AUTOMOBILE COMPONENT DESIGN

5TH SEMESTER –AUTOMOBILE ENGINEERING
LECTURE- NOTE

A

UNIT 1

INTRODUCTION

Instructional Objectives

- Explain what is design?
- Describe the machine and its designer,
- Illustrate the procedure of design,
- Know materials used in mechanical design, and
- Understand the considerations for manufacturing.

The subject Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time consuming one. From the study of existing ideas, a new idea has to be conceived. The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings. In the preparation of these drawings, care must be taken of the availability of resources in money, in men and in materials required for the successful completion of the new idea into an actual reality. In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing.

If the end product of the engineering design can be termed as mechanical then this may be termed as Mechanical Engineering Design. Mechanical Engineering Design may be defined as: **"Mechanical Engineering Design is defined as iterative decision making process to describe a machine or mechanical system to perform specific function with maximum economy and efficiency by using scientific principles, technical information, and imagination of the designer." A** designer uses principles of basic engineering sciences, such as Physics, Mathematics, Statics, Dynamics, Thermal Sciences, Heat Transfer, Vibration etc.

Classifications **of Machine** Design:

of existing designs. This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training. The desi gner only makes min o r alternation or modification in the existing designs of the product.

1. *Development design.* This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

2. *New design.* This type of design needs lot of research, technical ability and creative thinking. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design. The designs, depending upon the methods used, may be classified as follows:

(a) Rational design. This type of design depends upon mathematical formulae of principle of mechanics.

(b) Empirical design. This type of design depends upon empirical formulae based on the practice and past experience.

(c) ***Industrial design.*** This type of design depends upon the production aspects to manufacture any machine component in the industry.

(d) ***Optimum design.*** It is the best design for the given objective function under the specified constraints. It may be achieved by minimizing the undesirable effects.

(e) System design. It is the design of any complex mechanical system like a motor car.

lfj ***Element design.*** It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.

(g) ***Computer aided design.*** This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimization of a design.

General **Considerations in Machine** Design

Following are the general considerations in designing a machine component:

1. *Type of load and stresses caused by the load.* The load, on a machine component, may act in several ways due to which the internal stresses are set up. The various types of load and stresses are discussed later.

2. ***Motion of the parts or kinematics of the machine.*** The successful operation of any machine depends largely upon the simplest arrangement of the parts which will give the motion required.

The motion of the parts may be : *(a)* Rectilinear motion which includes unidirectional and reciprocating motions. *(b)* Curvilinear motion which includes rotary, oscillatory and simple harmonic. (c) Constant velocity. (Q Constant or variable acceleration.

3. ***Selection of materials.*** It is essential that a designer should have a thorough knowledge of the properties of the materials and their behaviour under working conditions. Some of the important characteristics of materials are: strength, durability, flexibility, weight, resistance to heat and corrosion, ability to cast, welded or hardened, machinability, electrical conductivity, etc. The various types of engineering materials and their properties are discussed later.

4. ***Form and size of the parts.*** The form and size are based on judgment. The smallest practicable cross-section may be used, but it may be checked that the stresses induced in the designed cross-section are reasonably safe. In order to design any machine part for form and size, it is necessary to know the forces which the part must sustain. It is also important to anticipate any suddenly applied or impact load which may cause failure.

5. ***Frictional resistance and lubrication.*** There is always a loss of power due to frictional resistance and it should be noted that the friction of starting is higher than that of running friction. It is, therefore, essential that a careful attention must be given to the matter of lubrication of all surfaces which move in contact with others, whether in rotating, sliding, or rolling bearings.

6. ***Convenient and economical features.*** In designing, the operating features of the machine should be carefully studied. The starting, controlling and stopping levers should be located on the basis of convenient handling. The adjustment for wear must be provided employing the various take up devices and arranging them so that the alignment of parts is preserved. If parts are to be changed for different products or replaced on account of wear or breakage, easy access should be provided and the necessity of removing other parts to accomplish this should be avoided if possible. The economical operation of a machine which is to be used for production or for the processing of material should be studied, in order to learn whether it has the maximum capacity consistent with the production of good work.

7. ***Use of standard parts.*** The use of standard parts is closely related to cost, because the cost of standard or stock parts is only a fraction of the cost of similar parts made to order. The standard or stock parts should be used whenever possible; parts for which patterns are already in existence such as gears, pulleys and bearings and parts which may be selected from regular shop stock such as screws, nuts and pins. Bolts and studs should be as few as possible to avoid the delay caused by changing drills, reamers and taps and also to decrease the number of wrenches required.

8. ***Safety of operation.*** Some machines are dangerous to operate, especially those which are speeded up to insure production at a maximum rate. Therefore, any moving part of a machine which is within the zone of a worker is considered an accident hazard and may be the cause of an injury. It is, therefore, necessary that a designer should always provide safety devices for the safety of the operator. The safety appliances should in no way interfere with operation of the machine.

9. ***Workshop facilities.*** **A** design engineer should be familiar with the limitations of this employer's workshop, in order to avoid the necessity of having work done in some other workshop. It is sometimes necessary to plan and supervise the workshop operations and to draft methods for casting, handling and machining special parts.

10. ***Number of machines to be manufactured.*** The number of articles or machines to be manufactured affects the design in a number of ways. The engineering and shop costs which are called fixed charges or overhead expenses are distributed over the number of articles to be manufactured. If only a few articles are to be made, extra expenses are not justified unless the machine is large or of some special design. An order calling for small number of the product will not permit any undue expense in the workshop processes, so that the designer should restrict his specification to standard parts as much as possible.

11. ***Cost of construction.*** The cost of construction of an article is the most important consideration involved in design. In some cases, it is quite possible that the high cost of an article may immediately bar it from further considerations. If an article has been invented and tests of handmade samples have shown that it has commercial value, it is then possible to justify the expenditure of a considerable sum of money in the design and development of automatic machines to produce the article, especially if it can be sold in large numbers. The aim of design engineer under all conditions should be to reduce the manufacturing cost to the minimum.

12. ***Assembling.*** Every machine or structure must be assembled as a unit before it can function. Large units must often be assembled in the shop, tested and then taken to be transported to their place of service. The final location of any machine is important and the design engineer must anticipate the exact location and the local facilities for erection.

Manufacturing considerations in Machine design

Manufacturing Processes

The knowledge of manufacturing processes is of great importance for a design engineer. The following are the various manufacturing processes used in Mechanical Engineering.

1. ***Primary*** *shaping* ***processes.*** The processes used for the preliminary shaping of the machine component are known as primary shaping processes. The common operations used for this process are casting, forging, extruding, rolling, drawing, bending, shearing, spinning, powder metal forming, squeezing, etc.

2. ***Machining processes.*** The processes used for giving final shape to the machine component, according to planned dimensions are known as machining processes. The common operations used for this process are turning, planning, shaping, drilling, boring, reaming, sawing, broaching, milling, grinding, hobbing, etc.

3. ***Surface finishing processes.*** The processes used to provide a good surface finish for the machine component are known as surface finishing processes. The common operations used for this process are polishing, buffing, honing, lapping, abrasive belt grinding, barrel tumbling, electroplating, super finishing, sheradizing, etc.

4. ***Joining processes.*** The processes used for joining machine components are known as joining processes. The common o p erat i o ns u s e d for this process are we lding, rivet ing, soldering, brazing, screw fastening, pressing, sintering, etc.

5. ***Processes effecting change in properties.*** These processes are used to impart certain specific properties to the machine components so as to make them suitable for particular operations or uses. Such processes are heat treatment, hot-working, cold-working and shot peening.

General Procedure in Machine Design

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways. However, the general procedure to solve a design problem is as follows:

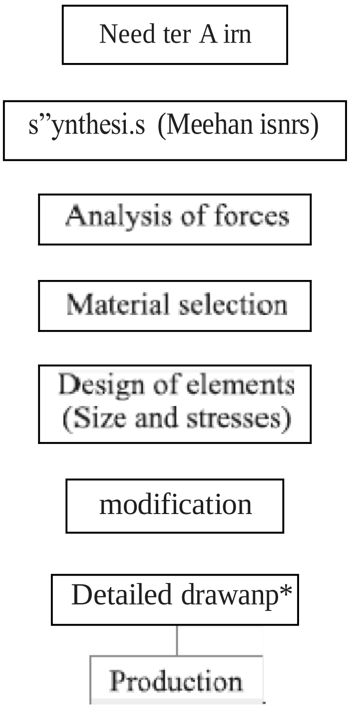


Fig.1. General Machine Design Procedure

1. ***Recognition of need.*** First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.

2. ***Synthesis {Mechanisms).*** Select the possible mechanism or group of mechanisms which will give the desired motion.

3. Analysis of forces. Find the forces acting on each member of the machine and the energy transmitted by each member.

4. ***Material selection.*** Select the material best suited for each member of the machine.

5. *Design of elements {Size and Stresses).* Find the size of each member of the machine by considering the force acting on the member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform than the permissible limit.

6. *Modification.* Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.

7. *Detailed drawing.* Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.

8. *Production.* The component, as per the drawing, is manufactured in the workshop. The flow chart for the general procedure in machine design is shown in Fig.1.

Standards and Standardization

Standards in Design:

Standard is a set of specifications, defined by a certain body or an organization, to which various characteristics of a component, a system, or a product should conform. The characteristics may include: dimensions, shapes, tolerances, surface finish etc.

Types of Standards Used In Machine Design:

Based on the defining bodies or organization, the standards used in the machine design can be divided into following three categories:

(i) Company Standards: These standards are defined or set by a company or a group of companies for their use.

(ii) National Standards: These standards are defined or set by a national apex body and are normally followed throughout the country. Like BIS, AWS.

(iii) International Standards: These standards are defined or set by international apex body and are normally followed throughout the world. Like ISO, IBWM.

Advantages:

- Reducing duplication of effort or overlap and combining resources
- Bridging of technology gaps and transferring technology
- Reducing conflict in regulations
- Facilitating commerce
- Stabilizing existing markets and allowing development of new markets
- Protecting from litigation

B.I.S DESIGNATIONS OF THE PLAIN CARBON STEEL:

Plain carbon steel is designated according to BIS as follows:

1. The first one or two digits indicate the 100 times of the average percentage content of carbon.
2. Followed by letter "C"
3. Followed by digits indicates 10 times the average percentage content of Manganese "Mn".

B.I.S DESIGNATIONS OF ALLOY STEEL:

Alloy carbon steel is designated according to BIS as follows:

1. The first one or two digits indicate the 100 times of the average percentage content of carbon.
2. Followed by the chemical symbol of chief alloying element.
3. Followed by the rounded off the average percentage content of alloying element as per international standards.
4. Followed by the chemical symbol of alloying elements followed by their average percentage content rounded off as per international standards in the descending order.
5. If the average percentage content of any alloying element is less than 1%, it should be written with the digits up to two decimal places and underlined.

Engineering materials and their properties:

The knowledge of materials and their properties is of great significance for a design engineer. The machine elements should be made of such a material which has properties suitable for the conditions of operation. In addition to this, a design engineer must be familiar with the effects which the manufacturing processes and heat treatment have on the properties of the materials. Now, we shall discuss the commonly used engineering materials and their properties in Machine Design.

Classification of Engineering Materials

The engineering materials are mainly classified as:

1. Metals and their alloys, such as iron, steel, copper, aluminum, etc.
2. Non-metals, such as glass, rubber, plastic, etc.

The metals may be further classified as:

(a) Ferrous metals and *(b)* Non-ferrous metals.

The ***ferrous zrietals** are those which have the iron as their main constituent, such as cast iron, wrought iron and steel.

The ***non-ferrous*** metals are those which have a metal other than iron as their main constituent, such as copper, aluminum, brass, tin, zinc, etc.

Selection of Materials for Engineering Purposes

The selection of a proper material, for engineering purposes, is one of the most difficult problems for the designer. The best material is one which serves the desired objective at the minimum cost. The following factors should be considered while selecting the material:

1. Availability of the materials,
2. Suitability of the materials for the working conditions in service, and
3. The cost of the materials.

The important properties, which determine the utility of the material, are physical, chemical and mechanical properties. We shall now discuss the physical and mechanical properties of the material in the following articles.

Physical Properties of Metals

The physical properties of the metals include luster, colour, size and shape, density, electric and thermal conductivity, and melting point. The following table shows the important physical properties of some pure metals.

Mechanical Properties of Metals

The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load. These mechanical properties of the metal include strength, stiffness, elasticity, plasticity, ductility, brittleness, malleability, toughness, resilience, creep and hardness. We shall now discuss these properties as follows:

1. Strength. It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is called stress.

2. Stiffness. It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.

3. **Elasticity.** It is the property of a material to regain its original shape after deformation when the external forces are removed. This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber.

4. Plasticity. It is property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.

5. Ductility. It is the property of a material enabling it to be drawn into wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper, aluminium, nickel, zinc, tin and lead.

6. Brittleness. It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Brittle materials when subjected to tensile loads snap off without giving any sensible elongation. Cast iron is a brittle material.

7. **Malleability.** It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it is not essential to be so strong. The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper and aluminium.

8. **Toughness.** It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed upto the point of fracture. This property is desirable in parts subjected to shock and impact loads.

9. **Machinability.** It is the property of a material which refers to a relative case with which a material can be cut. The machinability of a material can be measured in a number of ways such as comparing the tool life for cutting different materials or thrust required to remove the material at some given rate or the energy required to remove a unit volume of the material. It may be noted that brass can be easily machined than steel.

10. **Resilience.** It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume within

elastic limit. This property is essential for spring materials.

11. Creep. When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called **creep.** This property is considered in designing internal combustion engines, boilers and turbines.

12. Fatigue. When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as ***fatigue.** The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. This property is considered in designing shafts, connecting rods, springs, gears, etc.

13. Hardness. It is a very important property of the metals and has a wide variety of meanings. It embraces many different properties such as resistance to wear, scratching, deformation and machinability etc. It also means the ability of a metal to cut another metal. The hardness is usually expressed in numbers which are dependent on the method of making the test. The hardness of a metal may be determined by the following tests:

(a) Brinell hardness test,

(b) Rockwell hardness test,

(c) Vickers hardness (also called Diamond Pyramid) test, and

(d) Shore scleroscope.

Stress

When some external system of forces or loads acts on a body, the internal forces (equal and opposite) are set up at various sections of the body, which resist the external forces. This internal force per unit area at any section of the body is known as *unit **stress*** or simply a ***stress.*** It is denoted by a Greek letter sigma (o). Mathematically,

Stress, *a — P/A*

Where *P* — Force or load acting on a body, and

A — Cross-sectional area of the body.

In S.I. units, the stress is usually expressed in Pascal (Pa) such that 1 Pa = I N/m^2 In actual practice, we use bigger units of stress *i.e.* megapascal (MPa) and gigapascal (GPa), such that

$$1 \text{ MPa} = 1 \times 10^6 \text{ N/}^2 - 1 \text{ N/}^2$$

$$1 \text{ GPa} = 1 \times 10^9 \text{ N/}^2 - 1 \text{ kN/}^2$$

Strain

When a system of forces or loads act on a body, it undergoes some deformation. This deformation per unit length is known as ***unit strain*** or simply a ***strain.*** It is denoted by a Greek letter epsilon (s). Mathematically,

Strain, e= 6/ / f or 6f = e./

Where 6/ = Change in length of the body,

/= Original length of the body.

Tensile Stress and Strain

i —— —i i'm « • i'

«,j ›

Fig. Tensile stress and strain

When a body is subjected to two equal and opposite axial pulls *P* (also called tensile load) as shown in Fig. *(a),* then the stress induced at any section of the body is known as ***tensile stress*** as shown in Fig. *(b).* A little consideration will show that due to the tensile load, there will be a decrease in cross-sectional area and an increase in length of the body. The ratio of the increase in length to the original length is known as ***tensile strain.***

Let *P* — Axial tensile force acting on the body,
A — Cross-sectional area of the body,
/ = Original length, and 6/= Increase in length.

Then, Tensile stress, o,= *PIA* and tensile strain, e, = ó/ / /

Young's Modulus or Modulus of Elasticity

Hooke's law* states that when a material is loaded within elastic limit, the stress is directly proportional to strain, *i.e* ‘‘ oÈ' w = *E.r*

“ r H x 6!

where *E* is a constant of proportionality known as ***Young's modulus*** or ***modules of elasticity.*** In S.I. units, it is usually expressed in GPa *i.e.* GN/m° or kN/mm^2. It may be noted that Hooke's law holds good for tension as well as compression.

The following table shows the values of modulus of elasticity or Young's modulus (for the materials commonly used in engineering practice.

Values of ‘E’ for the commonly used engineering materials.

Material	Modulus of elasticity (E) GPa
Steel and Nickel	200 to 220
Wrought iron	190 to 200
Cast iron	100 to 160
Copper	90 to 110
Brass	80 to 90
Aluminium	60 to 80
Timber	10

Stress-Strain Curves

Properties are quantitative measure of materials behavior and mechanical properties pertain to material behaviors under load. The load itself can be static or dynamic. A gradually applied load is regarded as static. Load applied by a universal testing machine upon a specimen is closet example of gradually applied load and the results of tension test from such machines are the basis of defining mechanical properties. The dynamic load is not a gradually applied load — then how is it applied. Let us consider a load *P* acting at the center of a beam, which is simply supported at its ends. The reader will feel happy to find the stress (its maximum value) or deflection or both by using a formula from Strength of Materials. But remember that when the formula was derived certain assumptions were made. One of them was that the load *P* is gradually applied. Such load means that whole of P does not act on the beam at a time but applied in instalments. The instalment may be, say P/100 and thus after the l00th instalment is applied the load *P* will be said to be acting on the beam. If the whole of *P* is placed upon the beam, then it comes under the category of the dynamic load, often referred to as Suddenly Applied Load. If the load P falls from a height then it is a shock load. A fatigue load is one which changes with time. Static and dynamic loads can remain unchanged with time after first application or may alter with time (increase or reduce) in which case, they are fatigue load. A load which remains constantly applied over a long time is called creep load.

All Strength of Material formulae are derived for static loads. Fortunately the stress caused by a suddenly applied load or shock load can be correlated with the stress caused by gradually applied load. We will invoke such relationships as and when needed. Like stress formulae, the mechanical properties are also defined and determined under gradually applied loads because such determination is easy to control and hence economic. The properties so determined are influenced by sample geometry and size, shape and surface condition, testing machines and even operator. So the properties are likely to vary from one machine to another and from one laboratory to another. However, the static properties carry much less influence as compared to dynamic (particularly fatigue) properties. The designer must be fully aware of such influences because most machines are under dynamic loading and static loading may only be a dream.

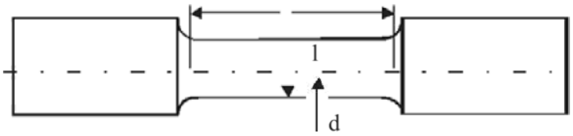
It is imperative at this stage to distinguish between **elastic constants** and mechanical properties. The elastic constants are dependent upon type of material and not upon the sample. However, strain rate (or rate of loading) and temperature may affect elastic constants. The materials used in machines are basically **isotropic** (or so assumed) for which two independent elastic constants exist whereas three constants are often used in correlating stress and strains. The three constants are Modules of Elasticity (, Modulus of Rigidity (G) and Poisson's Ratio (v). Any one constant can be expressed in terms of other two.

An isotropic material will have same value of *E* and *G* in all direction but a natural material like wood may have different values of *E* and *G* along fibres and transverse to fibre. Wood is non-isotropic. Most commonly used materials like iron, steel, copper and its alloys, aluminum and its alloys are very closely isotropic while wood and plastic are non-isotropic. The strength of material formulae are derived for isotropic materials only.

The leading mechanical properties used in design are ultimate tensile strength, yield strength, percent elongation, hardness, impact strength and fatigue strength. Before we begin to define them, we will find that considering tension test is the most appropriate beginning.

Tension Test

The tension test is commonest of all tests. It is used to determine many mechanical properties. A cylindrical machined specimen is rigidly held in two jaws of universal testing machine. One jaw is part of a fixed cross-head, while other joins to the part of moving cross-head. The moving cross-heads moves slowly, applying a gradually applied load upon the specimen.



— Figure 1.1 : Tension Test Specimen

The specimen is shown in Figure 1.1. The diameter of the specimen bears constant ratio with the gauge length which is shown is Figure 1.1 as distance between two gauge points marked at the ends of uniform diameter length. In a standard specimen = 5 . The diameter, *d*, and gauge length, /, are measured before the specimen is placed in the machine. As the axial force increases upon the specimen, its length increases, almost imperceptibly in the beginning. But if loading continues the length begins to increase perceptibly and at certain point reduction in diameter becomes visible, followed by great reduction in diameter in the local region of the length. In this localized region the two parts of the specimen appear to be separating as the machine continues to operate but the load upon the specimen begins to reduce. Finally at some lesser load the specimen breaks, with a sound, into two pieces. However, the increase in length and reduction of load may not be seen in all the materials. Specimens of some materials show too much of extension and some show too little. The reader must be conversant with the elastic deformation, which is recoverable and plastic deformation, which is irrecoverable. Both type of deformations occur during the test. The appearance of visible decrease in the diameter in the short portion of length (called necking) occurs when the load on the specimen is highest. The machines of this type have arrangement (devices) for the measurement of axial force, *P*, and increase in length, 6. The values of force, *P* and extensions, 6 can be plotted on a graph. Many machines have *x-y* recorder attached and direct output of graph is obtained. The stress is denoted by n and calculated as P/A where, *A* is the original area of cross-section. Although the area of cross-section of specimen begins to change as the deformations goes plastic, this reduction is seen at and after the maximum load. The separation or fracture into two pieces can be seen to have occurred on smaller diameter. Yet, the stress all through the test, from beginning to end, is represented by n = P/A. The strain is defined as the ratio of change in length at any load *P* and original length / and represented by c, i.e. c = ó// at all loads. Since *A* and / are constants hence nature of graph between *P* and 6 (load-extension) or between n and c (stress-strain) will be same. Figure 1.2 shows a stress-strain diagram, typically for a material, which has extended much before fracture occurred.

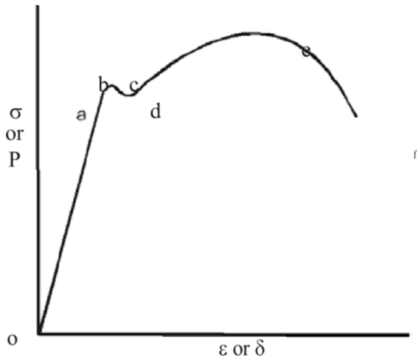


Figure 1.2 : **Typical** n — e **Diagram**

At first we simply observe what this diagram shows. In this diagram o is the starting point and *oa* is straight line. Along line *oa,* stress (n) is directly proportional to strain (s). Point *b* indicates the elastic limit, which means that if specimen is unloaded from any point between o and *b* (both inclusive) the unloading curve will truly retrace the loading curve. Behaviour of specimen material from point *b* to *c* is not elastic. In many materials all three points of *a, b* and c may coincide. At *c* the specimen shows deformation without any increase in load (or stress). In some materials (notably mild or low carbon steel) the load (or stress) may reduce perceptibly at *c,* followed by considerable deformation at the reduced constant stress. This will be shown in following section. However, in most materials *cd* may be a small (or very small) region and then stress starts increasing as if the material has gained strength. Of course the curve is more inclined toward c axis. This increase in stress from *d* to *e* is due to strain hardening. Also note again that *ob* is elastic deformation zone and beyond *b* the deformation is elastic and plastic — meaning that it is part recoverable and part irrecoverable. As the deformation increases plastic deformation increases while elastic deformation remains constant equal to that at *b.* If the specimen is unloaded from any point in the plastic deformation region the unloading curve will be parallel to elastic deformation curve as shown in Figure 1.3.

e

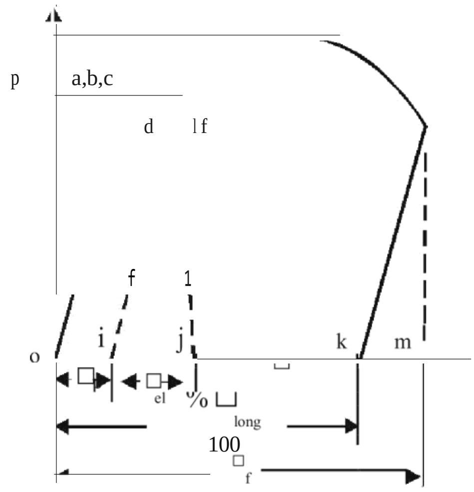


Figure 1.3 : n — e Diagram for a Ductile Material

Percent Elongation

From any point *g* the unloading will be along *gi* where *gi* is parallel to *oa. oi* is the strain which remains in the specimen or the specimen is permanently elongated by I *Ap*• The total strain at *g* when the specimen is loaded is oy *Gp Cel* Where c,¿ is recoverable part. At fracture, i.e. at point *f,* if one is able to control and unload the specimen just before fracture, the unloading will follow *f k.* The strain *ok* is an important property because deformation is defined as percent elongation. Hence, ok = ^{0}/oelongation 100. Percent elongation is important property and is often measured by placing two broken pieces together and measuring the distance between the gauge points. You can easily see that after the fracture has occurred, the specimen is no more under load, hence elastic deformation (which is equal to km) is completely recovered. However, in a so-called ductile material *km* << *om.* If the distance between gauge points measured on two broken halves placed together is *If,* then

% Elongation = ' ' x 100

The gauge length has pronounced effect on % elongation. Since the major amount of deformation occurs locally, i.e. over very small length smaller gauge length will result in higher % elongation. After 1/d > 5 the % elongation becomes independent of gauge length. % elongation is an indication of very important property of the material called ductility. The ductility is defined as the property by virtue of which a material can be drawn into wires which means length can be increased and diameter can be reduced without fracture. However, a ductile material deforms plastically before it fails. The property opposite to ductility is called brittleness. A brittle material does not show enough plastic deformation. Brittle materials are weak under tensile stress, though they are stronger than most ductile materials in compression

Ultimate Tensile Strength, Yield Strength and Proof Stress

The maximum stress reached in a tension test is defined as **ultimate tensile strength.** As shown in Figure 1.3 the highest stress is at point *e* and ultimate tensile stress (UTS) is represented by u- SOme authors represent it by S_u- The point c marks the beginning while *d* marks the end of yielding. c is called upper yield point while *d* is called the lower yield point. The stress corresponding to lower yield point is defined as the yield strength. For the purposes of machines, the part has practically failed if stress reaches yield strength, (n2), for this marks the beginning of the plastic deformation. Plastic deformation in machine parts is not permissible. Hence one may be inclined to treat Y as failure criterion. We will further discuss this later in the unit.

It is unfortunate to note that many practical materials show n — e diagrams which do not have such well defined yielding as in Figures 1.2 and 1.3. Instead they show a continuous transition from elastic to plastic deformation. In such cases yield strength (YN becomes difficult to determine. For this reason an alternative, called **proof** stress, is defined which is a stress corresponding to certain predefined strain. The proof stress is denoted by Op• A O — c diagram for a material, which shows no distinct yield is shown in Figure 1.5. The proof stress is determined corresponding to proof strain Up which is often called offset. By laying f•p on strain axis to obtain a point q on c axis and drawing a line parallel to elastic line to cut the n — c curve at p the proof stress Op is defined. Then Op is measured on stress axis. The values of proof strain or offset have been standardized for different materials by American Society for Testing and Materials *(ASTM).* For example, offset for aluminum alloys is 0.2%, same is for steels while it is 0.05% for cast iron (CI) and 0.35% for brass and bronze

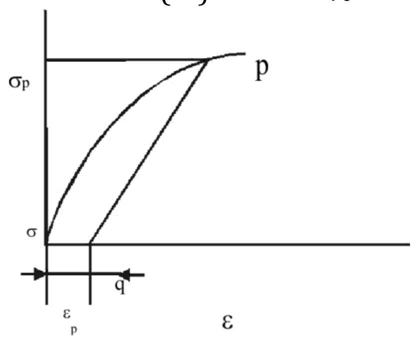


Figure 1.5: Proof Stress *(Op)* Corresponding to Offset *f.p*

Toughness and Resilience

Since the force, which pulls the tension test specimen, causes movement of its point of application, the work is done. This work is stored in the specimen and can be measured as energy stored in the specimen. It can be measured as area under the curve between load *(P)* and elongation (Af). In case of n — c curve area under the curve represents energy per unit volume.

Toughness is regarded as ability of a material to absorb strain energy during elastic and plastic deformation. The resilience is same capacity within elastic range. The maximum toughness will apparently be at fracture, which is the area under entire n — e diagram. This energy is called modulus of toughness. Likewise the maximum energy absorbed in the specimen within elastic limit is called modulus of resilience. This is the energy absorbed in the tension specimen when the deformation has reached point *a* in Figure 1.2. But since in most materials the proportional limit, elastic limit (points *a* and *b* in Figures 1.2 and 1.3) seem to coincide with yield stress as shows in Figure 1.3, the modules of resilience is the area of triangle as shown in Figure 1.6.

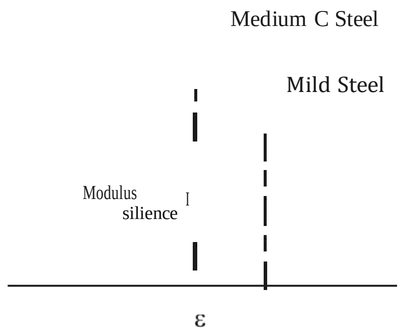


Figure 1.6 : Resilience and Toughness for Two Materials

It can be seen that modulus of resilience is greater for medium carbon steel than for mild steel, whereas modulus of toughness of two materials may be closely same. Medium carbon steel apparently has higher UTS and YS but smaller percent elongation with respect to mild steel. High modulus of resilience is preferred for such machine parts, which are required to store energy. Springs are good example. Hence, springs are made in high yield strength materials.

Stress Strain Diagram for Mild Steel

Mild steel as steel classification is no more a popular term. It was in earlier days that group of steel used for structural purposes was called mild steel. Its carbon content is low and a larger group of steel, named low carbon steel, is now used for the same purposes. We will read about steel classification later. Mild steel was perhaps developed first out of all steels and it was manufactured from Bessemer process by blowing out carbon from iron in a Bessemer converter. It was made from pig iron. The interesting point to note is that this steel was first studied through n — c diagram and most properties were studied with respect to this material.

The term **yield strength** (YS) is frequently used whereas yield behavior is not detectable in most steel varieties used today. It is mild steel, which very clearly shows yield behavior and upper and lower, yield points. Figure 1.7 shows a typical n — s diagram for mild steel.

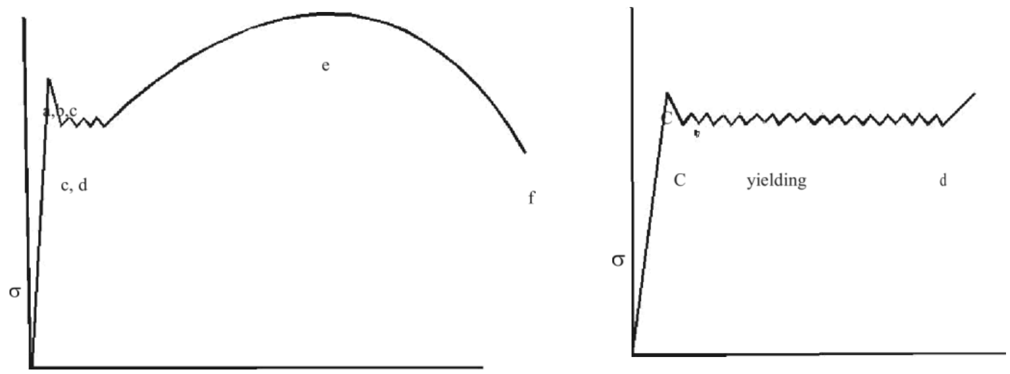


Figure 1.7 : CA — C Diagram for Mild Steel

The proportional limit, elastic limit and upper yield point almost coincide. *d* is lower yield point and deformation from *c'* to *d* is at almost constant stress level. There is perceptible drop in stress from c to c'. The deformation from c' to *d* is almost 10 times the deformation upto c. It can be seen effectively if strain is plotted on larger scale, as shown on right hand side in Figure 1.7, in which the e scale has been doubled.

The mechanism of yielding is well understood and it is attributed to line defects, dislocations.

The UTS normally increases with increasing strain rate and decreases with increasing temperature. Similar trend is shown by yield strength, particularly in low carbon steel.

Compression Strength

Compression test is often performed upon materials. lhe compression test on ductile material reveals little as no failure is obtained. Brittle material in compression shows specific fracture failure, failing along a plane making an angel greater than 45° with horizontal plane on which compressive load is applied. The load at which fracture occurs divided by area off-section is called compressive strength. For brittle material the stress-strain curves are similar in tension and compression and for such brittle materials as CI and concrete modulus of elasticity in compression are slightly higher than that in tension.

Torsional Shear Strength

Another important test performed on steel and CI is *torsion test.* In this test one end of specimen is rigidly held while twisting moment or torque is applied at the other end. The result of test is plotted as a curve between torque (and angle of twist or angular displacement 8. The test terminates at fracture. The T — 8 curves of a ductile material is very much similar to load extension or n — c curve of tensile test except that the torque does not reduce after attaining a maximum value but fracture occurs at maximum torque. It is because of the fact that there is no reduction in the sectional area of the specimen during the plastic deformation. The elastic limit in this case can be found as the point where straight line terminates and strain hardening begins, marked as point *b* in Figure 1.8. Mild steel will show a marked yielding while other ductile materials show a smooth transition from elastic to plastic deformation. The plastic deformation zone in torsion is much larger than in torsion because the plastic deformation beginning from outer surface and spreads inside while in tension the stress being uniform over the I-section the plastic deformation spreads over entire section at the same time.

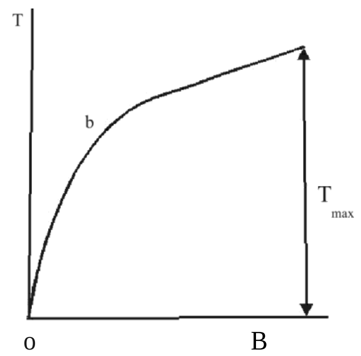


Figure 1.8 : Torque-twist Diagram in Torsion

The *modulus of rapture or ultimate torsional shear strength* is calculated form

$$\tau_u = \frac{3}{4}\frac{"max}{J}\frac{d}{2}$$

where Qmaxis maximum torque, y is polar moment of inertia of the specimen section of diameter *d*. From the *T* 8 diagram the slope of linear region can be found as proportional to modulus of rigidity, which is ratio of shearing stress to shearing strain.

Elastic Constants

Within elastic limit the stress is directly proportional to strain. This is the statement of Hooke's law and is true for direct (tensile or compressive) stress and strain as well as for shearing (including torsional shearing) stress and strain. The ratio of direct stress to direct strain is defined as *modulus of elasticity (E)* and the ratio of shearing stress and shearing strain is defined as *modulus of rigidity* (G). Both the modulus is called elastic constants. For isotropic material *E* and G are related with Poisson's ratio

$$\frac{E}{2\,(1\ \ v)}$$

Poisson's ratio which is the ratio of transverse to longitudinal strains (only magnitude) in tensile test specimen is yet another elastic constant. If stress n acts in three directions at a point it is called volumetric stress and produces volumetric strain. The ratio of volumetric stress to volumetric strain according to Hooke's law is a constant, called *bulk modulus* and denoted by *K*. It is important to remember that out of four elastic constants, for an isotropic material only two are independent and other two are dependent. Thus *K* can also be expressed as function of any two constants.

$$K \text{—} \frac{E}{3\,(1\ \ \ 2v)}$$

It may be understood that elastic constants *E* and *G* are not determined from tension or torsion test because the machines for these tests undergo adjustment of clearance and also some deformation, which is reflected in diagram ordinarily. The constants are determined from such devices, which show large deformation for comparatively smaller load. For example, *E* is determined by measuring deflection of a beam under a central load and G is determined by measuring deflection of a close-coiled helical spring an axial load. Poisson's ratio is normally not measured directly but is calculated from above equation.

Shear Stress and Strain

When a body is subjected to two equal and opposite forces acting tangentially across the resisting section, as a result of which the body tends to shear off the section, then the stress induced is called sAeor stress.

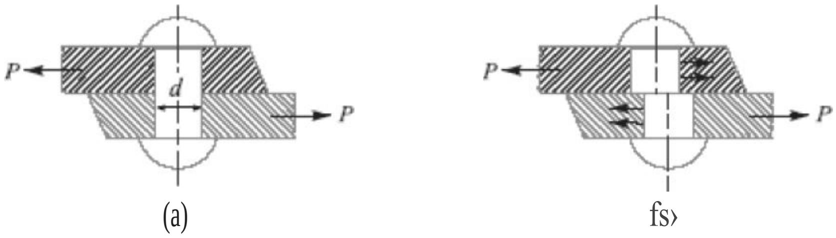


Fig. Single shearing of a riveted joint.

The corresponding strain is known as ***shear strain*** and it is measured by the angular deformation accompanying the shear stress. The shear stress and shear strain are denoted by the Greek letters tau (z) and phi (g) respectively. Mathematically,

$$\text{Shear stress, z} \quad \frac{\text{Tangential force}}{\text{Resisting area}}$$

Consider a body consisting of two plates connected by a rivet as shown in Fig. (n). In this case, the tangential force *P* tends to shear off the rivet at one cross-section as shown in Fig. *{b)*. It may be noted that when the tangential force is resisted by one cross-section of the rivel (or when shearing takes place at one cross-section of the rivet), then the rivets are said to be *ii* ***single shear.*** In such a case, the **area** resisting the shear off the **rivet,**

$$A \text{ ——} \frac{\pi}{4} \text{— } y\, d^2$$

And shear stress on the rivet cross-section

$$\frac{P}{A} \quad \frac{P}{\text{— x } d^2} \quad \frac{4P}{}$$

Now let us consider two plates connected by the two cover plates as shown in Fig. *baJ.* In this case, the tangential force *P* tends to shear off the rivet at two cross-sections as shown in Fig.

rivet (or when the shearing takes place at Two cross-sections of the rivet), then the rivets are said to be in *double xheor.* *I:n* such a case, the area resistuig the shear off the rivet,

$$A = 2 \times \frac{\pi}{4} \times d^2 \qquad \text{(For double shear)}$$

and shear stress on the rivet cross-section.

$$\frac{P}{A} = \frac{P}{2 \times \frac{\pi}{4} \times d^2} = \frac{2P}{\pi d^2}$$

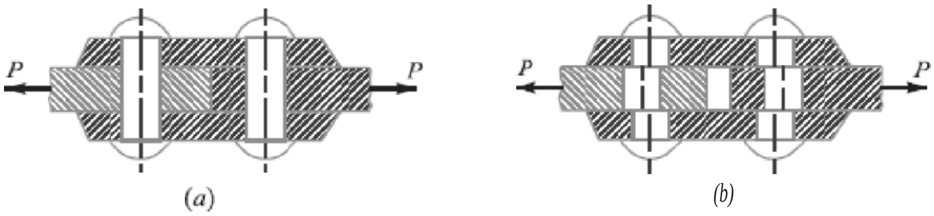


Fig. Double shearing of a riveted joint.

Nntes:

1. All lap joints and single cover biAt joints are in single shear, while the butt jouits with double cover plates are in double shear.
2. In case of shear, ltte area involved is parallel to the external force applied.
3. When the holes are to be punched or drilled in the metal plates, then the tools used to perform the operations must overcome the ultimate shearing resistance of the material to be cut. If a hole of diameter *'d'* ñ to be punched in a metal plate of thickness 'I', then the area to be sheared,

A = ml ^ r

And the maximum shear resistance of the tool or the force required to punch a hole,

- . • *zq* - *n d* • ' • lq

Where o Ultimate shear strength of the material of the plate.

Shear itlodutus or hfiodulus of Rigidi

It has been found experimentally that within the elastic limit, the shear stress is directly prnportional to shear slmin. Mathematically

$$\tau \propto \phi \quad \text{or} \quad \tau = C \, . \, \phi \quad \text{or} \quad \tau / \phi = C$$

Where, ‹= Shear stress,

‹p= Shear strain, and

C — Constant of proportionality, known as shear modulus or modulus of rigidity. It is also denoted by *N oT G.*

The following table shows the values of modulus of rigidity (for the materials in everyday use:

Values of C for the commonly used materials

Material	*Modulus of rigidity (C) GPa*
Steel	80 to 100
Wrought iron	80 to 90
Cast iron	40 to 50
Copper	30 to 50
Brass	30 to 50
Timber	10

Linear and Lateral Strain

Consider a circular bar of diameter *d* and length /, subjected to a tensile force *P* as shown in Fig. *(a).*

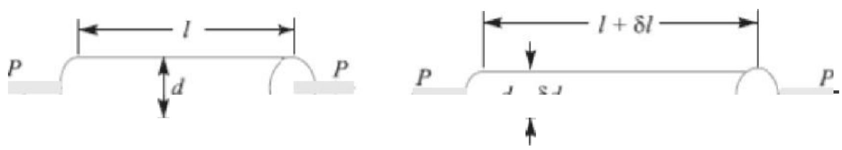


Fig. Linear and lateral strain.

A little consideration will show that due to tensile force, the length of the bar increases by an amount ó/ and the diameter decreases by an amount *òd,* as shown in Fig. *(b).* similarly, if the bar is subjected to a compressive force, the length of bar will decrease which will be followed by increase in diameter.

It is thus obvious, that every direct stress is accompanied by a strain in its own direction which is known as ***linear strain*** and an opposite kind of strain in every direction, at right angles to it, is known as ***lateral strain.***

Poisson's Ratio

It has been found experimentally that when a body is stressed within elastic limit, the lateral strain bears a constant ratio to the linear strain, Mathematically,

$$\frac{\text{LateralStrain}}{\text{LinearStrain}} = \text{Constant}$$

This constant is known as **Poisson's ratio** and is denoted by l/m or p.

Following are the values of Poisson's ratio for some of the materials commonly used in engineering practice.

Values of Poisson's ratio for commonly used materials

S.No.	*Material*	*Poisson 's ratio (1/m or)*
1	Steel Cast	0.25 to 0.33
2	iron Copper	0.23 to 0.27
3	Brass	0.31 to 0.34
4	Aluminium	0.32 to 0.42
5	Concrete	0.32 to 0.36
6	Rubber	0.08 to 0.18

Volumetric Strain

When a body is subjected to a system of forces, it undergoes some changes in its dimensions. In other words, the volume of the body is changed. The ratio of the change in volume to the original volume is known as ***volumetric strain.*** Mathematically, volumetric strain,

$$z \;—\; bV/V$$

Where 6U= Change in volume, and V = Original volume

Notes : 1. Volumetric strain of a rectangular body subjected to an axial force is given as

$$\varepsilon_v = \frac{}{V} = \varepsilon\left(1 - \frac{2}{m}\right); \text{ where } \varepsilon = \text{Linear strain.}$$

2. Volumetric strain of a rectangular body subjected to three mutually perpendicular forces is given by " ' ' ' + e + e ' . Where, e,, eg and e, are the strains in the directions x-axis, y-axls and z-axis respectively.

Bulk Modulus

When a body is subjected to three mutually perpendicular stresses, of equal intensity, then the ratio of the direct stress to the corresponding volumetric strain is known as ***bulk modulus.*** It is usually denoted by *K*. Mathematically, bulk modulus,

$$\frac{\text{Direct stress}}{\text{4"oliunetiic strain}} - \frac{n}{\text{ñ U."' U}}$$

Relation between Young's Modulus and Modulus of Rigidity

The Young's modulus (and modulus of rigidity (G) are related by the following relation,

$$\frac{iii.E}{fin + 1)} \quad \frac{E}{?\ (l + q)}$$

Principal Stresses and Principal Planes

In the previous chapter, we have discussed about the direct tensile and compressive stress as well as simple shear. Also we have always referred the stress in a plane which is at right angles to the line of action of the force. But it has been observed that at any point in a strained material, there are three planes, mutually perpendicular to each other which carry direct stresses only and no shear stress. It may be noted that out of these three direct stresses, one will be maximum and the other will be minimum. These perpendicular planes which have no shear stress are known as principal planes and the direct stresses along these planes are known as principal stresses. The planes on which the maximum shear known as planes of maximum shear.

Determination of Principal Stresses for a Member Subjected to Bi-axial Stress

When a member is subjected to bi-axial stress (i.e. direct stress in two mutually perpendicular planes accompanied by a simple shear stress), then the normal and shear stresses are obtained as discussed below:

Consider a rectangular body ABCD of uniform cross-sectional area and unit thickness subjected to normal stresses o1 and o2 as shown in Fig. (a). In addition to these normal stresses, a shear stress z also acts. It has been shown in books on 'Strength of Materials' that the normal stress across any oblique section such as EF inclined at an angle 8 with the direction of o2, as shown in Fig. (a), is given by

- " + '1 + " FOS *& T S fit /

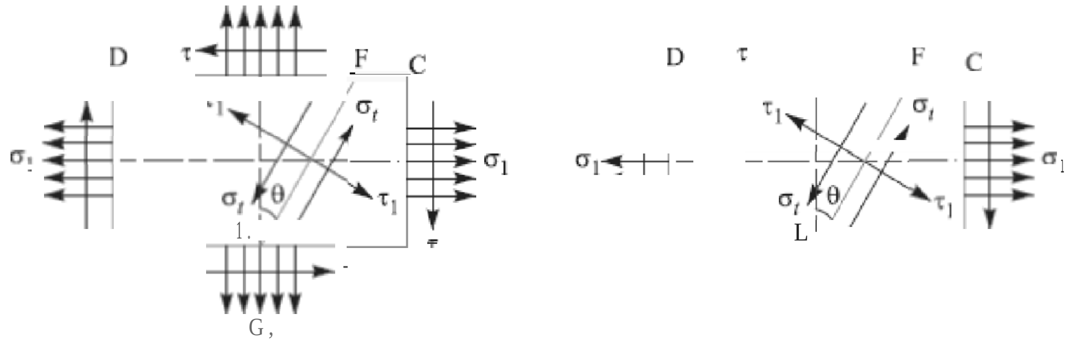
And tangential stress (i.e. shear stress) across the section EF,

Since the planes of maximum and minimum normal stress (i.e. principal planes) have no shear stress, therefore the inclination of principal planes is obtained by equating i1 = 0 in the above equation (ii), i.e.

$$\tau_1 \quad \frac{1}{2}(\sigma_1 - \sigma_2)\sin 2\theta - \tau\cos 2\theta \qquad ..(ii)$$

$$\frac{1}{2}(\sigma_1 - \sigma_2)\sin 2\theta - \tau\cos 2\theta = 0$$

$$an\ 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2} \qquad ...(iii$$



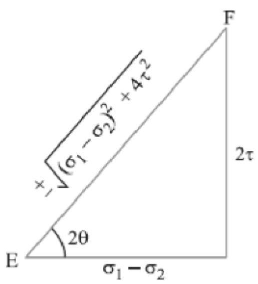
( ) Di re ct stre s s in tu o mutu.al ly prepeudicuLar planes acconipoiiiedby a simple shear stres s.

(b) D ifPCl SlfPSs iii one plane accooipanied by a simple she.u stress.

Fig. Principal stresses for a member subjected to bi-axial stress

We know that there are two principal planes at right angles to each other. Let 8 i and 8z be the inclinations of these planes with the normal cross-section. From the following Fig., we find that

$$\sin 2\theta = \pm \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$



$$\sin 2\theta_1 = + \frac{-}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

and $$\sin 2\theta_2 = - \frac{-}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

Also cos 28 = o' o' / (°i °) +

'° —'i' / (i :) +

und nos °0, $= - \frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$

The maximum and minimum principal stresses may now be obtained by substituting the values of sin 28 and cos 28 in equation (i).

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

So, Maximum principal (or normal) stress, and minimum principal (or noral) stress,

$$\sigma_{t2} = \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

The planes of maximum shear stress are at right angles to each other and are inclined at 45° to the principal planes. The maximum shear stress is given by one-half the algebraic difference between the principal stresses, i.e.

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

Notes: 1. when a member is subjected to direct stress in one plane accompanied by a simple shear stress, then the principal stresses are obtained by substituting o2 = 0 in above equations.

° + +

o * o'

i,,,, - +

2. In the above expression of ot2, the value of $\frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\tau^2}\right]$ is more than o1/2

Therefore the nature of ot2 will be opposite to that of ot1, i.e. if ot1 is tensile then ot2 will be compressive and vice-versa.

Application of Principal Stresses in Designing Machine Members

There are many cases in practice, in which machine members are subjected to combined stresses due to simultaneous action of either tensile or compressive stresses combined with shear stresses. In many shafts such as propeller shafts, C-frames etc., there are direct tensile or compressive stresses due to the external force and shear stress due to torsion, which acts normal to direct tensile or compressive stresses. The shafts like crank shafts, are subjected simultaneously to torsion and bending. In such cases, the maximum principal stresses, due to the combination of tensile or compressive stresses with shear stresses may be obtained. The results obtained in the previous article may be written as follows:

1. Maximum tensile stress,

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\tau^2}\right]$$

2. Maximum compressive stress,

3. Maximum shear stress,

°..«. °

Where, ct = Tensile stress due to direct load and bending,

xc = Compressive stress, and

i = Shear stress due to torsion.

Notes: 1. When i = 0 as in the case of thin cylindrical shell subjected in pressure, then otmax = ct.

2. When the shaft is subjected to an axial load (P) in addition to bending and twisting moments as in the propeller shafts of ship and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (ob). This will give the resultant tensile stress or compressive stress (ct or oc) depending upon the type of axial load (i.e. pull or push).

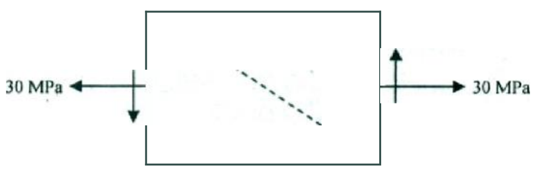
Problems:

1. A point in a structural member subjected to plane stress is shown in Fig.Q.1(b). Determine the following:

i) Normal and tangential stress intensities on plane MN inclined at 45°.

ii) Principal stresses and their direction

iii) Maximum shear stress and the direction of the planes on which it occurs.



a, =30Mpa a =20Mpa z, =15Mpa 0 = 45°

(i) Normal stress on plane MN (- **from I4B)**

$$,_A = \frac{\sigma_x + \sigma_v}{2} + \frac{\sigma_x - \sigma_v}{2} \cos 2\$ + z_{,,} \sin 2g$$

$$= \frac{30+20}{2} \sin 2\times 45 + 15\cos(2\times 45)$$

—25 N/mm2

Negative sign indicates T tends to produce CW rotation

(ii) Maximum principal stress

$$\text{———} \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\text{——} \frac{30—20}{2} + \sqrt{\left(\frac{30+20}{2}\right)^2 + 15^2} = 34.15a / mm' \text{ (Tensile)}$$

Minumim Principal stress

$$\frac{0 + 0,}{} \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau x^2 y}$$

— — 24, 1 55 N/mm2 (compressive)

Location

Angles at which the principal stresses act

$$\theta_{1,2} = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

where $ & $, are 90° apart

$$= \frac{1}{2}\tan^{-1}\frac{2 \times 15}{30 + 20}$$

8 = I5.48 & § = 105.48

(iii) Maxim um shear stress

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{30 - 20}{2}\right)^2 + 15^2} = z29.15N/mm'$$

Location:

Angies at which maximum shear stress act

1 ,= I|+ 45°

= I5.45 + 45 — 60.48

1,,= , +135''

= I5.48 + 135 = I50.48

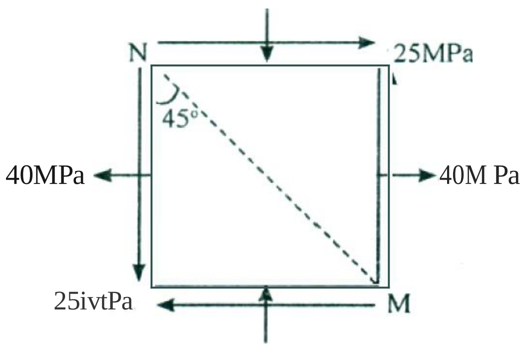
2. Point in a structural member is subjected to plane state of stree as shown in Fig. Q1(b). Determine the following:

i) Normal and tangential stress intensities at a angle of e 45°

ii) Principal stresses 0', and 0'2 and their directions.

iii) Maximum shear stress and its plane.

Solution:



Given, p, = 40MPa, o, = —30MPa, o, = 25MPa, 8 = 45°

(i) Normalstress on plane NON,

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{40 - 30}{2} + \frac{40 + 30}{2}\cos(2 \times 45) + 25\sin(2 \times 45)$$

= NON ,' mm'

Shcarstr¿sson plane kfN,

$$\tau_\theta = -\frac{\quad}{2}\text{'sin28tr cos20}$$

$$= -\frac{40+30}{2}\sin(2\times 45) + 25\cos(2\times 45)$$

= —3SN/ mm'

(ii) Max imum principal stress

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{40-30}{2} + \sqrt{\left(\frac{40+30}{2}\right)^2 + 25^2}$$

= 48.0 I l6N/mm' ('f< ›sile)

Minimum principal struss

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\frac{40-30}{2} \quad \sqrt{\left(\frac{40+30}{2}\right)^2 + 25^2}$$

= -3g.0I l6N / intn' (T#ns›/s)

Locutiun :

Angler at which principalstrcsaacts

$$\theta_1 = \frac{1}{2}\tan^{-1}\left[\frac{\quad}{\quad}\right]$$

$$= \frac{1}{2}\tan^{-1}\frac{2\times 25}{40+30}$$

8,-17.769•

8, •90a 17.769• •107.769"

(iii) Mexirnum5hGarstrcss,

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{40+30}{2}\right)^2 + 25^2}$$

- z 43.01 l6N / mm'

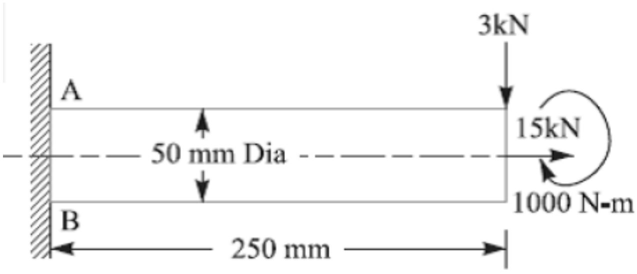
Location :

Aagleaal which maximumshearsircssacts:

8} e, + 46° = 17.769 + 4S = 62.769"

8} = e, + 135' = 17.769 ‹ 135= I8

3. A shaft, as shown in Fig., is subjected to a bending load of 3 kN, pure torque of 1000Nm and an axial pulling force of 15 kN stresses. Calculate the stresses at A and B.



Solution. Given : W = 3 kN = 3000 N ; T = 1000 N-m = 1 × 10^6 N-mm ; P = 15 kN = 15 × 10^3 N ; d = 50 mm; x = 250 mm

We know that cross-sectional area of the shaft,

$$A = \frac{\pi}{4} \times d^2$$

$$= \frac{\pi}{4} (50)^2 = 1964 \text{ mm}^2$$

∴ Tensile stress due to axial pulling at points A and B,

$$\sigma_o = \frac{P}{A} = \frac{15 \times 10^3}{1964} = 7.64 \text{ N/mm}^2 = 7.64 \text{ MPa}$$

Bending moment at points A and B,

$$M = W.x = 3000 \times 250 = 750 \times 10^3 \text{ N-mm}$$

Section modulus for the shaft,

$$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32} (50)^3$$

$$= 12.27 \times 10^3 \text{ mm}^3$$

∴ Bending stress at points A and B,

$$\sigma_b = \frac{M}{Z} = \frac{750 \times 10^3}{12.27 \times 10^3}$$

$$= 61.1 \text{ N/mm}^2 = 61.1 \text{ MPa}$$

This bending stress is tensile at point A and compressive at point B.

. Resultant tensile stress at point d.

$$\sigma_A = \sigma_b + \sigma_o = 61.1 + 7.64$$

= 6fi.74 MPa

and resultant compressive stress at point 2i.

crB — “ cr — 61.1 7.64 — S3.46 MPa

We knew that the shear stress at points A and *B* due to the torqiu transmitted,

$$\tau = \frac{I b U}{\pi d^3} = \frac{IN X) x 10‘}{\pi (50)^3} = \#0.74N/Rxn^2 = 4O7SMla \qquad \because \left(T = \frac{\pi}{16} \times \tau \times d^3 \right)$$

Strrssrs nt jioint A

We know' that maximum principal (or normal) stress at point N,

$$\sigma_{A(max)} = \frac{'A}{2} + \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\tau^2}\right]$$

68.74 -F 1/2 (8 74)^2 + 40 74)] 2

= 34.37 + 53.3 = 87.67 MPa (tensile) Ans.

Minimum principal (or nomial) stress ct point d.

$$\sigma_{A(min)} = \frac{\sigma_A}{2} - \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\tau^2}\right]$$

= 34.37 — 53.3 = — 18.93 MPa

18.93 MPa (compressive)

Ans. and iiiaximum shear stress at point d.

$$\tau_{A(max)} \quad 2\left[\sqrt{(\sigma_A)} + \tau^2\right] 4 \tfrac{1}{2}^2 \$ \left[\sqrt{(68.74)^2 + 4\,(40.74)^2}\right]$$

= 53.3 MPa Ans.

Strrssrs tit point B

We know' that maximum principa1(or nomial) stress at point *B.*

$$\sigma_{B(max)} = \frac{\sigma_B}{2} + \frac{1}{2} \quad [\ (\ )^2 \quad 4$$

53.46+ [(3 4 2 4 40 74)^2]

= 26.73 48.73 = 75.46 MPa (compressix e) Ans.

Minimum principal(or nomial) stress at point *B*,

$$\sigma_{B(min)} \quad \frac{}{2} \quad \frac{1}{2} B \left[\sqrt{(\sigma_B)^2 \ \cdot\ 4 - 2} + 4z^2\right]$$

= 26.73 —48.73 = —22 h4Pa

' 22 **/vtPa (tensile) Aas.**

and innxinnun shear stress at point *B.*

$$\frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\tau^2}\right] = \frac{1}{2}\left[\sqrt{(53.46)^2 + 4\,(40.74)^2}\right]$$

Design of Shaft, Keys, Couplings

Introduction

1.Shaft

- A shaft is a rotating machine element which is used to transmit power from one place to another.
- The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft.
- In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it.

  These members along with the forces exerted upon them causes the shaft to bending.

  In other words, we may say that a shaft is used for the transmission of torque and bending moment.
- The -various members are mounted on the shaft by means of keys or splines.
- The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used.

2. Axle

- An axle though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only.
- It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave.

3. Spindle

- A spindle a short shaft that imparts motion either to a cutting tool (e.g., drill press spindles) or work piece (e.g., lathe spindles).

Material Used for Shafts

- The material used for ordinary shafts is carbon steel of grades 40 C 8 , 45 C 8, 50 C 4, 50 C12.
- When a shaft of high strength is required, then an alloy steel such as nickel, nickel- chromium or chrome-vanadium steel is used.

Properties *of material* t/ser/ /'or *shans*

1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties.

est

The following two types of shafts are important from the subject point of view:

1. *Transmission shafts.*
   - These shafts transmit power between the source and the machines absorbing power.
   - The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts.
   - Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.
2. Machine shafts.

   These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

Standard Sizes of *Transmission Shafts*

The standard sizes of transmission shafts are:

- 25 mm to 60 mm with 5 mm steps; 60mm to 110mm with 10mm steps; 110mm to 140 mm with 15 mm steps; and 140 mm to 500 mm with 20 mm steps.
- The standard length of the shafts are 5 m, 6 m and 7 m.

Oesiqn *of shans*

The shafts may be designed on the basis of

1. Strength, and 2. Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered:

(a) Shafts subjected to twisting moment or torque only,

(b) Shafts subjected to bending moment only,

(c) Shafts subjected to combined twisting and bending moments, and

(d) Shafts subjected to axial loads in addition to combined torsional and bending loads.

Case (a) Shafts subjected to twisting moment or torque only

- When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that - 4

(i)

whee T' - Twisting moment (or torques acting upon the shaft,
= Polar moment of inertia of the shaft about the axis of rotation,
i = Torsional shear seess, and
r — Distance from neutral axis to the outer most fibre
d 12, where *d* is the diameter of the shaft.

We know that for round solid shaft, polar moment of inenia,

J u x //4
32

The equation *(i)* may now be written as

x d •' '• i» x i x *d'* . *(›i)*
32 /f* 2

Ftom this equation, we may determine the diameter of round solid shaft *[d .*

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^*\right]$$

where *d.* and d; = Outside and inside diameter of the shaft, and r - d / 2.

Substituting these values in equation (i), we have

T / (dq)⁴ - *d1* *d,* / 2 or *T* —– $\frac{\pi}{16}$ x $\left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right]$ ...*(iii)*

Let *k* -- Ratio of inside diameter and outside diameter of the shah
— *d, I d*

Now tiie equation *(iii)* may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o} \cdot 1 - \left(\frac{d_i}{d_o}\right)^4$$ „ π / t6* *Id,)'* (* *) ---('•›

From the equations (iii3 or *(iv1,* the outside and inside diameter of a hollow shaft may be determined.

It may be noted that

1. The hollow shafts are usually used iii marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft.

When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same, In other words, for the same material of both the shafts,

$T \quad \frac{\pi}{16} \quad \frac{\left[(d_o)^4 - (d_i)^4\right]}{\text{fig'}} \quad \frac{\pi}{16} \times \quad \times d'$

$(d_0 \quad (d)' \quad$ —· é $\quad$ or $\quad (dp)\ (1 - £^4) = d'$

dq

2. The twisting mcimerit {T1 may be obtained by using the following relation :

We know that the power transmitted (in watts) by the shaft,

$P = \frac{T\ N \times 2}{60}$ or F' $\quad 2 \quad N$

where T — Twisting moment in N-m, and

N -- Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment *(T)* is given by

$T > T_i \quad T\ )\ R$

wheve pi •fld *Th* — Tensions in the tight side and slack side of the belt respectively, and

fi = Radius of the pulley.

Case (b) Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$\cdot M \quad \sigma_b$

where *M* Bending moment,

= Moment of inertia of cross-sectional area of the shaft about the axis of rotation,

$£f_b$ — Bending stress, and

y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

" $\frac{\pi}{64}$ x d^4 and $\frac{d}{2}$

Substituting these values in equation *(i)*, we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad N — \frac{\pi}{32}\ b\ d'$$

From this equation, diameter of the solid shaft *(d1* may be obtained.

We also know that for a hollow shaft, moment of inertia,

" $\frac{/¿d')^4 — (d,)^4}{64}$ $\frac{(d)^4 (1 — 2^4)}{64}$ ...(where k = *d, fdp)*

and *y* *d, / 2*

Again substituting these values in equation (i), we have

$$\frac{M}{\frac{*}{64}(d)^4 (I — k^4)} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M -- \frac{\pi}{32} \times ab\ (d,)3\ (1 — k)$$

From this equation, the outside diameter of the shaft *(d ›* may be obtained.

Case (c) Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view :

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.
2. Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let z = Shear stress induced due to twisting moment, and

ab — Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2} \quad (\ j)^2 \quad 4 \quad ^2$$

Substituting the values of o$ and T ws have

$$"' \quad 2 \sqrt{\left(\frac{32\ 3\ f}{r\ d^3}\right)^2 + 4 \quad n\ d^3} \quad \frac{16}{r\ d'} \left[*\sqrt{M^2 + T^2} \right.$$

or $\frac{\pi}{16}$ ** ’o‘” *’ d^3 ñf -i- P

The expression 3f T is known as equivalent twisting moment and is denoted b¿ T,. The equivalent ***twisting*** *moment may be defined as that* ***twisting*** *moment, which when acting alone, produces the same shear stress (z) as the actual twisting moment. Ry limiting the maximum shear stress (z) equal to the allowable shear stress (z) for the material, the equation (i) may be written as*

$$T \quad ñ \quad + T' \quad 16 \text{ x z x r/}$$ ...(ii)

From this expression, diameter of the shaft /d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

¿, i p I (t)$ -i- 4 2 ..*liiil*

$$\frac{1}{2} \times \frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$\frac{32}{z\,d} \quad \frac{1}{2}(M + 3f^2 + 7^2)$$

or $\frac{\pi}{32}$ $o_b($) x d $\frac{1}{2}$ ñf + $\overline{ñf^2 + T'}$] : *(iv)*

The expression 2 *(M + M + }* is known as *equivalent bending moment* and is denoted by ñf,. The equivalent bending moment may be defined as that moment which when acting alone

produces the same tensile or compressive stress {ab1as the actual bending moment. By limiting the maximum normal stress ab gp i qual to the allowable bending stress (a p, then the equation *(iv)* may be written as

$$\frac{1}{2} QM + M^2 + F' = * a_b z d$$ . .*(v)*

From this expression, diameter of the shaft (‹I} may be evaluated.

In case of a hollow shaft, the equations *(ii)* and (v) may be written as

$$T_e \quad v' + \quad 16 \text{ X z } d)^3 (1 - 1^4)$$

$$M_e \quad \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{\pi}{32} \text{ x Zp } (d)^3 (1 \quad k^4)$$

- *It is suggested that diameter of* ffie *shaft may* be *obtained by using both* file *theories and* fhe larger offhe tW'o *values is adopted.*

Case (d) **Shafts Subjected to** Fluctuating **Loads**

- In the previous articles we have assumed that the shaft is subjected to constant torque and bending moment.
- But in actual practice, the shafts are subjected to fluctuating torque and bending moments.
- In order to design such shafts like line shafts and counter shafts, the combined shock and fatigue factors must be taken into account for the computed twisting moment (T) and bending moment (M).

Thus for a shaft subjected to combined bending and torsion, the equivalent twisting moment,

$$T, \quad (K \ \ x M) + (A \ x \ 7)^2$$

and equivalent bending moment,

$$3f, \quad \$xp \ x \ H + k(cq \ x \ If)^2 + (K, x \ T)^2$$

where *Kp* Combined shock and fatigue factor for bending, and

K, -- Combined shock and fatigue factor for torsion.

The following table shows the recommended values for Kp and *K,*.

Design of Shafts on the Basis of Rigidity

- Sometimes the shafts are to be designed on the basis of rigidity.
- The torsional rigidity is important in the case of camshaft of an IC. engine where the timing of the valves would be affected.
- The permissible amount of twist should not exceed 0.25° per meter length of such shafts.
- For line shafts or transmission shafts, deflections 2.5 to 3 degree per meter length may be used as limiting value.
- The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft.
- The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} \quad \frac{G \ . 8}{L} \quad \text{or} \quad 8 \quad \frac{T . L}{J-}$$

Where,

8 = Torsional deflection or angle of twist in radians,

T — Twisting moment or torque on the shaft,

I = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$$\frac{x \ d}{32}$$.(For solid shaft)

$-\frac{\pi}{32}$ (dV)’—(d,)’ .(For hollow shaft) 9

G = Modulus of rigidity for the shaft material, and

L — Length of the shaft.

Keys

J.7 /ntror/tJction

- A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them.
- It is always inserted parallel to the axis of the shaft.
  Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses.
- A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

*Y es of Keys

- The following types of keys are important from the subject point of view

1. Sunk keys.
2. Saddle keys,
3. Tangent keys,
4. Round keys, and
5. Splines.

1. Sunk Key

- The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley.
- The sunk keys are of the following types:

A. Rectangular sunk key.

- A rectangular sunk key is shown in following Fig.
- The usual proportions of this key are

Width of key,

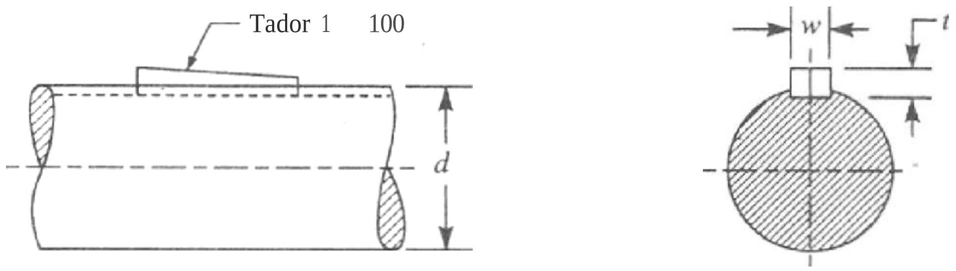
$w = d/4$ and

thickness of key,

$I = d/6$

Where

d = Diameter of the shaft or diameter of the hole in the hub.



- The key has taper I in 100 on the top side only.

B. Square sunk key.

- The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, i.e.

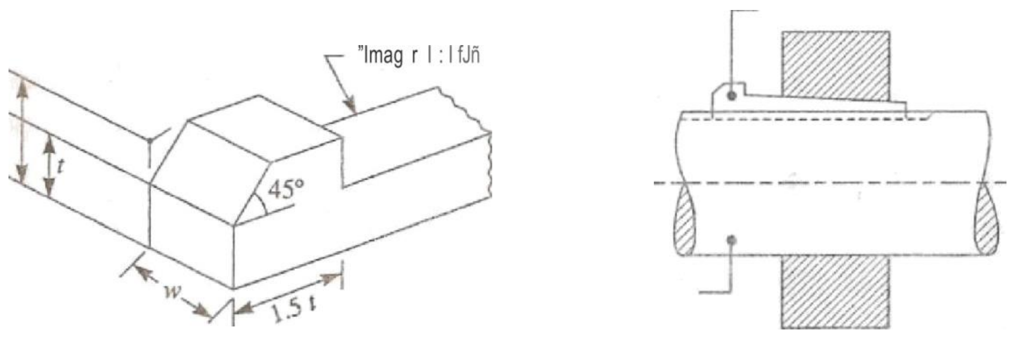
w =t = d/4

C. Parallel sunk key.

- The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout.
- It may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft.

D. Gib-head key.

- It is a rectangular sunk key with a head at one end known as gib head. It is
- usually provided to facilitate the removal of key.
- A gib head key is shown in following Fig. (a) and its use in shown in Fig. (b).

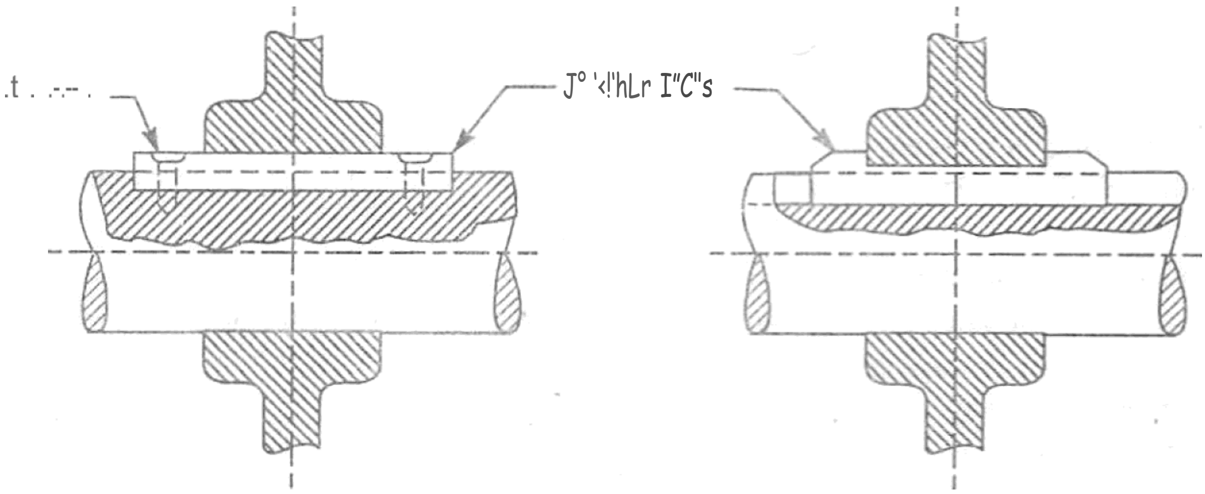


- The usual proportions of the gib head key are

Width, w = d/4 , and thickness at large end, I = d/6

E. Feather key.

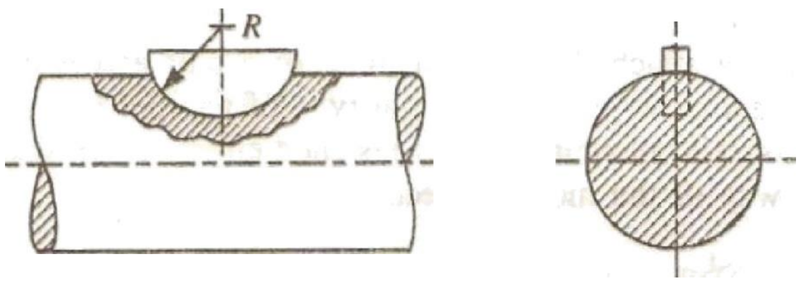
- ❖ A key attached to one member of a pair and which permits relative axial movement is known as feather key.
- ❖ It is a special type of parallel key which transmits a turning moment and also permits axial movement.
- ❖ It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.



- ❖ The feather key may be screwed to the shaft as shown in above Fig. (a) or it may have double gib heads as shown in above Fig. (b).
- ❖ The various proportions of a feather key are same as that of rectangular sunk key and gib head key.

F. Woodruff key.

- ❖ The woodruff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section in front view as shown in below Fig.
- ❖ A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.



- The main advantages of a woodruff key are as follows

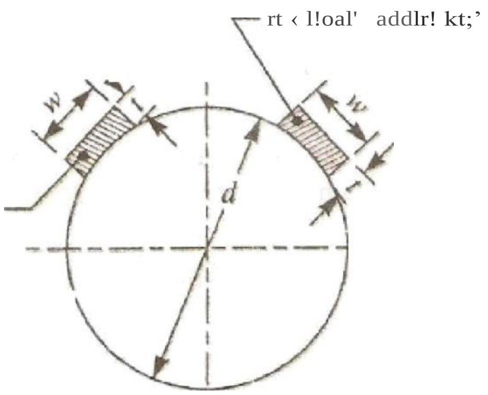
1. If accommodates itself to any taper in the hub or boss of the mating piece.
2. It is useful on tapering shaft ends, Its extra depth in the shaft "prevents any tendency to turn over in its keyway. -

- The disadvantages are

1. The depth of the keyway weakens the shaft.
2. It can not be used as a feather.

2. Saddle Key

- The saddle keys are of the following two types
    1. Flat saddle key. and
    2. Hollow saddle key.
- A flat saddle key is a taper key which fits in a keyway in the hub and is flat on the shaft as shown in following Fig.
  It is likely to slip round the shaft under load. Therefore it
  is used for comparatively light loads.

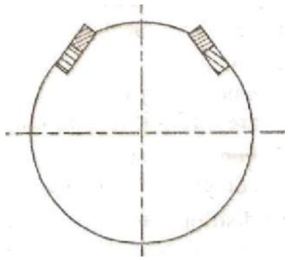


— = r

- A hollow saddle key is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft.
- Since hollow saddle keys hold on by friction, therefore these are suitable for light loads.
- It is usually used as a temporary fastening in fixing and setting eccentrics, cams etc.

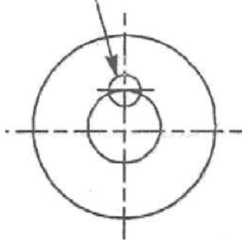
3. Tangent Keys

- The tangent keys are fitted in pair at right angles as shown in following Fig.
- Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.



4. Round Keys

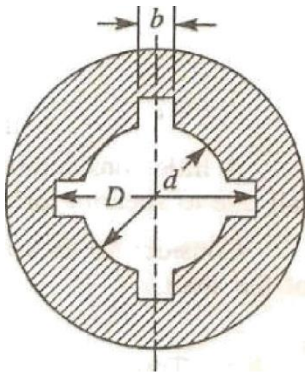
- The round keys, as shown in following Fig.



- These are circular in section partly in the shaft and partly in the hub.
- They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled.
- Round keys are usually considered to be most appropriate for low power drives.

5.Splines

- Sometimes, keys are made integral with the shaft which fits in the keyways broached in the hub.
- Such shafts are known as splined shafts as shown in following Fig.
- These shafts usually have four, six, ten or sixteen splines.
- The splined shafts are relatively stronger than shafts having a single keyway.



D = 1.25 *d* and b - 0.25 D

- The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions.
- By using splined shafts, we obtain axial movement as well as positive drive is obtained.

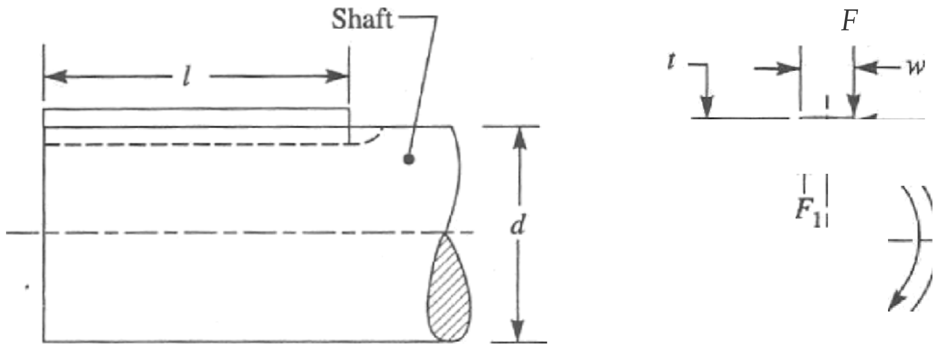
Oesiqn *of Sunk* Key

- When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key :

  1. Forces (F1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

  2. Forces (Fj due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.

❖ In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.



❖ Let

T — Torque transmitted by the shaft,

F — Tangential force acting at the circumference of the shaft,

d — Diameter of shaft,

I — Length of key, yr

= Width of key.

I = Thickness of key, and

oc — Shear and crushing stresses for the material of key.

❖ The usual proportions of this key are

Width of key, w = d/4 and

thickness of key, I = d/6

Where

d = Diameter of the shaft or diameter of the hole in the hub.

❖ A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

Case 1 Failure of key due to shearing

- Considering shearing of the key, the relation between tangential shearing force ,area resisting shearing and shear stress is

shear stress , == I ═══

Therefore , tangential force is

F = I x w x

Torque transmitted by the shaft,

d d

2 2

Case 2 Failure of key due to crushing

- Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

— Area resisting crushing x Crushing stress = *I* r 2 x a,

Torque transmittcd by the shaft,

T —— F X 2 d ! X 2 t X O, X d ( )

Key is equally strong in shearing and crushing

Thc ke; is equally' strong in shearing and crushing. if

$l \times w \times \tau \times \frac{}{2} - \times \frac{}{2} \times \sigma_c \times \frac{}{2}$.[Equating equations (i) and Ui))

r 2a

The permissible crushing stress for the usual key material is at least twice the permissible shearing stress.

Therefore from equation (iii), we have w = t.

In other words, a square key is equally strong in shearing and crushing.

Length of key .

In ordeI' to find the length of the key to transmit t'ull power of the shat't, the shearing strength of the key is equal to the torsi‹inal shear strength ‹it the shaft.

We kn‹ u thai the shearing strength of key,

T -— / x i” i 2 .(iv)

and torsional sheai stren ¿u th of the shaft,

$$T = \frac{}{16} \times \tau_1 \times d^3 \quad \ldots(v)$$

...(Tak ing 1 - fihcar stress for the shaft material)

Frnm c°quatinns (i›') anal (t'), e havc

/ x ›+' X — — — X I x &'
2 16

× $w \times \tau$ 2 x *i/τ = 1.5 71 d x (Taking w = J/4) .(v‹)

When the ke ' material is same as that ‹)f the shaft, then T = 1,.

1.571 d I equation (i•i”i

Effect of keywavOfi shaft

- A little consideration will show that the keyway cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft.
- It other words, the torsional strength of the shaft is reduced.
- The following relation for the weakening effect of the keyway is based on the experimental results by H.P. Moore.

$$e \quad 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right)$$

where e = Shaft strength fact‹ir. It is the ratio of the strength of the shaft 'ith keyway to the strength of lhe same shaft without keyway,

ii Width of keyway,

d —— Diameter' of shaft, and

li -— Depth tit keyw'ay ThickneSS Of kev fi) / 2

- It is usually assumed that the strength of the keyed shaft is 75% of the solid shaft, which is somewhat higher than the value obtained by the above relation.

Couplings

Introduction

- Shafts are usually available up to 7 metres length due to inconvenience in transport.
- In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

  Shaft couplings are used in machinery for several purposes, the most common of which are the following

Purpose of couplings

1. To provide for the connection of shafts of units that are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.
2. To provide for misalignment of the shafts or to introduce mechanical flexibility.
3. To reduce the transmission of shock loads from one shaft to another.
4. To introduce protection against overloads.
5. To alter the vibration characteristics of rotating units.

Requirements of a Good Shaft Coupling

I. It should be easy to connect or disconnect.
2. It should transmit the full power from one shaft to the other shaft without losses.
3. It should hold the shafts in perfect alignment.
4. It should reduce the transmission of shock loads from one shaft to another shaft.
5. If should have no projecting parts.

Types of Shafts Couplings

- shaft couplings are divided into two main groups as follows:

1. Rigid coupling. It is used to connect two shafts which are perfectly aligned. Following types of rigid coupling are important from the subject point of view

   (a) Sleeve or muff coupling.
   (b) Clamp or split-muff or compression coupling, and
   (c) Flange coupling.

2. Flexible coupling. It is used to connect two shafts having both lateral and angular misalignment. Following types of flexible coupling are important from the subject point of view:

   (a) Bushed pin type coupling,
   (b) Universal coupling, and

(c) Oldham coupling.

S/eeve *or muff coupling.*

- It is the simplest type of rigid coupling, made of cast iron.
- It consists of a hollow cylinder whose inner diameter is the same as that of the shaft.
- It is fitted over the ends of the two shafts by means of a gib head key, as shown in following Fig. The power is transmitted from one shaft to the other shaft by means of a key and a sleeve. It is, therefore, necessary that all the elements must be strong enough to transmit the torque.

Design Procedure

Step 1. Design of Shaft .

- Generally power transmitted by shaft is given , hence first of all find torque transmitted by shaft as

$$p — \frac{2\ TiNT}{6s}$$

T =.............N-m

$T = ... \times 10^3$ N-mm

Now as per torsion equation ,

T ?!- « d3

16

From above equation find (d)

Step 2. Proportions of sleeve

- The usual proportions of a cast iron sleeve coupling are as follows Outer

  diameter of the sleeve, D = 2 d + 13 mm

  and length of the sleeve, L = 3.5 d

Where,

d is the diameter of the shaft.

Step 3. Design of Key

- The usual proportions for rectangular key are

  Width of key, w = d/4 and

  thickness of key, t = d/6

  Where , d = Diameter of the shaft or diameter of the hole in the hub.

- And for square key proportions are

  Width of key, w = d/4 and

  thickness of key, t = d/4

  Where , d = Diameter of the shaft or diameter of the hole in the hub.

The length of the coupling key is atleast equal to the length of the *syeve (i.e. ,* 3.5 *d).* The coupling key is usually made into two parts so that the length of the key in each shaft,

$$L \quad \overline{2} \quad \frac{3.5\,d}{2}$$

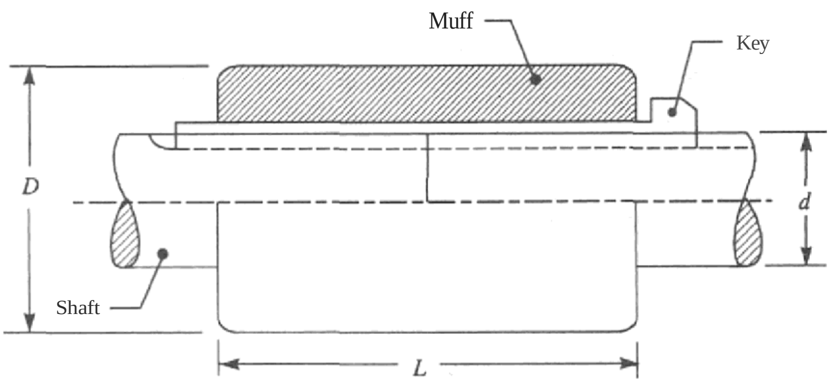
After fi xing the length of kcy in each shaft, the inciuced shearing and crushing stresses may be checked. We know that torque transmitted,

$$T — \;/\; H\; w\; H\; 1\; X\; \frac{d}{2}$$ (Considering shearing of the key)

$$= l \times \frac{t}{2} \times a, \times \frac{d}{2}$$ (Considering crushing of the key)

Step 4. Design of sleeve

- The sleeve is designed by considering it as a hollow shaft.



Let T - Torque to be transmitted by the coupling, and

i = Permissiblc shear stress for the material of the sleeve which is cast rion. The safe value of shear stress for cast iron may be taken as 14 MPa.

We know' that torquc transmitted by a hollow section,

$$T \text{ -- } \frac{\pi}{16} \text{ X } (p \left(\frac{D^4 - d''}{D} \right. \quad \frac{n}{16} \quad x \; D \quad (\text{-} \quad k4) \qquad . ('' .'' \; I — d\, I\, D)$$

From this expression, the induced shear stres5 in the sleeve may be checked.

Example 13.4. *Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.*

Solution. Given : P = 40 kW = 40 × 10^3 W ; N = 350 r.p.m. ; τ_s = 40 MPa = 40 N/mm^2; σ_{cs} = 80 MPa = 80 N/mm^2; τ_c = 15 MPa = 15 N/mm^2

The muff coupling is shown in Fig. 13.10. It is designed as discussed below :

1. *Design for shaft*

Let d = Diameter of the shaft.

We know that the torque transmitted by the shaft, key and muff,

$$T = \frac{P \times 60}{2\pi N} = \frac{40 \times 10^3 \times 60}{2\pi \times 350} = 1100 \text{ N-m}$$

$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 1100 \times 10^3 / 7.86 = 140 \times 10^3 \text{ or } d = 52 \text{ say } 55 \text{ mm } \textbf{Ans.}$$

2. *Design for sleeve*

We know that outer diameter of the muff,

$$D = 2\,d + 13 \text{ mm} = 2 \times 55 + 13 = 123 \text{ say } 125 \text{ mm } \textbf{Ans.}$$

and length of the muff,

$$L = 3.5\,d = 3.5 \times 55 = 192.5 \text{ say } 195 \text{ mm } \textbf{Ans.}$$

Let us now check the induced shear stress in the muff. Let τ_c be the induced shear stress in the muff which is made of cast iron. Since the muff is considered to be a hollow shaft, therefore the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125}\right]$$

$$= 370 \times 10^3 \; \tau_c$$

$$\therefore \quad \tau_c = 1100 \times 10^3 / 370 \times 10^3 = 2.97 \text{ N/mm}^2$$

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm^2, therefore the design of muff is safe.

3. *Design for key*

From Table 13.1, we find that for a shaft of 55 mm diameter,

Width of key, w = 18 mm **Ans.**

Since the crushing stress for the key material is twice the shearing stress, therefore a square key may be used.

∴ Thickness of key, $t = w$ = 18 mm **Ans.**

We know that length of key in each shaft,

$$l = L/2 = 195/2 = 97.5 \text{ mm } \textbf{Ans.}$$

Let us now check the induced shear and crushing stresses in the key. First of all, let us consider shearing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3 \; \tau_s$$

$$\therefore \quad \tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$$

N‹ow c‹›nsideri n* crushinp• of the key. We kni»v that tr›rque transmitted *(T)*,

$$1100 \times 10^3 \quad l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2}$$

24.1 x 10 o,.,

u ., 1 I OO x 10 / 24.1 x 10 = 4ñ.6 N/mm

Since the inducc d shear and crushing stresses are less than the pcrmic•sib1e stresses, therefore the design ct' key is safe.

Flange coupling

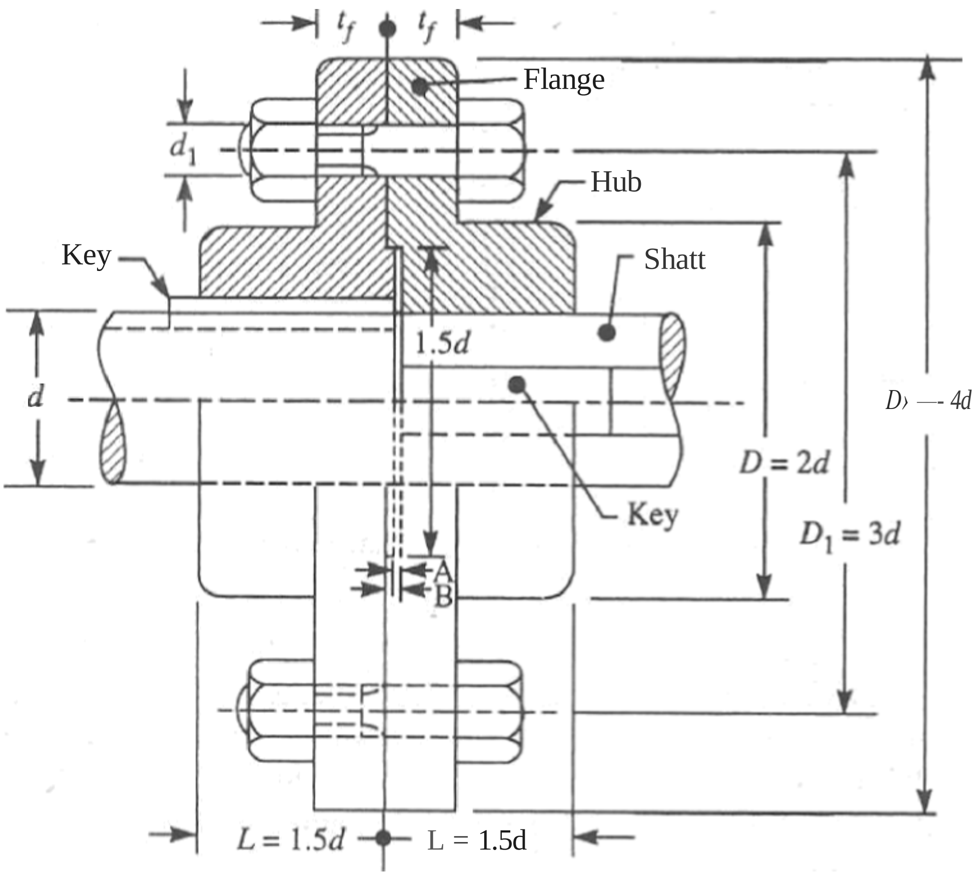
- A flange coupling is having two separate cast iron flanges.
- Each flange is mounted on the shaft end and keyed to it.
- The faces are turned up at right angle to the axis of the shaft.

One of the flange has a projected portion and the other flange has a corresponding recess. This helps to bring the shafts into line and to maintain alignment. The

- two flanges are coupled together by means of bolts and nuts.
- The flange coupling is adopted to heavy loads and hence it is used on large shafting. The flange
- couplings are of the following three types

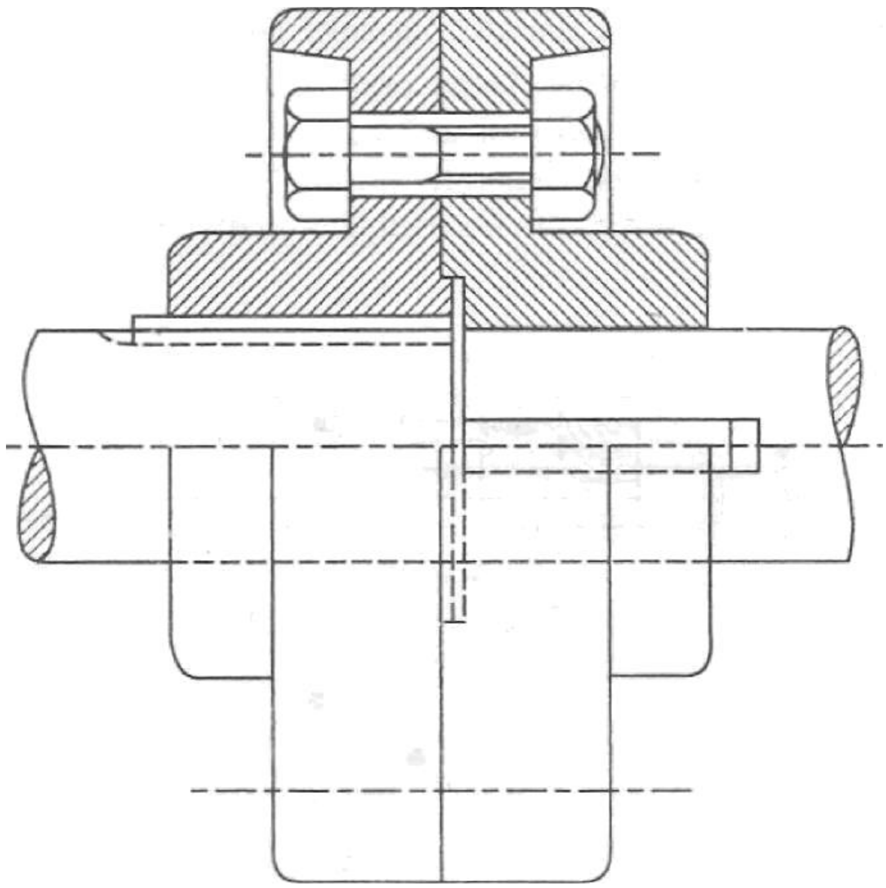
1. Unprotected type flange coupling.

- In an unprotected type flange coupling, as shown in following Fig. each shaft is keyed to the boss of a flange with a counter sunk key and the flanges are coupled together by means of bolts.
- Generally, three, four or six bolts are used.
- The keys are staggered at right angle along the circumference of the shafts in order to divide the weakening effect caused by keyways.



2. Protected type flange coupling.

- In a protected type flange coupling, as shown in following Fig.the protruding bolts and nuts are protected by flanges on the two halves of the coupling, in order to avoid danger to the workman. The thickness of the protective circumferential flange (tp) IS taken as 0.25 d. The other proportions of the coupling are same as for unprotected type flange coupling.



Design Procedure of Flange Coupling

Consider a flange coupling as shown in above Fig.

Let ,

d —— Dianiclcr of shaft or inner diameter of hub,

f — Outer diamctcr of hub,

I = Nominal or ‹outside diameter ct" bolt,

N, = Diameter of bolt circlc,

n = Number of bcilis,

i = Thickness of flange,

1 , i3 and i = A1ion'ablc shear stress for shaft, bolt and key material respectivcly

i,. = Allowable shear stress for the flange material i.c., cast iron,

o, , , and o„ = Allozatble crushing stress for bolt and key material respectively.

Step 1. Design of Shaft

- Generally power transmitted by shaft is given , hence first of all find torque transmitted by shaft as

$$p' \quad \frac{2\ TiNT}{60}$$

T =............N-m

T =........x 10^3 N-mm

Now as per torsion equation

T - — < d’
16

From above equation find (d)

Step 2 The usual proportions for both an unprotected and protected type cast iron flange couplings, are as follows

If d is the diameter of the shaft or inner diameter of the hub, then Outside diameter of hub,

D = 2d

Length of hub, L = 1.5 d

Pitch circle diameter of bolts,

D1 = 3d

Outside diameter of flange,

D2 =4d

Thickness of flange, If = 0.5 d

Number of bolts = 3, for d upto 40 mm

= 4, for d upto 100 mm

= 6, for d upto ISO mm

Step 3. Design of Key

- The key is designed with usual proportions and then checked for shearing and crushing stresses.
- The material of key is usually the same as that of shaft.
- The length of key is taken equal to the length of hub.
- The usual proportions for rectangular key are Width

  of key, w = d/4 and

  thickness of key, I = d/6

  Where , d = Diameter of the shaft or diameter of the hole in the hub.
- And for square key proportions are

  Width of key, w = d/4 and

  thickness of key, I = d/4

  Where , d = Diameter of the shaft or diameter of the hole in the hub.

The length of key is taken equal to the length of hub 1 = L

After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted,

$T \quad l \times w \times \tau \times \frac{-}{2}$ (Considering shearing of the key)

x x $\frac{d}{2}$

X2 * 2 (Considering crushing of the key)

Step 4 Hub design

d")ic hub is designed by considering it as a hollow shaft, transmitting the same toi'que *(T)* as that ofi a so1'0 shaft.

$$T \quad \frac{\pi}{16} \times \frac{D^4 - d^4}{D}$$

The t›uicr diaiiietc•r of hub is usuall y taken as twice the diiimeler of shat't. Thcreforc 1'rom the abo *c* relati‹›n, the inducc‹l shears ng sPress in the hub may be checkeii.

"FI c lcngtli r›f huh (/-) in taken as I .1 1.

Step 5 Design of Flange

sign for flange

The flange at the junction of the hub is under shear while transmitting the torque. Therefore, the troque transmitted,

T = Circumference of hub × Thickness of flange × Shear stress of flange × Radius of hub

$$\pi D \times t_f \times \tau_c \times \frac{D}{2} = \frac{\pi D^2}{2} \times \tau_c \times t_f$$

The thickoes•s Of flange is iisually taken as half th. diameter ol shaft. Tf:.-rcfore trod .h* nice'e relation, the induced shearing stress In the t3ange ma›' bc checked.

Step 6 Design of Bolts

h, g e | g ti fOt“ ISO Its stress dine to the tort".e transmitted. The number Of JlIS "i“he bolts are subjected to shear n) depends upon the diameter of Shaf. aCd ihG itc li Circle diameter Of bOliS (ñ) iS ta eel US “. d, ’ñ'e 1‹nev' that

Load on each bolt $= \frac{\pi}{4} (d_1)^2 \tau_b$

Tc:a! load on i11 the bOlts

π

and torque transmitted, $T = \frac{\pi}{4} (d_1)^2 \tau_b \times n \times \frac{D_1}{2}$

From this equation, the diameter of bolt (d_1) may be obtained. Now the diameter of bolt may be checked in crushing.

We know that area resisting crushing of all the bolts

and crushing strength of all the bolts

$(n \times d_1 \times t_f)\, \sigma_{cb}$

Torque, $T = (n \times d_1 \times t_f \times \sigma_{cb}) \frac{D_1}{2}$

From above equation , the induced crushing stress in the bolt may be checked .

EXample: Design a rigid flange coupling to transmit a torque of 250 N-m between two co-axial shafts. The shaft is made of alloy steel, flanges out of cast iron and bolts out of steel Four bolts are used to couple the flanges. The shafts are keyed to the flange hub. The permissible stresses are given below:

She‹rr *sfre.*i.f *OTl Shuft*	*1()t) MP i*
Dearing or crt/s/ffff, q *str ess on shaft*	*25a MPa*
.Shear stress oii key s	*1t)t) LlP‹i*
Peari•8 •tress on ke;	*250 MPn*
Shearing strrss oil CRst iroii	*200 MPa*
Shear stress oti holis	*It)() MP i*

After designing the various elements, make a neat sketch of the assembly indicating the important dimensions. The stresses developed in the various members may be checked if thumb rules are used for fixing the dimensions.

Soution. Given : T = 250 N-m = 250 × 10^3 N-mm ; n = 4 ; τ_s = 100 MPa = 100 N/mm^2 ; σ_{cs} = 250 MPa = 250 N/mm^2 ; τ_k = 100 MPa = 100 N/mm^2 ; σ_{ck} = 250 MPa = 250 N/mm^2 ; τ_c = 200 MPa = 200 N/mm^2; τ_b = 100 MPa = 100 N/mm^2

The cast iron flange coupling of the protective type is designed as discussed below :

1. *Design for hub*

First of all, let us find the diameter of the shaft (d). We know that the torque transmitted by the shaft (T),

$$250 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 100 \times d^3 = 19.64\ d^3$$

$$\therefore \quad d^3 = 250 \times 10^3 / 19.64 = 12\ 729 \quad \text{or} \quad d = 23.35 \text{ say } 25 \text{ mm } \textbf{Ans.}$$

We know that the outer diameter of the hub,

$$D = 2\,d = 2 \times 25 = 50 \text{ mm}$$

and length of hub, $L = 1.5\,d = 1.5 \times 25 = 37.5$ mm

Let us now check the induced shear stress in the hub by considering it as a hollow shaft. We know that the torque transmitted (T),

$$250 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \left[\frac{(50)^4 - (25)^4}{50}\right] = 23\ 013\ \tau_c$$

$$\therefore \quad \tau_c = 250 \times 10^3 / 23\ 013 = 10.86 \text{ N/mm}^2 = 10.86 \text{ MPa}$$

Since the induced shear stress for the hub material (*i.e.*, cast iron) is less than 200 MPa therefore the design for hub is safe.

2. *Design for key*

From Table 13.1, we find that the proportions of key for a 25 mm diameter shaft are :

Width of key, w = 10 mm **Ans.**

and thickness of key, t = 8 mm **Ans.**

The length of key (l) is taken equal to the length of hub,

$$\therefore \quad l = L = 37.5 \text{ mm } \textbf{Ans.}$$

Let us now check the induced shear and crushing stresses in the key. Considering the key in shearing. We know that the torque transmitted (T),

$$250 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 37.5 \times 10 \times \tau_k \times \frac{25}{2} = 4688\ \tau_k$$

$$\therefore \quad \tau_k = 250 \times 10^3 / 4688 = 53.3 \text{ N/mm}^2 = 53.3 \text{ MPa}$$

Considering the key in crushing. We know that the torque transmitted (T),

$$250 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 37.5 \times \frac{8}{2} \times \sigma_{ck} \times \frac{25}{2} = 1875\ \sigma_{ck}$$

$$\therefore \quad \sigma_{ck} = 250 \times 10^3 / 1875 = 133.3 \text{ N/mm}^2 = 133.3 \text{ MPa}$$

Since the induced shear and crushing stresses in the key are less than the given stresses, therefore the design of key is safe.

3. *Design for flange*

The thickness of the flange (t_f) is taken as 0.5 d.

$$\therefore \quad t_f = 0.5\,d = 0.5 \times 25 = 12.5 \text{ mm } \textbf{Ans.}$$

Let us n›w check the induced shear stress in the flange by consldering the flange at the junk lion of the hub in shear. We know' that the torque transmitted T),

250 x 10' u' x i, x = 50 x i, x 12.5 49 094 i,.

i, = 250 x II)" / 49 094 = 5.1 N/mm° = 5.1 MPa

Sincc the induced shear stress in the flange of cost iron is less than 2f)[] MPa, therefore design of 1)anp•e is safe.

I. i d —— Nt›mina1 diamcter of bolts. M'e know that the pitch circlc diamctcr tif bolts,

It –– 3 d —— 3 x 25 = 75 mm Ans.

The bolts ai'c suhjccicd to shear stress due to the torque transmitted. We knt›w tha' torqu* tr:in sna ittcd (T .

$$250 \times 10' \quad \frac{\pi}{4} (d_1)^2 \tau_b \times n \frac{D_1}{2} \frac{\pi}{4} (d_1)^2 \; 100 \times 4 \times \frac{}{2} \quad 11\ 780\ (d_1)^2$$

(d,) 250 x 1 I) / 11 750 — 21.22 or d — 4.éa lTNn

As.ruining coarse threads, the nearest standard size of the bolt is Al 6. Ans. Other' proportions of the flange are taken as follows :

Oiticr diametcr of thc flangc,

D = 4 d –– 4 x ?5 = 1(Xi mm Ans. "l"hic

kncss tJf thC F°otcctive circuits fcrc ntia[llangc,

fp = 0.25 d = 0.2? x 25 = 6.25 mm Ans.

Flexible Coupling

- We have already discussed that a flexible coupling is used to join the ends of shafts when they are not in exact alignment.
- Following are the different types of flexible couplings

  I. Bushed pin flexible coupling,

  2. Oldham's coupling, and

  3. Universal coupling.

I. Bushed pin flexible coupling

- A bushed-pin flexible coupling, as shown in following Fig. is a modification of the rigid type of flange coupling. The coupling bolts are known as pins. The rubber or leather bushes are used over the pins.
- The two halves of the coupling are dissimilar in construction.
- A clearance of 5 mm is left between the face of the two halves of the coupling.
- There is no rigid connection between them and the drive takes place through the medium of the compressible rubber or leather bushes.