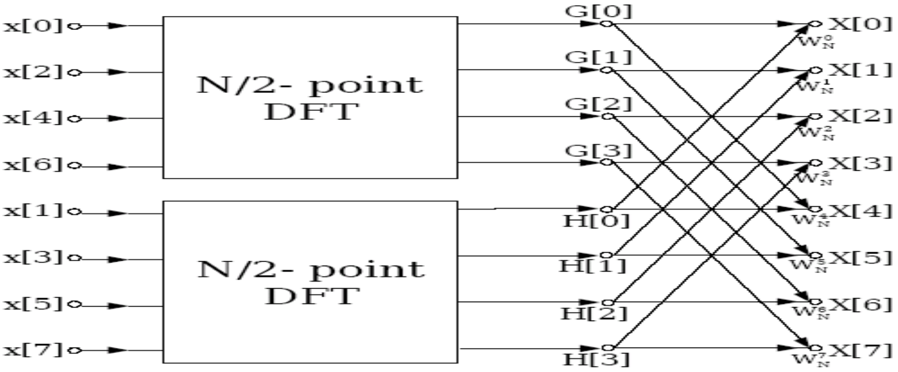


Figure 1. Decimation in time of a length-N DFT into two length-$\frac{N}{2}$ DFTs followed by a combining stage.

Whereas direct computation of all N DFT frequencies according to the DFT equation would require $N2$ complex multiples and $N2-N$

complex additions (for complex-valued data), by reusing the results of the two short-length DFTs as illustrated in Figure, the computational cost is now

New Operation Counts

- $2(N2)2+N=N22+N$

- complex multiplies
- $2N2(N2-1)+N=N22$

- complex additions

This simple reorganization and reuse has reduced the total computation by almost a factor oftwo over direct DFT computation!

Additional Simplification

A basic butterfly operation is shown in Figure, which requires only $N2$

twiddle-factor multiplies per stage. It is worthwhile to note that, after merging the twiddle factors toa single term on the lower branch, the remaining butterfly is actually a length-2 DFT! The theory of multi-dimensional index maps shows that this must be the case, and that FFTs of any factorable length may consist of successive stages of shorter-length FFTs with twiddle-factor multiplications in between.

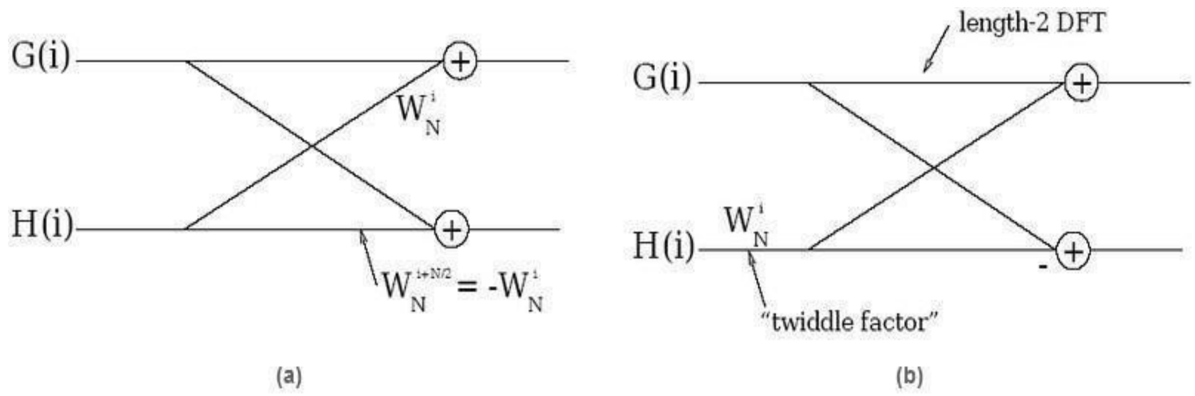


Figure 2. Radix-2 DIT butterfly simplification: both operations produce the same outputs

Radix-2 decimation-in-time FFT

The same radix-2 decimation in time can be applied recursively to the two length $N2$

DFTs to save computation. When successively applied until the shorter and shorter DFTs reach length-2, the result is the radix-2 DIT FFT algorithm.

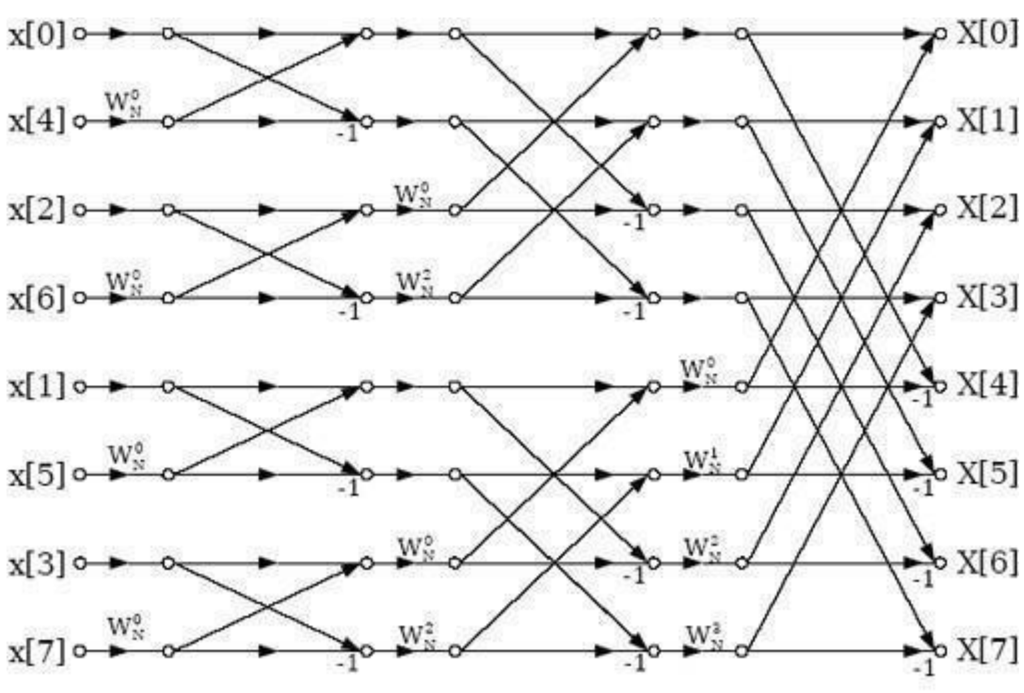


Figure 3. Radix-2 Decimation-in-Time FFT algorithm for a length-8 signal

The full radix-2 decimation-in-time decomposition illustrated in Figure using the simplified butterflies involves $M=\log_2 N$ stages, each with $N2$ butterflies per stage. Each butterfly requires 1 complex multiply and 2

adds per butterfly. The total cost of the algorithm is thus

Computational cost of radix-2 DIT FFT

- $N2\log_2 N$

- complex multiplies
- $N\log_2 N$

- complex adds

This is a remarkable savings over direct computation of the DFT. For example, a length-1024 DFT would require 1048576

complex multiplications and 1047552 complex additions with direct computation, but only 5120 complex multiplications and 10240

complex additions using the radix-2 FFT, a savings by a factor of 100 or more. The relative savings increase with longer FFT lengths, and are less for shorter lengths.

Modest additional reductions in computation can be achieved by noting that certain twiddle factors, namely Using special butterflies for $W0N$

, W_N^{2N}, W_N^{4N}, W_N^{8N}, W_{3N}^{8N}

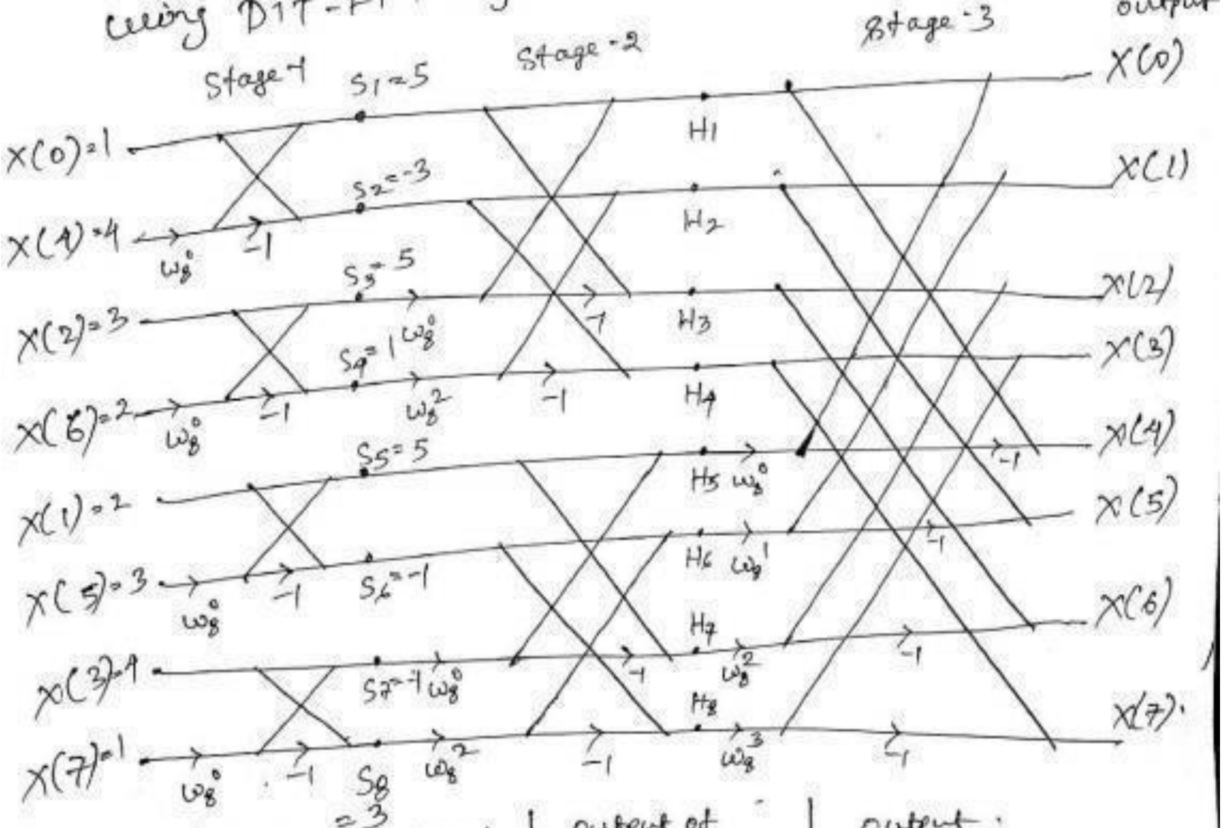
, require no multiplications, or fewer real multiplies than other ones. By implementing special butterflies for these twiddle factors as discussed in FFT algorithm and programming tricks, the computational cost of the radix-2 decimation-in-time FFT can be reduced to

- $2N\log_2 N - 7N + 12$

- real multiplies
- $3N\log_2 N - 3N + 4$ real additions

Example:

Q. Find the DFT of sequence: $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$ using DIT-FFT algorithm.



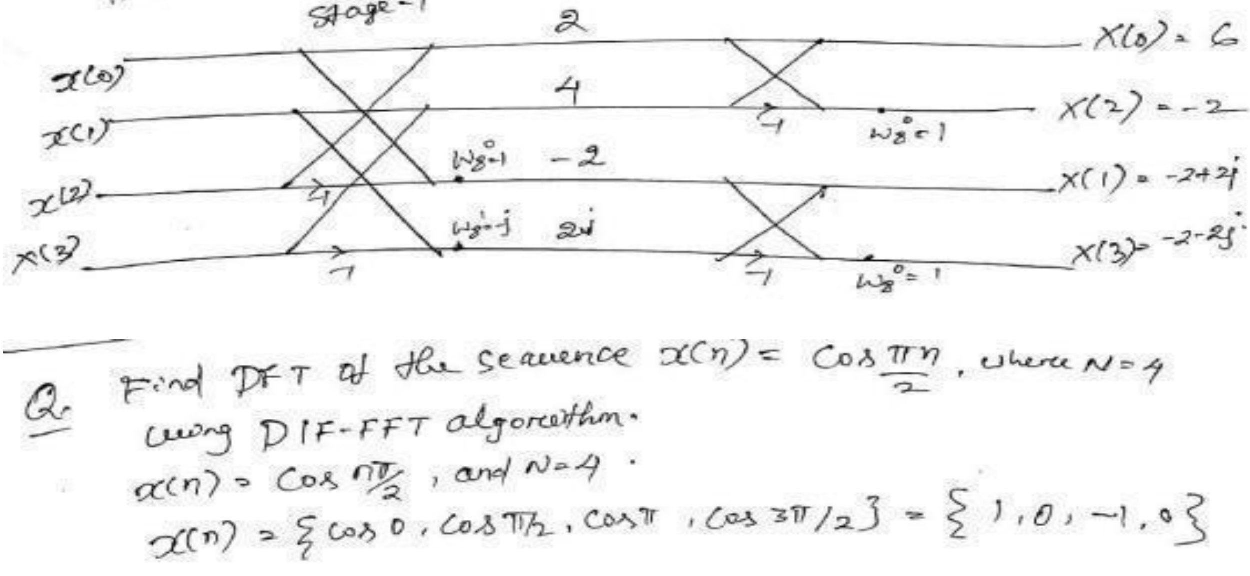
Input	Output of stage-1	Output of stage-2	Output
$x(0) = 1$	$S_1 = 1+4 = 5$	$H_1 = S_1 + W_8^0 S_3 = 10$	$X(0) = H_1 + W_8^0 H_5 = 10+10=2$
$x(4) = 4$	$S_2 = 1-4 = -3$	$H_2 = S_2 + W_8^2 S_4 = -3-j$	$X(1) = H_2 + W_8^1 \cdot H_6 = -5.828 - j2.414$

$x(2)=3$	$S_3 = 3+2=5$	$H_3 = S_1 - W_8^0 S_3 = 0$	$X(2) = H_3 + W_8^2 H_7 = 0$
$x(6)=2$	$S_4 = 3-2 = 1$	$H_4 = S_2 - W_8^2 S_4 = -3+j$	$X(3) = H_4 + W_8^3 H_8 = -0.172 - j0.414$
$x(1)=2$	$S_5 = 2+3=5$	$H_5 = S_5 + W_8^0 S_7 = 10$	$X(4) = H_1 - W_8^0 H_5 = 10-10 = 0$
$x(5)=3$	$S_6 = 2-3 = -1$	$H_6 = S_6 + W_8^2 S_8 = -1-3j$	$X(5) = H_2 - W_8^1 H_6 = -0.172 + j0.414$
$x(3)=4$	$S_7 = 4+1=5$	$H_7 = S_5 - W_8^0 S_7 = 5-5 = 0$	$X(6) = H_3 - W_8^2 H_7 = 0-(-j)0 = 0$
$x(7)=1$	$S_8 = 4-1 = 3$	$H_8 = S_6 - W_8^2 S_8 = -1+3j$	$X(7) = H_4 - W_8^3 H_8 = -5.828 + j2.414$

$$X(K) = \{20, -5.828 - j2.414, 0, -0.172 - j0.414, 0, -0.172 + j0.414, 0, -5.828 + j2.414\}$$

Q. Find the DFT of the Sequence $x(n) = \{0, 1, 2, 3\}$ using DIF-FFT algorithm.

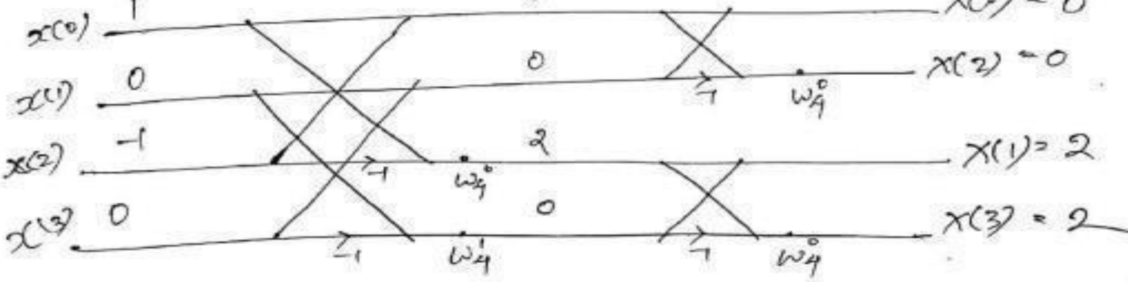
Here $N=4$.



Q. Find DFT of the Sequence $x(n) = \cos\frac{\pi n}{2}$, where $N=4$ using DIF-FFT algorithm.

$x(n) = \cos\frac{n\pi}{2}$, and $N=4$.

$x(n) = \{\cos 0, \cos\pi/2, \cos\pi, \cos 3\pi/2\} = \{1, 0, -1, 0\}$

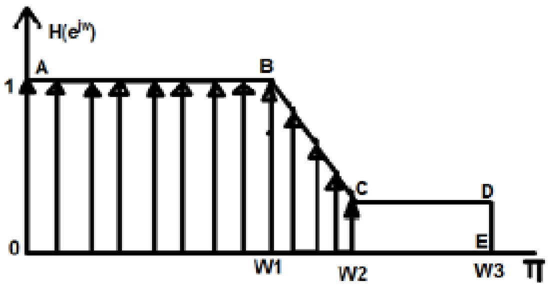


Applications of FFT algorithms

- The **fast Fourier transform** (**FFT**) is a computationally efficient method of generating a **Fourier transform**. The main **advantage** of an **FFT** is speed, which it gets by decreasing the number of calculations needed to analyze a waveform.
- It covers **FFTs**, frequency domain filtering, and **applications** to video and audio signal processing.
- As fields like communications, speech and image processing, and related areas are rapidly developing, the **FFT** as one of the essential parts in digital signal processing has been widely used.
- In the fields like cross-correlation, matched filtering, system identification, power spectrum estimation, and coherence function measurement.
- Used to create additional measurement functions such as frequency response, impulse response, coherence, amplitude spectrum, and phase spectrum.

Introduction to digital filters.(FIR Filters)& General considerations

FIR filters can be useful in making computer-aided design of the filters. Let us take an example and see how it works. Given below is a figure of desired filter.



While doing computer designing, we break the whole continuous graph figures into discrete values. Within certain limits, we break it into either 64, 256 or 512 *and so on.*

we have taken limits between -π to +π. We have divided it into 256 parts. The points can be represented as H0

, H1,….up to H256

. Here, we apply IDFT algorithm and this will give us linear phase characteristics.

Sometimes, we may be interested in some particular order of filter. Let us say we want to realize the above given design through 9th order filter. So, we take filter values as h0, h1, h2….h9. Mathematically, it can be shown as below

$$H(e^{j\omega}) = h_0 + h_1 e^{-j\omega} + h_2 e^{-2j\omega} + \ldots\ldots\ldots\ldots + h_9 e^{-9j\omega}$$

Where there are large number of dislocations, we take maximum points.

For example, in the above figure, there is a sudden drop of slopping between the points B and C. So, we tryto take more discrete values at this point, but there is a constant slope between point C and D. There we take less number of discrete values.

For designing the above filter, we go through minimization process as follows;

$$H(e^{j\omega_1}) = h_0 + h_1 e^{-j\omega_1} + h_2 e^{-2j\omega_1} + ... + h_9 e^{-9j\omega_1}$$

$$H(e^{j\omega_2}) = h_0 + h_1 e^{-j\omega_2} + h_2 e^{-2j\omega_2} + ... + h_9 e^{-9j\omega_2}$$

Similarly,

$$(e^{j\omega_{1000}}) = h_0 + h_1 e H^{-j\omega_{1000}} h_2 e^{-2j\omega_{1000}} + ... + h_9 + e^{-9j\omega_{1000}}$$

Representing the above equation in matrix form, we have –

$$\begin{bmatrix} H(e^{j\omega_1}) \\ . \\ . \\ H(e^{j\omega_{1000}}) \end{bmatrix} = \begin{bmatrix} e^{-j\omega_1} & \dots & e^{-j9\omega_1} \\ . & & . \\ . & & . \\ e^{-j\omega_{1000}} & \dots & e^{j9\omega_{1000}} \end{bmatrix} \begin{bmatrix} h_0 \\ . \\ . \\ h_9 \end{bmatrix}$$

Let us take the 1000×1 matrix as B, 1000×9 matrix as A and 9×1 matrix as $h^\wedge$

So, for solving the above matrix, we will write

$$h^\wedge = [A^T A]^{-1} A^T B$$

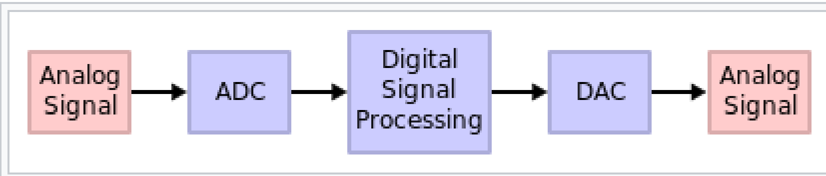
$$= [A^{*T} A]^{-1} A^{*T} B$$

where A^* represents the complex conjugate of the matrix A.

Introduction to DSP architecture, familiarisation of different types of processor

- Digital signal processing algorithms typically require a large number of mathematical operations to be performed quickly and repeatedly on a series of data samples.
- Signals (perhaps from audio or video sensors) are constantly converted from analog to digital, manipulated digitally, and then converted back to analog form.
- Many DSP applications have constraints on latency; that is, for the system to work, the DSP operation must be completed within some fixed time, and deferred (or batch) processing is not viable.
- Most general-purpose microprocessors and operating systems can execute DSP algorithms successfully, but are not suitable for use in portable devices such as mobile phones and PDAs because of power efficiency constraints.

- A specialized DSP, however, will tend to provide a lower-cost solution, with better performance, lower latency, and no requirements for specialised cooling or large batteries.



A typical digital processing system

Architecture

Software architecture

- By the standards of general-purpose processors, DSP instruction sets are often highly irregular; while traditional instruction sets are made up of more general instructions that allow them to perform a wider variety of operations, instruction sets optimized for digital signal processing contain instructions for common mathematical operations that occur frequently in DSP calculations.
- Both traditional and DSP-optimized instruction sets are able to compute any arbitrary operation but an operation that might require multiple ARM or x86 instructions to compute might require only one instruction in a DSP optimized instruction set.

Instruction sets

- multiply–accumulates (MACs, including fused multiply–add, FMA) operations

  - used extensively in all kinds of matrix operations
    - convolution for filtering
    - dot product
    - polynomial evaluation
  - Fundamental DSP algorithms depend heavily on multiply–accumulate performance
    - FIR filters
    - Fast Fourier transform (FFT)

- related instructions:

  - SIMD
  - VLIW

Data instructions

- Saturation arithmetic, in which operations that produce overflows will accumulate at the maximum (or minimum) values that the register can hold rather than wrapping around (maximum+1 doesn't overflow to minimum as in many general-purpose CPUs, instead it stays at maximum). Sometimes various sticky bits operation modes are available.
- Fixed-point arithmetic is often used to speed up arithmetic processing.
- Single-cycle operations to increase the benefits of pipelining.

Program flow

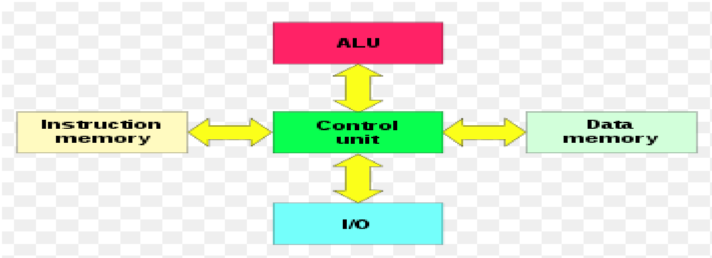
- Floating-point unit integrated directly into the datapath
- Pipelined architecture
- Highly parallel multiplier–accumulators (MAC units)
- Hardware-controlled looping, to reduce or eliminate the overhead required for looping operations

Hardware architecture

- In engineering, hardware architecture refers to the identification of a system's physical components and their interrelationships.
- This description, often called a hardware design model, allows hardware designers to understand how their components fit into a system architecture and provides to software component designers important information needed for software development and integration.
- Clear definition of a hardware architecture allows the various traditional engineering disciplines (e.g., electrical and mechanical engineering) to work more effectively together to develop and manufacture new machines, devices and components.
- DSPs are usually optimized for streaming data and use special memory architectures that are able to fetch multiple data or instructions at the same time, such as the Harvard architecture or Modified von Neumann architecture, which use separate program and data memories

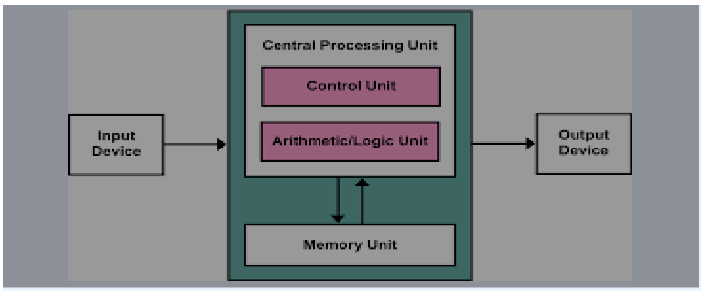
Harvard architecture

Harvard architecture is **used** primarily for small embedded computers and signal processing. Commonly **used** within CPUs to handle the cache. Not only data but also instructions of programs are stored within the same memory. This makes it easier to re-program the memory.



Von Neumann architecture

The von Neumann architecture—also known as the von Neumann model or Princeton architecture—is a computer architecture based on a 1945 description by John von Neumann and others in the FirstDraft of a Report on the EDVAC.



Short Questions With Answers

1. Define FFT.

Ans- A *fast Fourier transform* (*FFT*) is an algorithm that computes the discrete Fourier transform (DFT) of a sequence, or its inverse (IDFT).

2. What are the Applications of FFT ?

Ans-

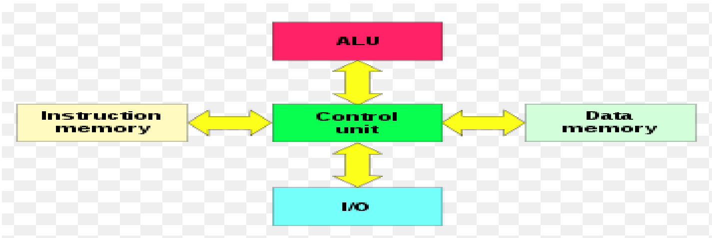
- Digital signal processing algorithms typically require a large number of mathematical operations to be performed quickly and repeatedly on a series of data samples.
- Signals (perhaps from audio or video sensors) are constantly converted from analog to digital, manipulated digitally, and then converted back to analog form.

3. Define FIR.

Ans- The Frequency-Domain **FIR** Filter block implements frequency-domain, **fast Fourier** transform (**FFT**)-based filtering to filter a streaming input signal. In the time domain, the filtering operation involves a convolution between the input and the impulse response of the finite impulse response (**FIR**) filter.

4. What is Harvard Architecture ?

Ans- **Harvard architecture** is **used** primarily for small embedded computers and signal processing. Commonly **used** within CPUs to handle the cache. Not only data but also instructions of programsare stored within the same memory. This makes it easier to re-program the memory.



5. What is Von Neumann Architecture ?

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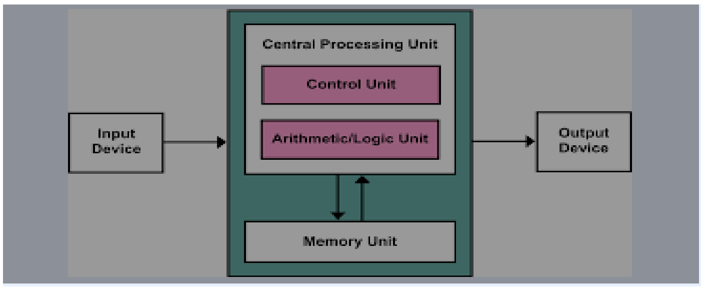

Long Questions

1.

Find the DFT of the sequence :
$x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$ using DIF-FFT algorithm

2. Explain the Radix-2 Algorithm.

3. Write short note on DSP Architecture.

4. What are the different applications of FFT Algorithm.

5. Write short note on FIR Filter.

6. What are the different FFT Algorithms? Explain each in detail.

THE END