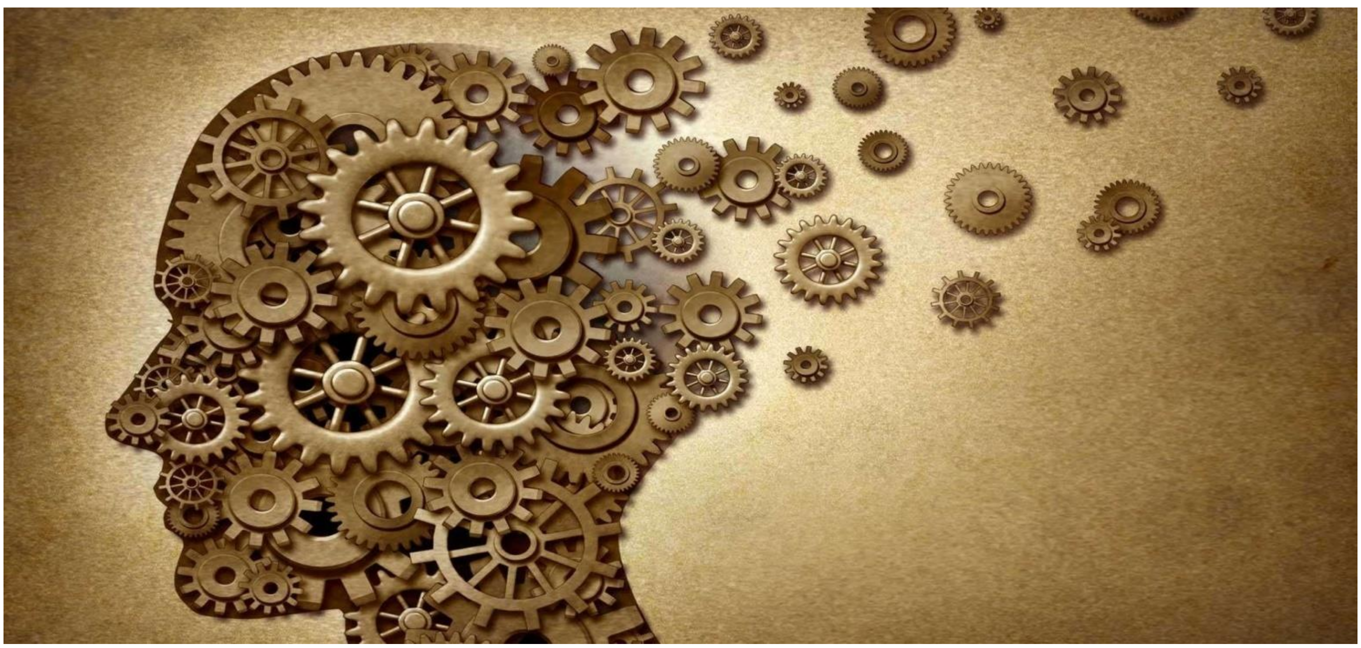
Department of Basic Science And Engineering

Subject : Engineering Mechanics

COMMON FOR ALL 1ST AND 2ND SEMESTER BRANCH



NILASAILA INSTITUTE OF SCIENCE AND TECHNOLOGY

SERGARH,BALASORE

Chapater - 1.0

FUNDAMENTALS OF ENGINEERING MECHANICS

Mechanics:-

It is defined as that branch of science, which describes and predicts the conditions of rest or motion of bodies under the action of forces. Engineering mechanics applies the principle of mechanics to design, taking into account the effects of forces.

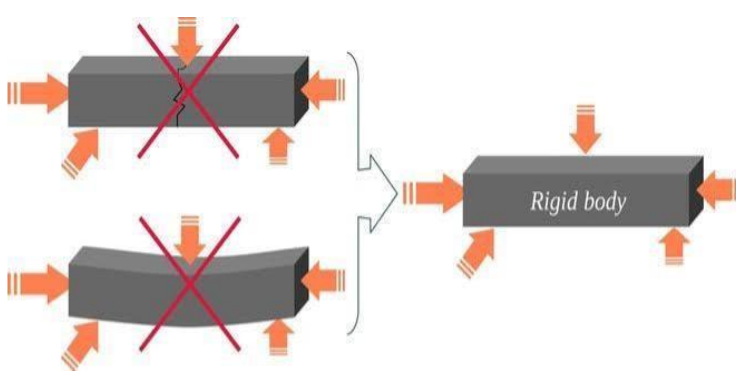
Statics :-

It is the branch which deals with the forces and their effects on an object or a body at rest.

- For example, if we have an object or a body at rest and we deal with the forces and their effects that are acting on the body than we are dealing with static branch of engineering mechanics.

Rigid body:-

A rigid body is defined as a definite quantity of matter, the parts of which are fixed in position relative to each other. Physical bodies are never absolutely but deform slightly under the action of loads. If the deformation is negligible as compared to its size, the body is termed as rigid.



Dynamics

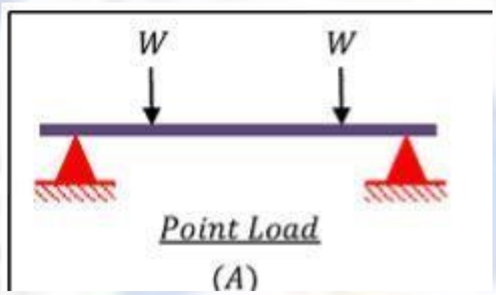
It deals with study of motion and forces on a body which is in motion. Subcomponent ofdynamics is kinematics.

Force :-

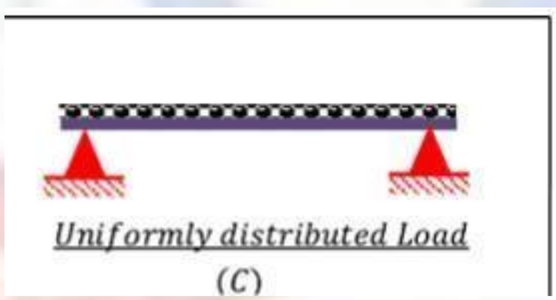
Force may be defined as any action that tends to change the state of rest or motion of abody to which it is applied. The three quantities required to completely define force are called itsspecification or characteristics. So the characteristics of a force are:

1. Magnitude 2. Point of application 3. Direction of application

Concentrated force/point load

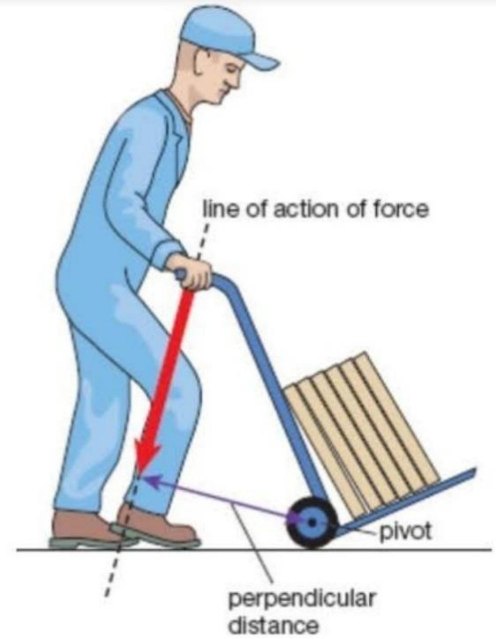


Distributed force



Line of action of force:-

The direction of a force is the direction, along a straight line through its point of application in which the force tends to move a body when it is applied. This line is called line of action of force.

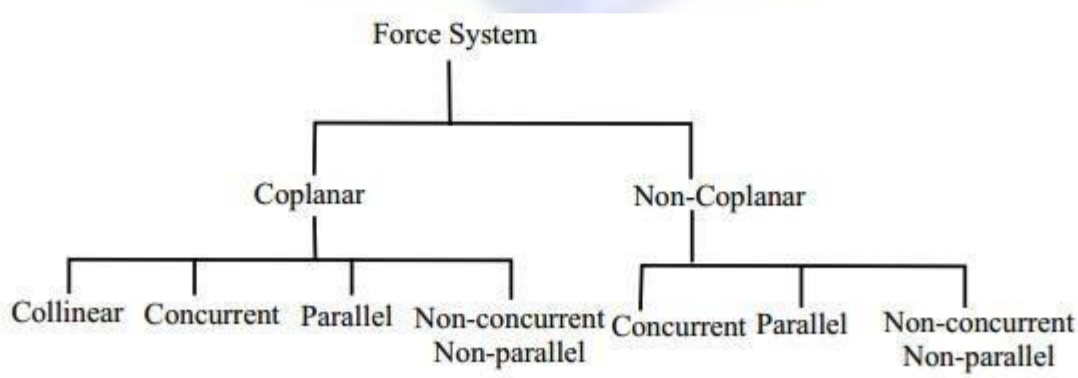


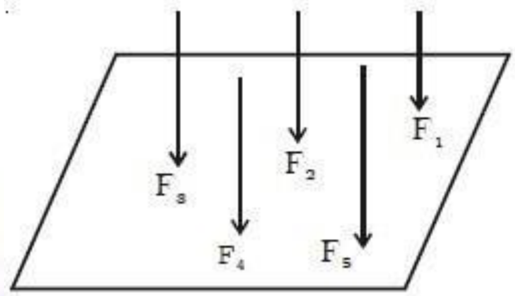
Perpendicular means 'at right angles' — so in the equation use the shortest distance between the pivot and the line along which the force acts

Classification of force system

Coplanar Force System:

The forces, the lines of action of which lie on the same plane, are known as coplanar forces.

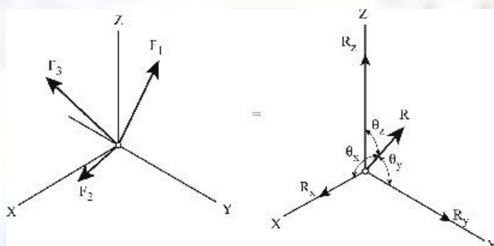




Coplanar forces

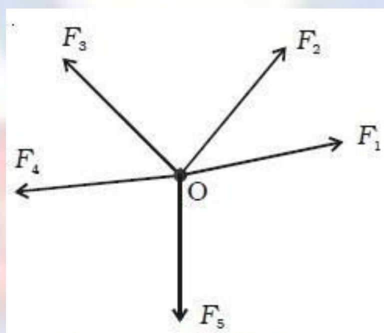
Non-Coplanar Force System:

The forces, the lines of action of which do not lie on the same plane, are known as non-coplanar force system.



Concurrent Forces:

All forces of this kind, which act at one point, are known as concurrent forces.



Concurrent forces

Coplanar-Concurrent System:

All such forces the line of action of which lies in one plane and they meet at one point are called as coplanar-concurrent force system.

Coplanar-Parallel Force System:

If the lines of action of forces are parallel to each other and they lie in same plane then the system is known as coplanar-parallel forces system.

Coplanar-Collinear Force System:

All forces of this kind whose line of action lies in one plane lie along single line then it is called as coplanar collinear force system.

Non-concurrent Coplanar Forces System:

All forces of this kind whose line of action lies in one plane but they do not meet at one point, are called as non concurrent coplanar force system.

Characteristics of force:

1. Force can introduce motion in a body.
2. Force can change direction of a moving body.
3. Force can change size of an object.
4. Force is a vector quantity.
5. Force can stop a moving body.
6. Force has a unit of Newton(N) in M.K.S system and dyne in C.G.S system.
7. They are of two types: contact and non-contact forces.

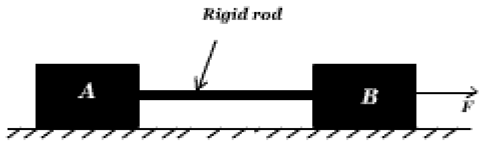
Principle of transmissibility of forces

Principle of transmissibility states that the state of rest or of motion of a rigid body will be unaltered if a force acting on the body will be replaced by another force of the same magnitude and direction but acting anywhere on the body along the same line of action of the applied forces.

In other words, we can also write here the principle of transmissibility as the external effects of the force will be independent of the point of application of the force along the line of action of the force.

Let us understand here with the help of following example.

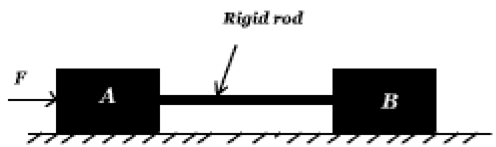
Let us consider here two rigid blocks A and B as displayed here in following figure. Let us assume that these two rigid blocks are joined with each other with the help of a rigid rod as shown in the figure.



Let us think that a force F is applied over the object B and hence system i.e. both rigid blocks A and B and the rigid rod will move towards right direction. In other words, we are applying here the pulling force and system is moving towards right direction under the action of this pulling force F.

Now, we have replaced the force with same magnitude and direction i.e. F and the force is acting now on the object A but in the same line of action.

What will be the external effect of this force? Let us see here in the following figure.



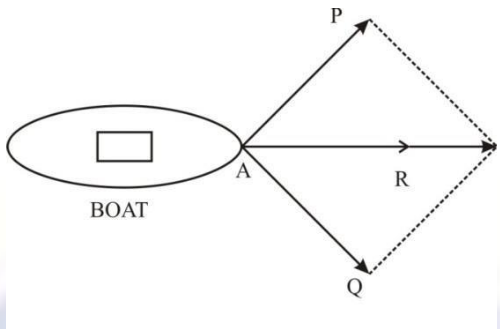
System i.e. rigid blocks A and B with rigid rod will move in right direction. In other words, we are applying here the pushing force and system is moving towards right direction under the action of this pushing force F.

PRINCIPLE OF SUPERPOSITION OF FORCES

This principle states that the combined effect of force system acting on a particle or a rigid body is the sum of effects of individual forces.

Consider two forces P and Q acting at A on a boat as shown in Fig.3.1. Let R be the resultant of these two forces P and Q. According to Newton's second law of motion, the boat will move in the direction of resultant force R with acceleration proportional to R. The same motion can be obtained when P and Q are applied simultaneously.

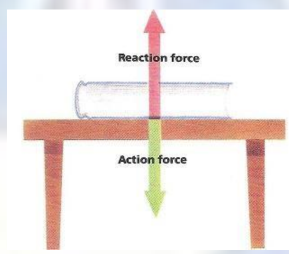
Therefore, we have noted here one very important observation i.e. pulling force or pushing force will produce the same effect.



Principle of Superposition

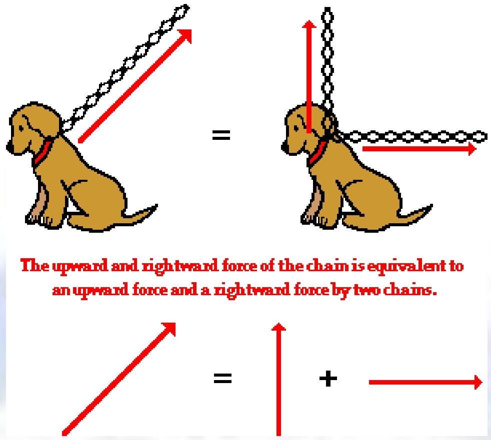
Action Forces

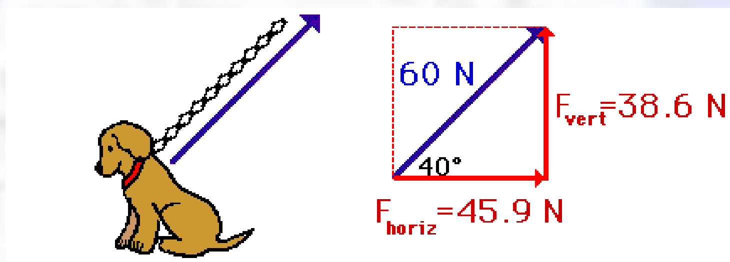
Forces always act in pairs and always act in opposite directions. When you push on an object, the object pushes back with an equal force. Think of a pile of books on a table. The weight of the books exerts a downward force on the table. This is the action force. The table exerts an equal upward force on the books. This is the reaction force. Note that the two forces act on different objects. The action force acts on the table, and the reaction force acts on the books.



RESOLUTION OF A FORCE INTO COMPONENTS

A given force F can be resolved into (or replaced by) two forces, which together produces the same effects that of force F. These forces are called the components of the force F. This process of replacing a force into its components is known as resolution of a force into components. A force can be resolved into two components, which are either perpendicular to each other or inclined to each other. If the two components are perpendicular to one another, then they are known as rectangular components and when the components are inclined to each other, they are called as inclined components.





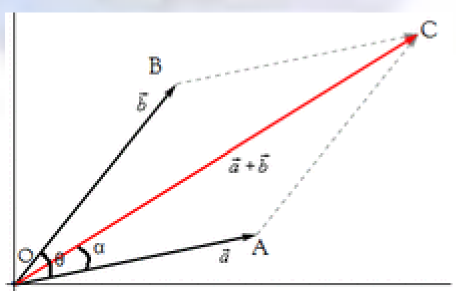
$$\sin 40° = \frac{F_{vert}}{60\ N} \qquad \cos 40° = \frac{F_{horiz}}{60\ N}$$

$$F_{vert} = 60\ N \times \sin 40° \qquad F_{horiz} = 60\ N \times \cos 40°$$

$$F_{vert} = 38.6\ N \qquad F_{horiz} = 45.9\ N$$

Composition of Forces

The process of finding out the resultant force of a number of given forces is called the composition/compounding of forces.



Parallelogram Law

"If two forces acting simultaneously on a particle is represented in magnitude and direction by two adjacent sides of a parallelogram, their resultant may be represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection."

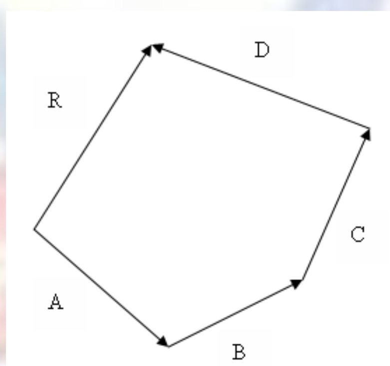
$$Resultant, R = \sqrt{P^2 + Q^2 + 2PQcos\theta}$$

$$\alpha = \tan^{-1}\left(\frac{Q\,sin\theta}{P + Q\,cos\theta}\right)$$

Where α = angle of resultant

Polygon law of forces

If a number of forces acting simultaneously on a particle be represented in magnitude and direction by the sides of a polygon taken in order, their resultant may be represented in magnitude and direction by the closing side of the polygon taken in opposite order.

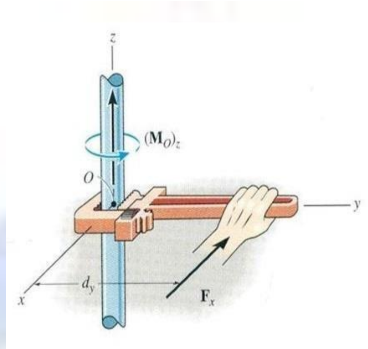


Where A, B, C and D are the forces

and R = Resultant force

Moment of Force:

The moment of a force : is the ability of the force to produce turning or twisting about an axis or point or line.



Mathematically:

Moment of a force = Applied Force x Perpendicular Distance

M = F x D

F = Force

D = Perpendicular Distance between the point of action and Moment center

Its S.I unit is N-m (Newton metre)

Resultant moment exerted by a force system.

Suppose that N forces (1) (2) ( ) , ... N FF F act at positions (1) (2) ( ) , ,... N rr r . The resultant moment of the force system is simply the sum of the moments exerted by the forces. You can calculate the resultant moment by first calculating the moment of each force, and then adding all the moments together (usingvector sums).

Just one word of caution is in order here – when you compute the resultant moment, you must take moments about the same point for every force

The Physical Significance of a Moment

A force acting on a solid object has two effects: (i) it tends to accelerate the object (making the object's center of mass move); and (ii) it tends to cause the object to rotate.

1. The moment of a force about some point quantifies its tendency to rotate an object about that point.

2. The magnitude of the moment specifies the magnitude of the rotational force.

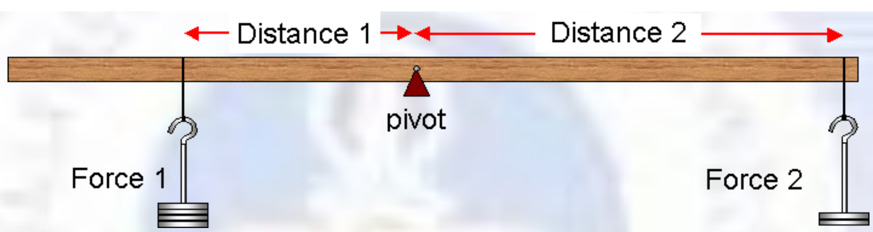
3. The direction of a moment specifies the axis of rotation associated with the rotational force, following the right hand screw convention.

Sign convention of Moment of a force

1. If the Moment produces a Clockwise rotation,Then we use Positive sign with that moment.

2. If the Moment produces Anti-clokwise Rotation then we use Negative Sign with that moment.

Law of moments



When an object is balanced (in equilibrium) the sum of the clockwise moments is equal to the sum of the anticlockwise moments.

Moment of Force F1 = Moment of Force F2

Force 1 x its distance from pivot = Force 2 x distance from the pivot

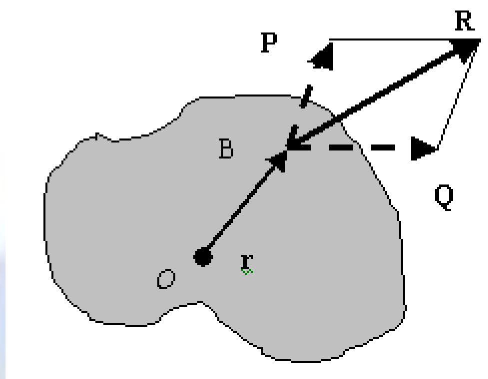
$$F1\ d1 = F2\ d2$$

Varignon's Theorem:

Moment of a force about any point is equal to the sum of the moments of the components of that force about the same point.

To prove this theorem, consider the force R acting in the plane of the body shown in Figure The forces P and Q represent any two nonrectangular components of R.

The moment of R about point O is $M_0 = r \times R$



Because R = P+Q, we may write

r × R = r × (P +Q)

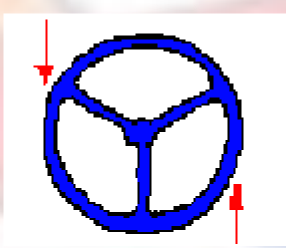
Using the distributive law for cross products, we have

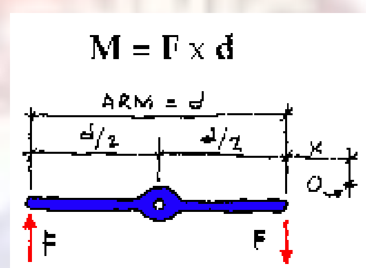
$M_0 = r \times R = r \times P + r \times Q$

which says that the moment of R about O equals the sum of the moments about O of its components P and Q . This proves the theorem.

Couples

A special case of moments is a couple. A couple consists of two parallel forces that are equal in magnitude, opposite in sense and do not share a line of action. It does not produce any translation, only rotation. The resultant force of a couple is zero. BUT, the resultant of a couple is not zero; it is a pure moment.





For example, the forces that two hands apply to turn a steering wheel are often (or should be) a couple. Each hand grips the wheel at points on opposite sides of the shaft. When they apply a force that is equal in magnitude yet opposite in direction the wheel rotates. If both hands applied a force in the same direction, the sum of the moments created by each force would equal zero and the wheel would not rotate. Instead of rotating around the shaft, the shaft would be loaded with a force tending to cause a translation with a magnitude of twice F. If the forces applied by the two hands were unequal, there would again be an unbalanced force creating a translation of the "system." A pure couple always consists of two forces equal in magnitude.The moment of a couple is the product of the magnitude of one of the forces and the perpendicular distance between their lines of action.

$$M = F \times d$$

Characteristics of the Couples

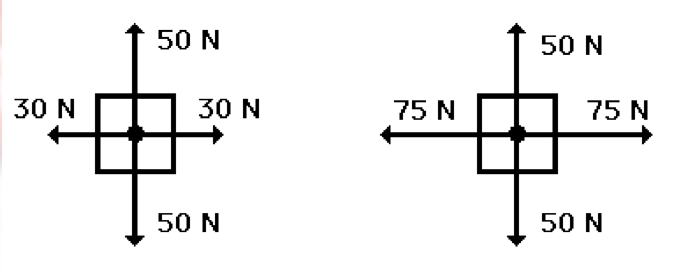
- As the two forces that make up the couple are equal and opposing, the couple does not create translational motion.
- When it operates on a body, the net resulting force on the body is zero.
- Since, the algebraic sum of the moments of the two forces around any point in their plane is not zero, it causes pure rotational motion in the body.
- A couple's moment about any point in its plane is constant in size and direction.

Chapter 2

Equilibrium

Equilibrium:

When all the forces that act upon an object are balanced, then the object is said to be in a state of equilibrium. The forces are considered to be balanced if the rightward forces are balanced by the leftward forces and the upward forces are balanced by the downward forces. This however does not necessarily mean that all the forces are equal to each other.



These two objects are at equilibrium since the forces are balanced. However, the forces are not equal.

Conditions for Equilibrium

The two conditions that are important for equilibrium:-

- The sum or resultant of all external forces acting on the body must be equal to zero.
- The sum or resultant of all external torques from external forces acting on the object must be zero.

The two conditions given here must be simultaneously satisfied in equilibrium. In essence, for an object to be in equilibrium, it should not experience any acceleration (linear or angular). So both the net force and the net torque on the object must be zero.

First Condition of Equilibrium

For an object to be in equilibrium, it must be experiencing no acceleration. This means that both the net force and the net torque on the object must be zero. Here we will discuss the first condition, that of zero net force.

In the form of an equation, this first condition is:

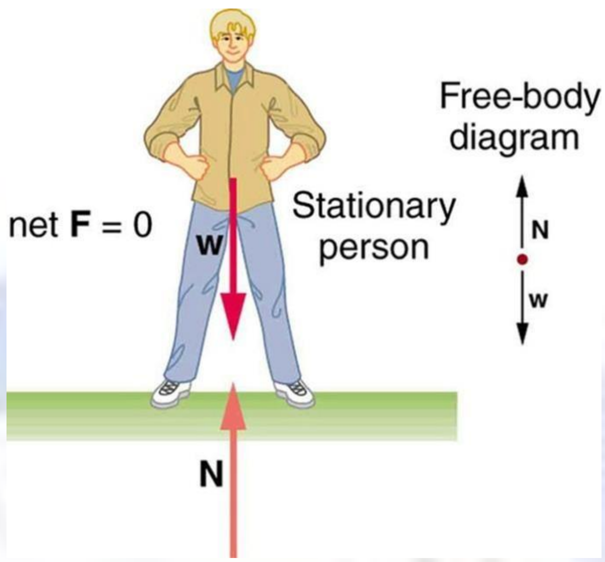
$$F_{net}=0$$

In order to achieve this conditon, the forces acting along each axis of motion must sum to zero. For example, the net external forces along the typical x– and y-axes are zero. This is written as

$$\text{net } F_x = 0 \text{ and net } F_y = 0$$

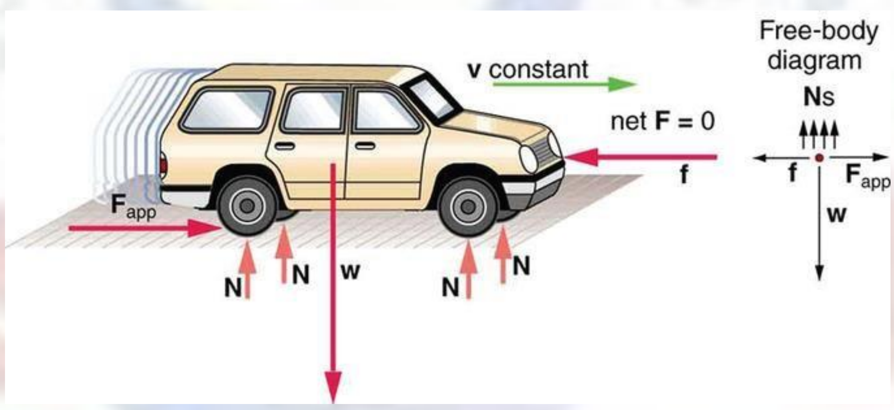
The condition Fnet = 0 must be true for both static equilibrium, where the object’s velocity is zero, and dynamic equilibrium, where the object is moving at a constant velocity.

Below, the motionless person is in static equilibrium. The forces acting on him add up to zero. Both forces are vertical in this case.



This motionless person is in static equilibrium.

Below, the car is in dynamic equilibrium because it is moving at constant velocity. There are horizontal and vertical forces, but the net external force in any direction is zero. The applied force between the tires and the road is balanced by air friction, and the weight of the car is supported by the normal forces, here shown to be equal for all four tires.

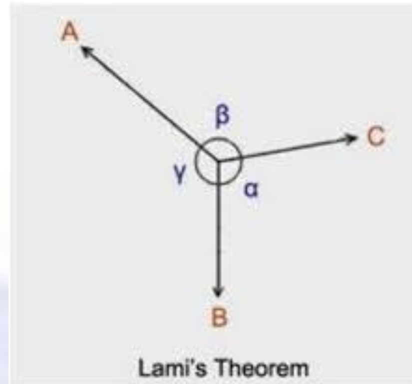


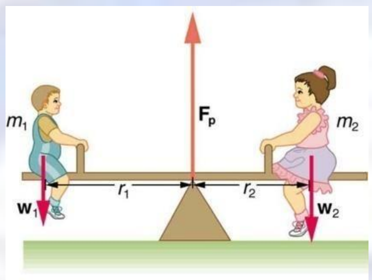
This car is in dynamic equilibrium because it is moving at constant velocity. The forces in all directions are balanced.

Second Condition

The second condition of static equilibrium says that the net torque acting on the object must be zero.

A child's seesaw, shown in, is an example of static equilibrium. An object in static equilibrium is one that has no acceleration in any direction. While there might be motion, such motion is constant.Two children on a seesaw: The system is in static equilibrium, showing no acceleration in any direction.If a given object is in static equilibrium, both the net force and the net torque on the object must be zero.





The system is in static equilibrium, showing no acceleration in any direction.

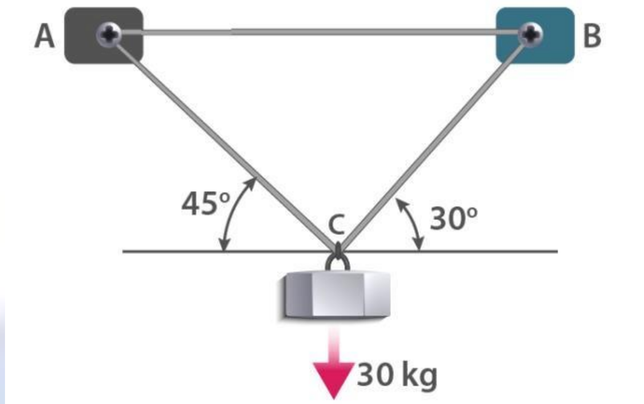
Lami's Theorem

Lami's Theorem states, "When three forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the other two forces". Referring to the above diagram, consider three forces A, B, C acting on a particle or rigid body making angles α, β and γ with each other.

In the mathematical or equation form, it is expressed as,

$$\frac{A}{sin\alpha} = \frac{B}{sin\beta} = \frac{C}{sin\gamma}$$

Que 1.An iron block of mass 30 kg is hanging from the two supports A and B as shown in the diagram. Determine the tensions in both the ropes.



Solution:

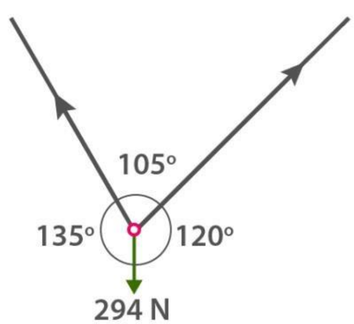
Given, m = 30 kg, W = mg = 30 x 9.8 = 294 N

Let's draw an FBD i.e. Free Body Diagram for the given condition. C be the point of suspension from where the iron block is hanging.

Using Lami's Formula we get,

$\frac{T_{AC}}{sin(120^o)} = \frac{T_{BC}}{sin(135^o)} = \frac{294}{sin(105^o)}$

We get required angles to apply Lami's Theorem as,



Therefore,

$\frac{T_{AC}}{sin(120^o)} =$

i.e. $T_{AC} =$ $263.566N$

Similarly,

$\frac{T_{BC}}{sin(120^o)} = \frac{294}{sin(105^o)}$

i.e. $T_{BC} = \frac{294\ X\ sin(135^o)}{sin(105^o)} = \frac{294\ X\ 0.707}{0.966} = 215.2N$

So, the required tensions along the shown directions are 263.566 N and 215.2 N respectively.

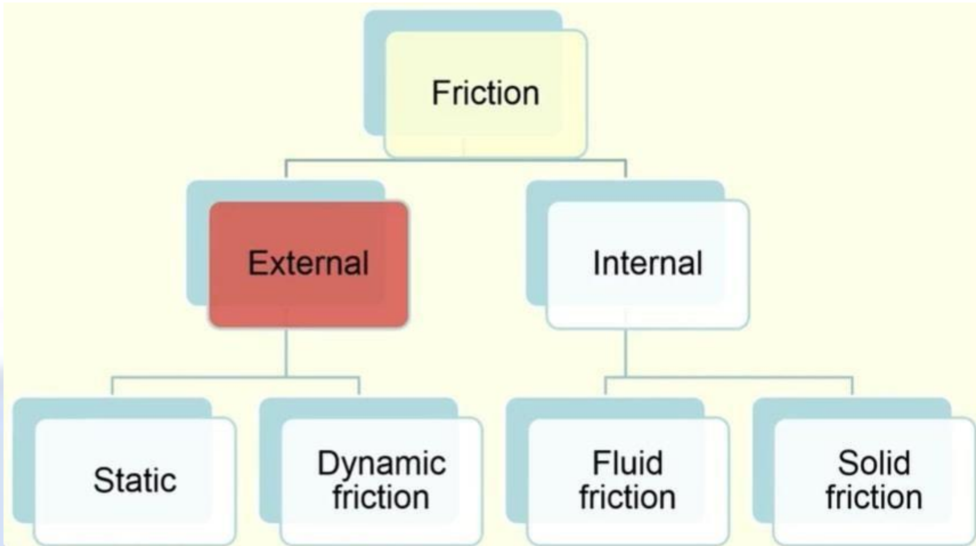
Chapter 3

FRICTION

Friction:

Friction is the contact resistance exerted by one body when the second body moves or tends to move past the first body. Friction is a retarding force that always acts opposite to the motion or to the tendency to move.

Types of Friction:



Static friction:

In static friction, the frictional force resists force that is applied to an object, and the object remains at rest until the force of static friction is overcome

Dynamic friction:

When the force acting on the body is greater than the limiting friction, then the body comes into motion. The friction now acting between the surfaces of contact is dynamic friction.

Advantages of Friction:

- Friction is responsible for many types of motion
- It helps us walk on the ground
- Brakes in a car make use of friction to stop the car
- Asteroids are burnt in the atmosphere before reaching Earth due to friction.
- It helps in the generation of heat when we rub our hands.

Disadvantages of Friction:

- Friction produces unnecessary heat leading to the wastage of energy.
- The force of friction acts in the opposite direction of motion, so friction slows down the motion of moving objects.
- Forest fires are caused due to the friction between tree branches.
- A lot of money goes into preventing friction and the usual wear and tear caused by it by using techniques like greasing and oiling.

coefficient of friction,

coefficient of friction is the ratio of the frictional force resisting the motion of two surfaces in contact to the normal force pressing the two surfaces together. It is usually symbolized by the Greek letter mu (μ). Mathematically, $\mu = F/N$, where F is the frictional force and N is the normal force. Because both F and N are measured in units of force (such as newtons or pounds), the coefficient of friction is dimensionless.

$$\mu = F/N,$$

Angle of friction:

It is the angle which the resultant of the limiting friction and the normal reaction makes with the normal reaction. Angle of repose: It is the minimum angle that an inclined plane makes with the horizontal when a body placed on it just begin to slide down.

R is the resultant of normal reaction *N* and force of friction *f*.

$$R = \sqrt{N^2 + f^2}$$

ϕ is the angle of friction

$$\tan\phi = \frac{f}{N}$$

or

$$\phi = \tan^{-1}\frac{f}{N}$$

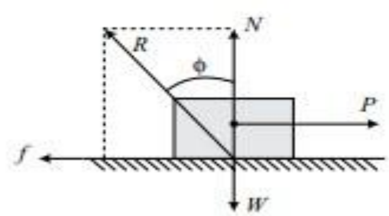
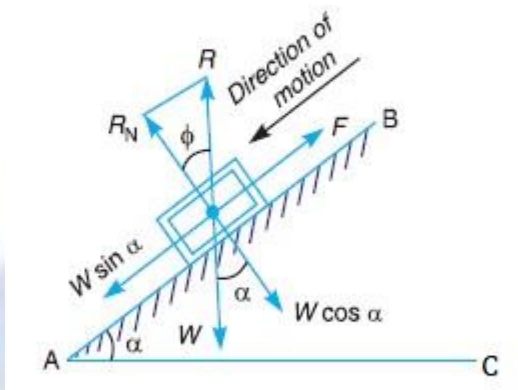


Fig. 2.4 Angle of friction

The angle of repose

The angle of repose or angle of sliding is defined as the minimum angle of inclination of a plane with the horizontal such that a body placed on the plane just begins to slide down. Its value depends on the material and nature of the surfaces in contact



From the above figure we can say that, AB is an inclined plane such that a body placed on it just begins to slide down. ∠BAC = α = Angle of repose.

The various force involved are:

1. Weight (mg), of the body acting vertically downwards.
2. Normal reaction (RN), acting perpendicular to AB.
3. Force of friction (F), acting upon the plane AB.

Now, mg can resolve into two rectangular components: mg cosα opposite to R and mg sinα opposite to F. In equilibrium:

$$F = mg \sin\alpha \quad \dots (1)$$

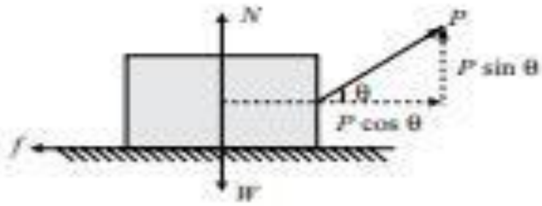
$$N = mg \cos\alpha \quad \dots (2)$$

Dividing equations (1) by (2), we get: $f/N = mg \sin\alpha / mg \cos\alpha = \mu$,

i.e. $\mu = \tan\alpha$

Hence, the coefficient of limiting friction between any two surfaces in contact is equal to the tangent of the angle of repose between them.

Equilibrium of Bodies on Horizontal Plane:



$$\Sigma F_x = 0$$

$\therefore$ $$f = P\cos\theta = \mu N \qquad ...(1)$$

$$\Sigma F_y = 0$$

$$N = W + P\cos\theta \qquad ...(2)$$

From equations (1) and (2)

$$P\cos\theta = \mu N$$

$$= \mu(W - P\sin\theta)$$

$$P\cos\theta + \mu P\sin\theta = \mu W$$

Now, $$\mu = \tan\phi = \frac{\sin\phi}{\cos\phi}$$

$$P\left(\cos\theta + \frac{\sin\phi}{\cos\phi}\sin\theta\right) = \frac{\sin\phi}{\cos\phi}.W$$

$$P(\cos\theta\cos\phi + \sin\theta\sin\phi) = W\sin\phi$$

$$P\cos(\theta - \phi) = W\sin\phi$$

$$P = \frac{w\sin\phi}{\cos(\theta - \phi)}$$

For P to be minimum, $\cos(\theta - \phi)$ should be maximum

$\therefore$ $$\cos(\theta - \phi) = 1$$

$$\theta = \phi$$

Minimum value of P = $\mathbf{W\sin\phi}$

or $= \mathbf{W\sin\theta}$ **Ans.**

Chapter 4

Centroid and Moment of Inertia

A body may be considered to be made up of a number of minute particles having weights having weights w1, w2, w3,…,wnwhich are attracted towards the centre of body. As the particles are considered negligible in comparison to body, all the forces are considered to be parallel to each other. The resultant of all these forces acting at a point known as Centre of Gravity (C.G)

.

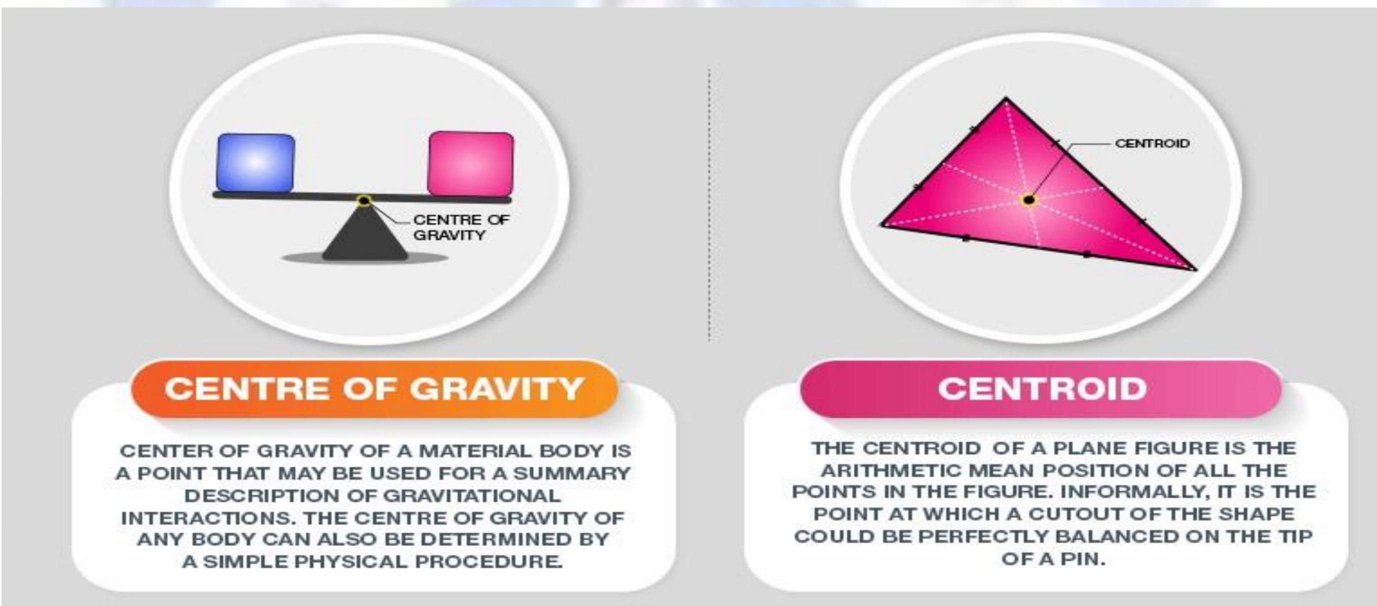
CENTROID DEFINITION:

Centroid is the centre point or geometric centre of a plane figure like triangle, circle, quadrilateral, etc. The method of finding centroid is same as finding C.G of a body.

CENTRE OF GRAVITY (C.G):

Centre of Gravity of a body is a fixed point with respect to the body, through which resultant of weights of all particles of the body passes, at any plane

.



METHODS FOR CENTRE OF GRAVITY

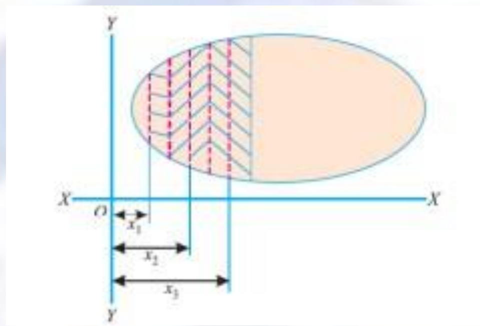
The centre of gravity (or centroid) may be found out by any one of the following two methods:

1. By geometrical considerations
2. By moments

. By graphical method.

CENTRE OF GRAVITY BY MOMENTS

Consider a body of mass M whose centre of gravity is required to be found out. Divide the body into small masses, whose centers of gravity are known as shown in Fig. 6.9. Let m1, m2, m3...; etc. be the masses of the particles and (x1, y1), (x2, y2), (x3, y3), ... be the co-ordinates of the centers of gravity from a fixed point O



Let a‾ nd ‾be the co-ordinates of the centre of gravity of the body. From the principle of moments, we know that

$$M\,\overline{x} = m_1 x_1 + m_2 x_2 + m_3 x_3 \ .....$$

$$\overline{x} = \frac{\Sigma mx}{M}$$

$$\overline{y} = \frac{\Sigma my}{M},$$

$$M = m_1 + m_2 + m_3 + \ .....$$

AXIS OF REFERENCE

The centre of gravity of a body is always calculated with reference to some assumed axis known as axis of reference. The axis of reference, of plane figures, is generally taken as the lowest line of the figure for calculating y and the left line of the figure for calculating ‾.

CENTRE OF GRAVITY OF PLANE FIGURES

The centre of area of plane geometrical figures is known as centroid, and coincides with the centre of gravity of the figure. It is a common practice to use centre of gravity for centroid and vice versa.

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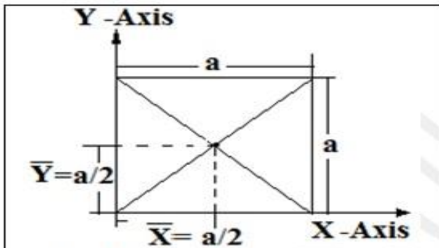
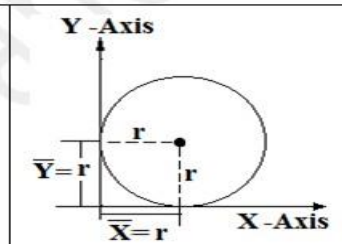
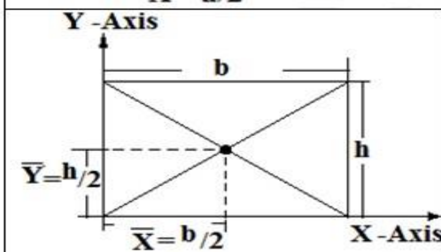
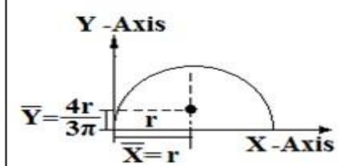
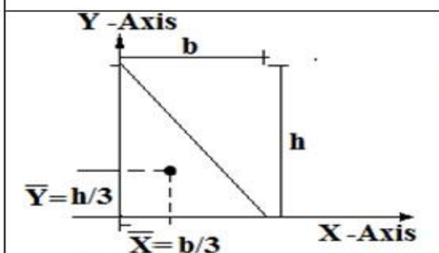
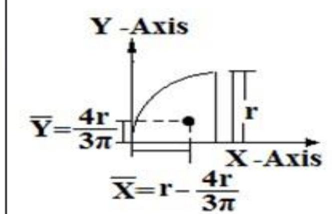
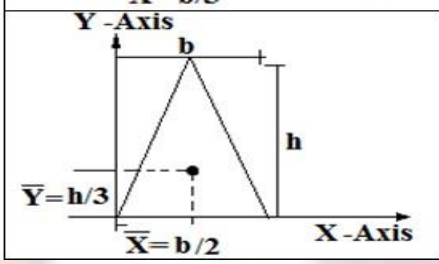
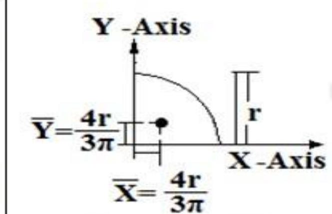
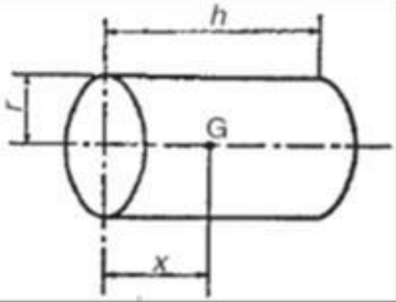
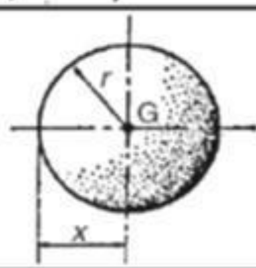
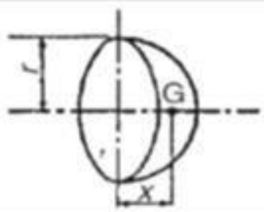
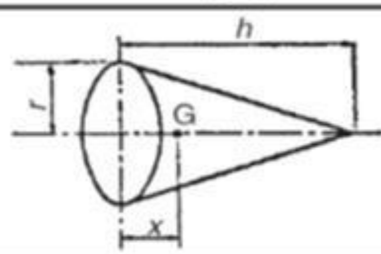
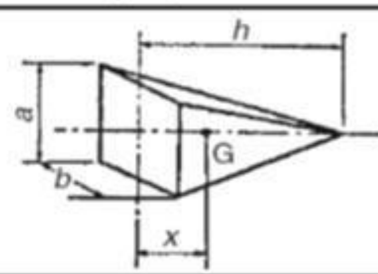
Centroid of basic geometrical figures

	Square: $\overline{X} = \frac{a}{2}$ $\overline{Y} = \frac{a}{2}$		Circle $\overline{X} = r$ $\overline{Y} = r$
	Rectangle $\overline{X} = \frac{b}{2}$ $\overline{Y} = \frac{h}{2}$		Semi - Circle $\overline{X} = r$ $\overline{Y} = \frac{4r}{3\pi}$
	Right Angle Triangle $\overline{X} = \frac{b}{3}$ $\overline{Y} = \frac{h}{3}$		Quarter - Circle $\overline{X} = r - \frac{4r}{3\pi}$ $\overline{Y} = \frac{4r}{3\pi}$
	Equilateral Triangle $\overline{X} = \frac{b}{3}$ $\overline{Y} = \frac{h}{3}$		Quarter - Circle $\overline{X} = \frac{4r}{3\pi}$ $\overline{Y} = \frac{4r}{3\pi}$

Shape of volume		Position of centre of gravity (G) at distance *x* from the end shown	*Volume*
Cylinder		$h/2$	$\pi r^2 h$
Sphere		r	$4\pi r^3/3$
Hemisphere		$3r/8$	$2\pi r^3/3$
Cone		$h/4$	$\pi r^2 h/3$
Pyramid		$h/4$	$abh/3$

Position of centre of gravity (G) for some common solids.

CENTRE OF GRAVITY OF SYMMETRICAL SECTIONS

Section, whose centre of gravity is required to be found out, and is symmetrical about X-X axis or Y-Y axis the procedure for calculating the centre of gravity of the body is to calculate either ‾or .‾ This is due to the reason that the centre of gravity of the body will lie on the axis of symmetry

Question:Find the centre of gravity of a channel section 100 mm × 50 mm × 15 mm.

Solution: As the section is symmetrical about X-X axis, therefore its centre of gravity will lie on this axis.Now split up the whole section into three rectangles ABFJ, EGKJ and CDHK as shown in fig.

Let the face AC be the axis of reference.

(i) *Rectangle ABFJ*

$$a_1 = 50 \times 15 = 750 \text{ mm}^2$$

and $$x_1 = \frac{50}{2} = 25 \text{ mm}$$

(ii) *Rectangle EGKJ*

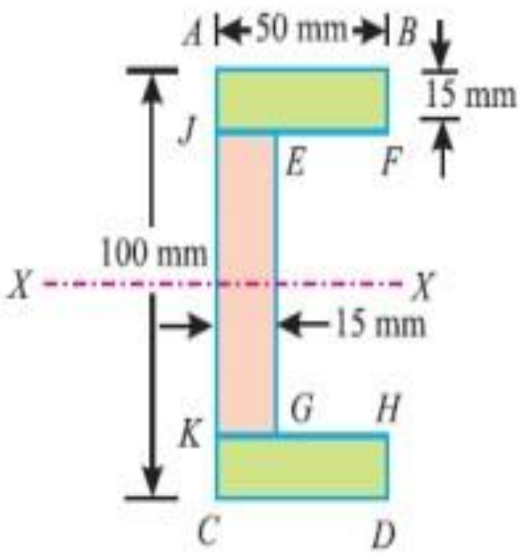
$$a_2 = (100 - 30) \times 15 = 1050 \text{ mm}^2$$

and $$x_2 = \frac{15}{2} = 7.5 \text{ mm}$$

(iii) *Rectangle CDHK*

$$a_3 = 50 \times 15 = 750 \text{ mm}^2$$

and $$x_3 = \frac{50}{2} = 25 \text{ mm}$$



We know that distance between the centre of gravity of the section and left face of the section AC,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(750 \times 25) + (1050 \times 7.5) + (750 \times 25)}{750 + 1050 + 750} = 17.8 \text{ mm} \quad \textbf{Ans.}$$

CENTRE OF GRAVITY OF UNSYMMETRICAL SECTIONS

Sometimes, the given section, whose centre of gravity is required to be found out, is not symmetrical either about X-X axis or Y-Y axis. In such cases, we have to find out both the values of ‾x and y‾

Question:Find the centroid of an unequal angle section 100 mm × 80 mm × 20 mm.

Solution :

As the section is not symmetrical about any axis, therefore we have to find out the values of x and y for the angle section. Split up the section into two rectangles as shown in Fig Let left face of the vertical section and bottom face of the horizontal section be axes of reference.

(*i*) *Rectangle* 1

$$a_1 = 100 \times 20 = 2000 \text{ mm}^2$$

$$x_1 = \frac{20}{2} = 10 \text{ mm}$$

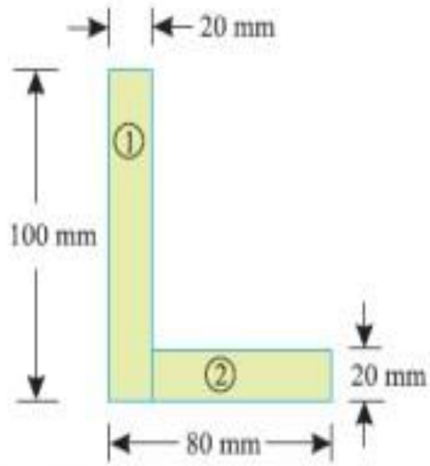
and $$y_1 = \frac{100}{2} = 50 \text{ mm}$$

(*ii*) *Rectangle* 2

$$a_2 = (80 - 20) \times 20 = 1200 \text{ mm}^2$$

$$x_2 = 20 + \frac{60}{2} = 50 \text{ mm}$$

and $$y_2 = \frac{20}{2} = 10 \text{ mm}$$



We know that distance between centre of gravity of the section and left face,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{2000 + 1200} = 25 \text{ mm} \quad \textbf{Ans.}$$

Similarly, distance between centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200} = 35 \text{ mm} \quad \textbf{Ans.}$$

MOMENT OF INERTIA:

Moment of a force (P) about a point, is the product of the force and perpendicular distance (x) between the point and the line of action of the force (i.e. P.x).

If this moment is again multiplied by the perpendicular distance (x) between the point and the line of action of the force i.e. $P.x(x) = Px^2$, then this quantity is called moment of inertia.

CALCULATION OF MOMENT OF INERTIABY INTEGRATION METHOD:

The moment of inertia of an area may be found out by the method of integration:

Consider a plane figure, whose moment of inertia is required to be found out about X-X axis and Y-Y axis as shown in Fig 4.12. Let us divide the whole area into a no. of strips. Consider one of these strips.

Let dA= Area of the strip

x = Distance of the centre of gravity of the strip on X-X axis and

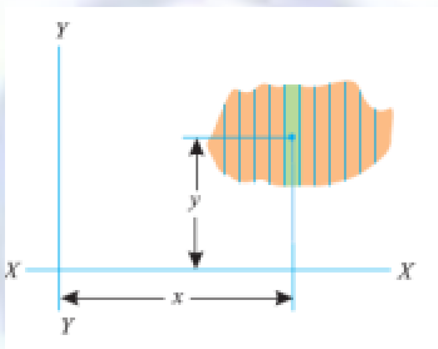
y = Distance of the centre of gravity of the strip on Y-Y axis.

We know that the moment of inertia of the strip about Y-Y axis = $dA.x^2$

Now the moment of inertia of the whole area may be found out by integrating above equation.

I,e

$$I_{YY} = \Sigma dA.x^2$$
$$I_{XX} = \Sigma dA . y^2$$



Unit: It depends on units of area and length

If area=m^2

, length =m then, M.I=m^4

If area=mm^4

, length=mm then, M.I=mm^4

THEOREM OF PERPENDICULAR AXIS

If IXX and IYY be the moments of inertia of a plane section about two perpendicular axis meeting at O, the moment of inertia IZZ about the axis Z-Z, perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by:

$$I_{ZZ} = I_{XX} + I_{YY}$$

Proof:

Consider a small lamina (P) of area da having co-ordinates as x and y along OX and OY two mutually perpendicular axes on a plane section as shown in Fig 4.13

Now consider a plane OZ perpendicular to OX and OY.

Let (r) be the distance of the lamina (P) from Z-Z axis such that

$$OP = r.$$

From the geometry of the figure, we find that

$$r^2 = x^2 + y^2$$

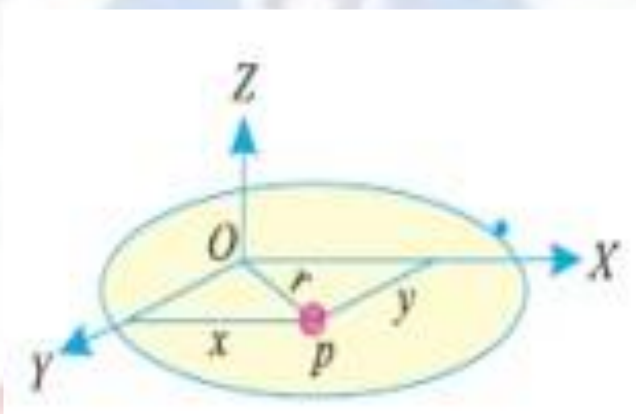
We know that the moment of inertia of the lamina P about X-X axis,

$I_{XX} = da.\, y^2$ ...[I = Area × (Distance)^2]

Similarly, $I_{YY} = da.\, x^2$

And $I_{zz} = da.r^2 = da\,(x^2 + y^2)$ ....($r^2 = x^2 + y^2$)

$= da.x^2 + da.y^2 = I_{xx} + I_{yy}$



THEOREM OF PARALLEL AXIS

It states, If the moment of inertia of a plane area about an axis through its centre of gravity is denoted by IG, then moment of inertia of the area about any other axis AB, parallel to the first, and at a distance h from the centre of gravity is given by:

$$I_{AB} = I_G + ah^2$$

Where I_{AB}= Moment of inertia of the area about an axis AB,

I_G= Moment of Inertia of the area about its centre of gravity

a = Area of the section, and

h = Distance between centre of gravity of the section and axis AB.

Proof

Consider a strip of a circle, whose moment of inertia is required to be found out about a line AB as shown in Fig.4.14

Let δa= Area of the strip

y = Distance of the strip from the centre of gravity the section and

h = Distance between centre of gravity of the section and the axis AB.

Moment of inertia of the whole section aboutan axis passing through the centre of gravity of the section $=\delta a.\ y^2$

and moment of inertia of the whole section about an axis passing through its centre of gravity,

$$IG=\Sigma\delta a.y^2$$

Moment of inertia of the section about the axis AB,

$$I_{AB}= \Sigma\delta a(h + y)^2$$

$$= \Sigma\delta a(h^2+ y^2+ 2\ h\ y)$$

$$= (\Sigma h^2.\ \delta a) + (\Sigma y2.\delta a) + (\Sigma 2\ h\ y\ .\delta a)$$

$$= a\ h^2+ IG+ 0$$

$$=a\ h^2+ IG$$

Chapter 5

Simple Machine

SIMPLE MACHINE:

A simple machine is a device by which heavy load can be lifted by applying less effort as compared to the load. A simple machine makes a difficult task easy by multiplying or redirecting the force in a single movement.

e.g. Heavy load of car can be lifted with the help of simple screw jack by applying small force.

COMPOUND MACHINE:

Compound machine is a device which may consists of number of simple machines. A compound machine may also be defined as a machine which has multiple mechanisms for the same purpose.

e.g. In a crane, one mechanism (gears) are used to drive the rope drum and other mechanism (pulleys) are used to lift the load. Thus, a crane consists of two simple machines or mechanisms i.e. gears and pulleys. Hence, it is a compound machine.

Simple Lifting Machine:

Simple lifting Machines.

A simple machine is a mechanical device which can change. the direction and magnitude of a force or effort and makes. work easier.

TERMINOLOGY IN SIMPLE LIFTING MACHINE:

(M.A, V.R. & Efficiency and relation between them)

Effort:It may be defined as, the force which is applied so as to overcome the resistance or to lift the load. It is denoted by „P‟.

Load: The weight to be lifted or the resistive force to be overcome with the help of a machine is called as load (W).

Velocity Ratio (V.R.): It is defined as the ratio of distance travelled by the effort (Y) to the distance travelled by the load (X)

$$V.R. = \frac{\text{distance travelled by the effort}}{\text{distance travelled by the load}} = \frac{Y}{X}$$

Mechanical Advantage: It is defined as the ratio of load to be lifted to the effort applied.

$$M.A. = \frac{\text{Load}}{\text{effort}} = \frac{W}{P}$$

input: The amount of work done by the effort is called as input and is equal to the product of effort and distance travelled by it.

Input = P x Y

Where , P= Effort and Y= distance travelled by the effort

Output: The amount of work done by the load is called as output and is equal to the product of load and distance travelled by it.

Output = W xX

Where , W= Load and X= distance travelled by the load

Efficiency: The ratio of output to input is called as efficiency of machine and it is denoted by Greek letter eta (η)

$$\eta = \frac{Output}{Input} = \frac{W x X}{P x Y} = \frac{M.A}{V.R}$$

MAXIMUM MECHANICAL ADVANTAGE (MAX. M.A.):

We know that

$M.A = \frac{W}{P}$ we know $P = mW + C$

$$M.A = \frac{W}{mW+C}$$

Dividing numerator and denominator by W

$$M.A = \frac{1}{m+\frac{C}{W}}$$

As C<<<W the ratio $\frac{C}{W}$ is very small so by neglecting it the M.A will be maximum

$$M.A_{max} = \frac{1}{m}$$

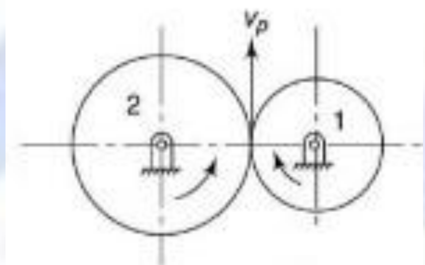
MAXIMUM EFFICIENCY:

The ratio of $M.A_{max}$ to the V.R. is called as maximum efficiency.

$$\eta_{max} = \frac{M.A\ max}{V.R.} = \frac{1}{m} \times \frac{1}{V.R}$$

SIMPLE GEAR DRIVE:

Gears are used to transmit power from one shaft to another shaft. Gear use no intermediate link or connector and transmit the motion by direct contact. In the following figure two gear are engaged and rotational motion can be transferred from one gear to other gear.



V_p= tangential velocity at point of point of contact of two gear

$V_p = \omega_1 r_1 = \omega_2 r_2$

$= \pi d_1 N_1 = \pi d_2 N_2$

$= \frac{N_1}{N_2} = \frac{d_2}{d_1}$............(1)

N=angular velocity in rpm

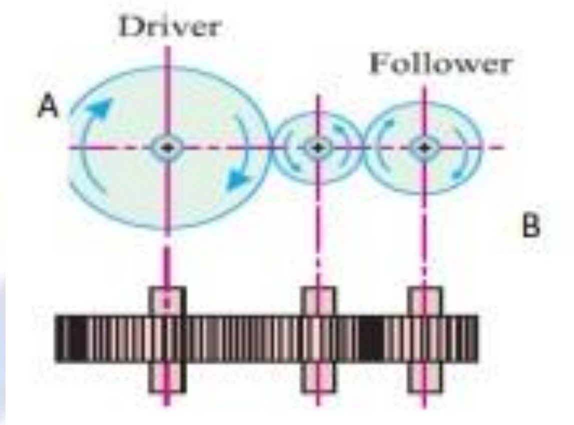
ω=angular velocity in (rad/s)

d=pitch circle diameter.

SIMPLE GEAR TRAIN:

In simple gear train each shaft supports one gear. A simple gear drive is that gear drive in which all the gears lie in the same plane. Figure show a simple gear drive in which gear A, I1 and B lie in the same plane. The gear A is driver gear and gear B is follower gear. Gear I1 is idle gear.

The function of ideal gear is to fill the gap between first gear and last gear and some time it is used to change the direction of rotation of first and last gear



VELOCITY RATIO OF A SIMPLE GEAR TRAIN :

Now consider a simple train of wheels with one intermediate wheel as shown in Figure

Let N1 = Speed of the driver

T1 = No. of teeth on the driver

d1 = Diameter of the pitch circle of the driver

N2, T2, d2 = Corresponding values for the intermediate wheel, and

N3, T3, d3 = Corresponding values for the follower.

p= Pitch of the two wheels.

We know that the pitch of the driver.

$$p=\frac{\pi d_1}{T1} \qquad \text{.....(2)}$$

Similarly pitch of the ideal gear

$$p=\frac{\pi d_2}{T_2} \qquad \text{.....(3)}$$

Similarly pitch of the follower

$$p=\frac{\pi d_3}{T_3} \qquad \text{.....(4)}$$

Since pitch of mating gear are same

Equating eq^n(2) and eq^n(3)

$$\frac{d_1}{d_2} = \frac{T_1}{T_2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Equatingeq^n(3) and eq^n(4)

$$\frac{d_2}{d_3} = \frac{T_2}{T_3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

From eq^n (1) & (5)

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Similarly

$$\frac{N_2}{N_3} = \frac{T_3}{T_2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Multiplying eq^n(5) & (6)

$$\frac{N_1}{N_3} = \frac{T_3}{T_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

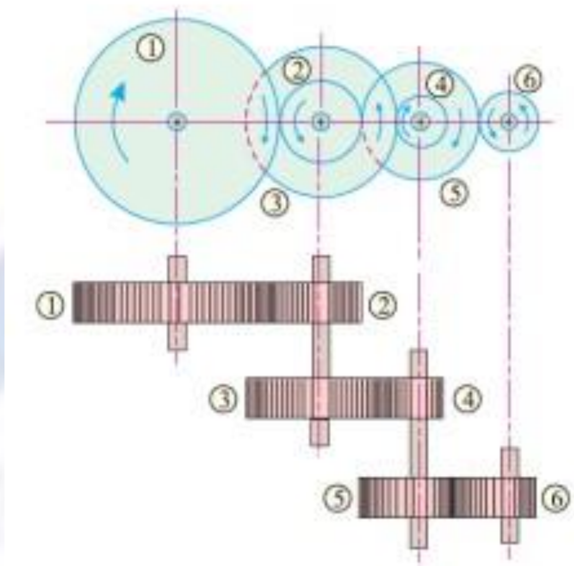
VELOCITY RATIO :

It is the ratio between the velocities of the driver and the follower.

$$\text{Velocity ratio} = \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

COMPOUND GEAR TRAIN:

When series of gears are connected in such a way that two or more gears are mounted on same shaft or rotate about an axis with same angular velocity it is known as compound gear train.



N1 = Speed of the driver 1

T1 = No. of teeth on the driver 1,

Similarly

N2, N3, N4,,N 5 &N6 = Speed of the respective wheels

T2, T3,T4 ,T5&T6 = No. of teeth on the respective wheels.

Since the gears 1 in mesh with the gear 2, therefore

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \text{ .... } (8)$$

Similarly

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \text{ .... } (9)$$

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \text{ .... } (10)$$

Multiplying eq^n (8) (9) & (10)

$$\frac{N_1}{N_2} X \frac{N_3}{N_4} X \frac{N_5}{N_6} = \frac{T_2}{T_1} X \frac{T_4}{T_3} X \frac{T_6}{T_5}$$

Velocity ratio of compound gear train

$$\frac{N_1}{N_6} = \frac{T_2}{T_1} X \frac{T_4}{T_3} X \frac{T_6}{T_5} (\text{ as} N_2 = N_3 \text{ and } N_4 = N_5)$$

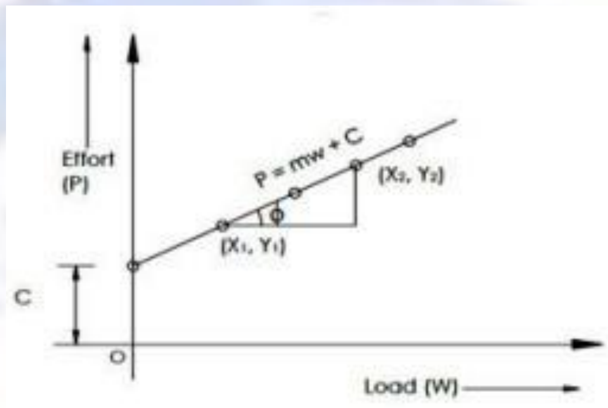
$$= \frac{\text{Product of the teeth on the followers}}{\text{Product of the teeth on the drivers}}$$

LAW OF MACHINE:

The equation which gives the relation between load lifted and effort applied in the form of a slope and intercept of a straight line is called as Law of a machine.

$$P = m W + C$$

Where, P = effort applied, W = load lifted, m = slope of the line and C = y – intercept of the straight line.



$$m = \tan\Phi = \frac{Y_2 - Y_1}{X_2 - X_1}$$

It has been observed that, the graph of load v/s effort is a straight line cuts the Y-axis giving the intercept „C" which indicates the effort lost on friction.

It must be noted that, if the machine is an ideal machine, the straight line of the graph will pass through the origin.

REVERSIBLE MACHINE:

When a machine is capable of doing some work in the reverse direction even on removal of effort, it is called as reversible machine.

e.g. simple pulley used to lift load W with effort P

MAXIMUM MECHANICAL ADVANTAGE (MAX. M.A.):

We know that

$M.A = \frac{W}{P}$ we know P = mW+ C

$M.A = \frac{W}{mW+C}$

Dividing numerator and denominator by W

$M.A = \frac{1}{m+\frac{C}{W}}$

As C<<<W the ratio $\frac{C}{W}$ is very small so by neglecting it the M.A will be maximum

$M.A_{max} = \frac{1}{m}$

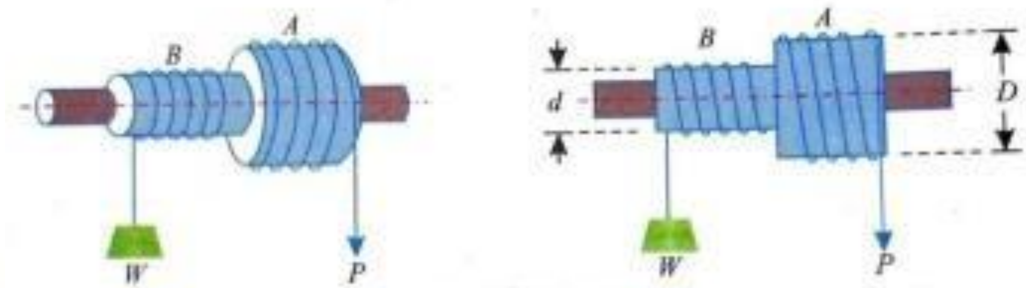
MAXIMUM EFFICIENCY:

The ratio of $M.A_{max}$ to the V.R. is called as maximum efficiency.

$\eta_{max} = \frac{M.A\ max}{V.R.} = \frac{1}{m} \times \frac{1}{V.R}$

SIMPLE WHEEL AND AXLE:

In simple wheel and axle, effort wheel and axle are rigidly connected to each other and mounted on a shaft. A string is wound round the axle so as to lift the load (W) another string iswound round the effort wheel in opposite direction so as to apply the effort (P) as shown in the figure.



Simple wheel and axle.

Let, W = Load lifted

P = Effort Applied

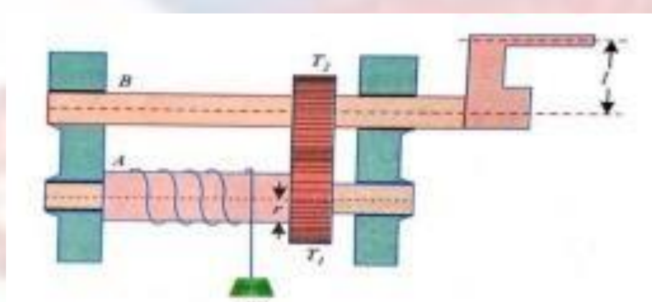
D = Diameter of the effort wheel

d = diameter of the load axle

When the effort wheel completes one revolution, the effort moves through a distance equal to the circumference of the effort wheel (D) and simultaneously the load moves up through a distance equal to the circumference of the load axle (d).

$$V.R. = \frac{\text{Distance travelled by theEffort}}{\text{Distance travelled by the load}} = \frac{\pi D}{\pi d} = \frac{D}{d}$$

SINGLE PURCHASE CRAB WINCH:



Single purchase crab winch.

In single purchase crab winch, a rope is fixed to the drum and is wound a few turns round it.
The free end of the rope carries the load W. A toothed wheel A is rigidly mounted on the load drum. Another toothed wheel B, called pinion, is geared with the toothed wheel A as shown in Fig. 5.6The effort is applied at

the end of the handle to rotate it.

Let T_1 = No. of teeth on the main gear (or spur wheel) A, T_2= No. of teeth on the pinion B,

I = Length of the handle,

r = Radius of the load drum. W = Load lifted, and

P = Effort applied to lift the load.

We know that,

Distance moved by the effort in one revolution of the handle=2+rl

No. of revolutions made by the pinion =1

No. of revolutions made by the load drum=T/Tt

Distance moved by the load =2nr X T_2/T_1

V.R.= Distance travelled by theEffort / Distance travelled by the load =

Output _M.A Input V.R

$$\eta = \frac{Output}{Input} = \frac{M.A}{V.R}$$

DOUBLE PURCHASE CRAB WINCH:

A double purchase crab winch is an improved form of a single purchase crab winch, in whichthe velocity ratio is intensified with the help of one more spur wheel and a pinion. In a double purchase crab winch, there are two spur wheels of teeth T_1 and T_2 and T_3as well as two pinions of teeth T_2 and Th.The arrangement of spur wheels and pinions are such that the spur wheel with T_1 gears with the pinion of teeth T_2 Similarly, the spur wheel with teeth T_3gears with the pinion of the teeth T„ The effort is applied to a handle as shown in figure

Let T_1 and T_3 = No. of teeth of spur wheels, T_2 and T‹ = No. of teeth of the pinions

I = Length of the handle,

r = Radius of the load drum,

W = Load lifted, and

P = Effort applied to lift the load, at the end of the handle. Distance moved by the effort in one revolution of the handIe=2nI

CHAPTER 6

DYNAMICS

KINEMATICS AND KINETICS

KINEMATICS: It is that branch of Dynamics, which deals with motion of bodies without considering the forces causing motion.

KINETICS: It is that branch of Dynamics, which deals with motion of bodies and the Forces causing the motion. It predicts the type of motion by a given force system.

PRINCIPLES OF DYNAMICS:

NEWTON'S LAWS OF MOTION

(a) First Law of motion: It states, "Everybody continues in its state of rest or of uniform motion in a straight line, unless compelled by some external force to change that state".
This law can also termed as law of inertia.

(b) Second law of motion: It states,"The rate of change of momentum is directly proportional to the impressed force and takes place in the same direction in which the impressed force acts".

It relates to the rate of change of momentum and the external force.

Let, m= mass of the body

u= initial velocity of the body v= final velocity of the body a= constant acceleration

t= time in seconds in which the velocity changes from u to v F= force that changes the velocity from u to v in t seconds

For the body moving in straight line, Initial momentum = mu

Final momentum = mv

According to Newton's Second law of motion

Rate of change of momentum impressed force Fx ma

F=k x ma

Where k= a constant of proportionality

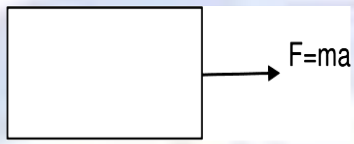
If a unit force is chosen to act on a unit mass of 1kg to produce unit acceleration of 1m/s^2 then,

$$F=ma= \text{Mass} \times \text{Acceleration}$$

The SI unit of force is Newton, briefly written as N

(c) Third law of motion: It states," To every action, there is always an equal and opposite reaction".
If a body exerts a force P on another body, the second body will exert the same force P on the first body in the opposite direction. The force exerted by first body is called action where as the force exerted by the second body is called reaction.

MOTION OF PARTICLE ACTED UPON BY A CONSTANT FORCE



The motion of a particle acted upon by a constant force is governed by Newton's second law of motion.

If a constant force, F= ma is applied on a particle of mass 'm', then the particle will move with a uniform acceleration 'a'.

EQUATIONS OF MOTION

Let, u= initial velocity of the body v= final velocity of the body
s= distance travelled by the body in motion a= acceleration of the body
t= time taken by the body

The equations of motion are: $v=u+at$

$s=ut+eat*$

$v2- u2=2as$

RECOIL OF GUN

According to Newton's third law of motion, when a bullet is fired from a gun, the opposite reaction of the bullet is known as the recoil of gun.

Let, M = mass of the gun
V = Velocity of the gun with which it recoils m = mass of the bullet
v = velocity of the bullet after firing

Now, momentum of the bullet after firing = mv Momentum of the gun = MV

Equating equations above we get,

$mv = MV$

This relation is known as law of Conservation of Momentum.

WORK

When force acts on a body and the body undergoes some displacement, then work is said to be done. The amount of work done is equal to the product of force and displacement in the direction of force.

Let, P = force acting on the body

and s = distance through which the body moves Then work done, W = P s

Sometimes the force and displacement are not collinear.

In such a case, work done is expressed as the product of the component of the force in the direction of motion and the displacement.

Hence, work done W = P cos6 • s

If 6 = 90°, cos 6 = 0 and there will be no work done i.e. if force and displacement are at right angles to each other, work done will be zero.

Similarly, work done against the force is taken as negative.

When the point of application of the force moves in the direction of motion of the body, work is said to be done by the force.

Work done by the force is taken as +ve.

POWER

Power is defined as the rate of doing work.

SI units, the unit of power is watt (briefly written as W) which is equal to 1 N-m/s or 1 J/s.It is also expressed in Kilowatt (KW), which is equal to 103 W and Megawatt (MW) which is equal to 106 W. In case of engines, the following two terms are commonly used for power.

INDICATED POWER: It is the actual power generated in the engine cylinder

BRAKE POWER: It is the amount of power available at the engine shaft

Efficiency of engine is expressed as the ratio of brake power to the indicated power. It is also called Mechanical efficiency of an engine

ENERGY

Energy may be defined as the capacity for doing work.

Since energy of a machine is measured by the work it can do, therefore unit of energy is same as that of work.

In S.I system, energy is expressed in Joules or Kilojoules. There are two types of mechanical energy.

1. POTENTIAL ENERGY: It is the energy possessed by a body by virtue of its position. A body at some height above the ground level possesses potential energy. If a body of mass(m) is raised to a height (h) above the ground level, the work done in raising the body is = Weight of the body x distance through which it moved = (mg)h = mgh

   This work (equal to mgh) is stored in the body as potential energy.

   The body, while coming down to its original level, can do work equal to mgh. Potential energy is zero when the body is on the earth.

2. KINETIC ENERGY: It is the energy possessed by a body by virtue of its motion. We can measure kinetic energy of a body by finding the work done by the body against external force to stop it.

   Let, m= Mass of the body

   u= Velocity of the body at any instant P= External force applied a=Constant Retardation of the body

   s= distance travelled by the body before coming to rest

$$K.E = \frac{1}{2} mv^2$$

MOMENTUM AND IMPULSE

MOMENTUM: It is the product of mass and velocity of a body. It represents the energy of motion stored in a moving body. If, m = mass of a moving body in kg

v = velocity of the body in m/sec,

Momentum of the body = mv kg-m/sec

IMPULSE: It is defined as the product of force and time during which the force acts on the body. According to the second law of motion,

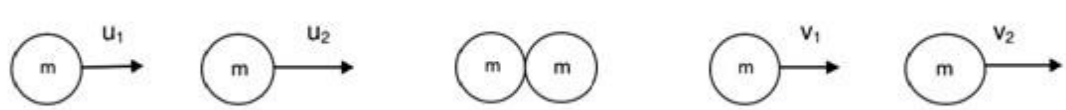
$$, \text{Impulse}, I = F x\, t = mv - mu$$

i.e. impulse is equal to change in momentum

LAW OF CONSERVATION OF LINEAR MOMENTUM

It states that "the total momentum of two bodies remains constant after their collision or any other mutual action. And no external forces act on the bodies, the algebraic sum of their momentum along any direction

is constant. Momentum along a straight line is called linear momentum



If a body of mass m1 moving with velocity u1 collides with another body of mass m moving with velocity u2.

Let v1 and v2 be the velocities of the bodies after collision.

We have:

total momentum before collision = $m_1u_1 + m_2u_2$

Total momentum after collision = $m_1v_1 + m_2v_2$

Now, according to the law of conservation of linear momentum,

. Momentum before collision = momentum after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

COLLISION OF ELASTIC BODIES

Collision means the interaction or the contact between two bodies for a short period of time. The bodies produce impulsive forces on each other during collision.

The act of collision between two bodies that takes place in a short period of time and during which the bodies exert very large forces on each other, is known as impact.

The bodies come to rest for a moment immediately after collision. During the phenomenon of collision, the bodies tend to compress each other.

The bodies tend to regain their actual shape and size after impact, due to elasticity. The process of getting back the original shape is called restitution.

The time of compression is the time taken by the two bodies in compression, immediately after collision and the time of restitution is the time of regaining the original shape after collision. The period of collision is the sum of the time of compression and restitution.

NEWTON'S LAW OF COLLISION OF ELASTIC BODIES AND COEFFICIENT OF RESTITUTION

Newton's law of collision of elastic bodies states that "when two moving bodies collide with each other, their velocity of separation bears a constant ratio to their velocity of approach".

Let us consider two bodies A and B of masses m1 and m2 respectively move along the same line and produce direct impact.

Let u1 = initial velocity of body A u1 - initial velocity of body B

v1 - final velocity of body A after collision v2 = final velocity of body B after collision

The impact will take place when us› u2

Hence the velocity of approach = U1— U2

After impact, the separation of the two bodies will take place if vt> v/ Hence the velocity of separation = v2 - v,

According to Newton's law of Collision of Elastic bodies,

m(*2 - *1)   m(U1— U2)

»(v2 - v1)= e (u1— u2)

where, e = a constant of proportionality known as coefficient of restitution.

The value of 'e' lies between 0 and 1.

If e = 0 , it indicates that the two bodies are inelastic. If e=1, it indicates that the two bodies are perfectly elastic.